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# HYDRAULICS

## DIMENSIONAL ANALYSIS AND HYDRAULIC SIMILARITY


### TOPIC 4.2

by

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Chapter 4: Dimensional Analysis and Hydraulic Similarity by N Adilah A A Ghani

Communitising Technology



## DIMENSIONAL ANALYSIS AND HYDRAULIC SIMILARITY

4.2

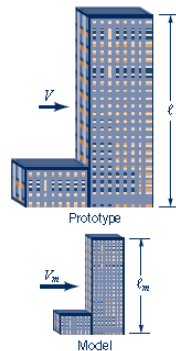
- Hydraulic Similarity

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## 4.2 : HYDRAULIC SIMILARITY

### 4.2.1 : Hydraulic Scale Model



A **model** is a representation of a physical system that may be used to predict the behavior of the system in some desired respect.

The physical system for which the predictions are to be made is called the **prototype**.

#### a) Undistorted Model

Models which are geometrically similar to their prototypes.

- ❖ Linear dimensions (horizontal & vertical directions) are similar in ratios.

#### EXAMPLE 4.4 (Undistorted Method)

1. A model of spillway is constructed to a scale of 1:30 in a flume. The length of the spillway is 30m. If the discharge over the spillway at the head of 6m is  $443.6\text{m}^3/\text{s}$ , calculate the corresponding head and discharge of the model required for this model study.

2. A dam 35m long is to discharge water at the rate of  $114\text{m}^3/\text{s}$  under the head of 2.7m. Find the length of the model and head of water if the supply available in the laboratory is 30L/s.

## b) Distorted Model

Models which are geometrically not similar to their prototypes.

- ❖ Linear dimensions (horizontal & vertical directions) are different in ratios.

### EXAMPLE 4.5 (Distorted Method)

A diversion weir 240m long has discharging capacity of 250m<sup>3</sup>/s under a head of 1.2m. A model of the weir is to be constructed in laboratory where the available channel is 3m wide and 500mm deep. Design the suitable model for the weir, if the water available in the laboratory is 25 L/s.

## 4.2.2 : Types of Similarity

### a) Geometric Similarity

The ratio of the corresponding linear dimensions of a model and a prototype are equal:

$$L_r = L_m / L_p$$

Where:

$L_r$  = length ratio

$L_p$  = length prototype

$L_m$  = length model

## b) Kinematic Similarity

Similarity of motion in a model and prototype:

$$V_r = V_m / V_p$$

where :

$V_r$  = velocity ratio

$V_p$  = velocity prototype

$V_m$  = velocity model

## c) Dynamic Similarity

Similarity between forces in a model and prototype:

$$F_r = F_m / F_p$$

Where:

$F_r$  = force ratio

$F_p$  = force prototype

$F_m$  = force model

### EXAMPLE 4.6

A prototype of a volume of 1000m<sup>3</sup> is investigated by a hydraulic model 1.0m<sup>3</sup> in volume. If the length of the model is 2.25m, what is the length of the prototypes?

Solution:

$$V_r = L_r^3 = V_m / V_p = 1/1000$$

Where  $V_r$  is the volume ratio of the model volume to the prototype volume. Therefore,

$$L_r = 1/10$$

$$L_r = L_m / L_p = 2.25 / L_p$$

$$L_p = 2.25 \times 10 = 22.5\text{m}$$

### EXAMPLE 7.6 Reynolds Number Similarity

**GIVEN** Model tests are to be performed to study the flow through a large check valve having a 2-ft-diameter inlet and carrying water at a flowrate of 30 cfs as shown in Fig. E7.6a. The working fluid in the model is water at the same temperature as that in the prototype. Can plane geometric similarity exist between model and prototype, and the model inlet diameter is 3 in.

**FIND** Determine the required flowrate in the model.

**SOLUTION**

To ensure dynamic similarity, the model tests should be run so that

$$Re_m = Re$$

or

$$\frac{V_m D_m}{\nu_m} = \frac{V D}{\nu}$$

where  $V$  and  $D$  correspond to the inlet velocity and diameter, respectively. Since the same fluid is to be used in model and prototype,  $\nu = \nu_m$ , and therefore

$$\frac{V_m}{V} = \frac{D}{D_m}$$

The discharge,  $Q$ , is equal to  $VA$ , where  $A$  is the inlet area, so

$$\frac{Q_m}{Q} = \frac{V_m A_m}{VA} = \left(\frac{D}{D_m}\right)^3 \left[\frac{(\pi A) V_m^3}{(\pi A) V^3}\right]$$

$$= \frac{D_m^3}{D^3}$$

and for the data given

$$Q_m = \left(\frac{3}{24}\right)^3 (30 \text{ ft}^3/\text{s})$$

$$Q_m = 3.75 \text{ cfs} \quad (\text{Ans})$$

**COMMENT** As indicated by the foregoing analysis, to maintain Reynolds number similarity using the same fluid in model and prototype, the required velocity scale is inversely proportional to the length scale, that is,  $V_m/V = (D_m/D)^{-1}$ . This strong influence of the length scale on the velocity scale is shown in

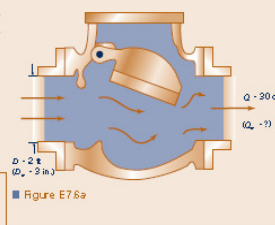


Figure E7.6a

Fig. E7.6b. For this particular example,  $D_m/D = 0.125$ , and the corresponding velocity scale is 8 (see Fig. E7.6b). Thus, with the prototype velocity equal to  $V = (30 \text{ ft}^3/\text{s})/(\pi A/4) = 9.50 \text{ ft/s}$ , the required model velocity is  $V_m = 76.4 \text{ ft/s}$ . Although this is a relatively large velocity, it could be attained in a laboratory facility. It is to be noted that if we tried to use a smaller model, say one with  $D_m = 1 \text{ in.}$ , the required model velocity is  $249 \text{ ft/s}$ , a very high velocity that would be difficult to achieve. These results are indicative of one of the difficulties encountered in maintaining Reynolds number similarity—the required model velocities may be impractical to obtain.

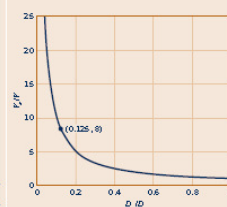


Figure E7.6b

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