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Finite Element Analysis

Dynamic Analysis
Natural Modes and Frequencies

by

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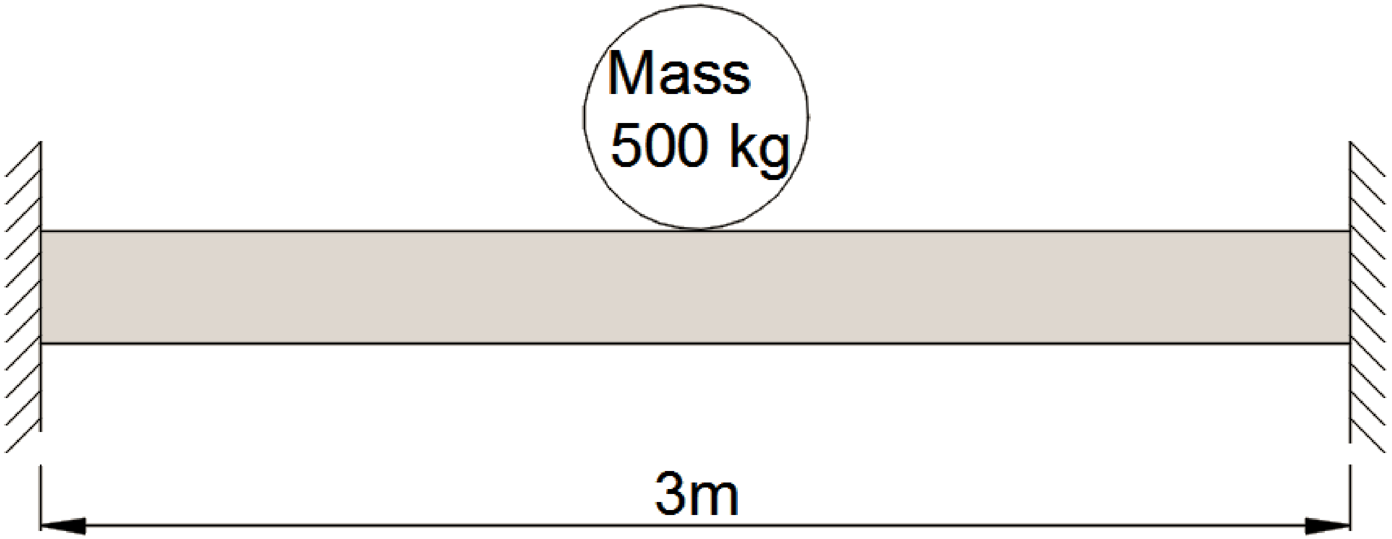
Lesson Outcomes

- At the end of this lesson, the student should be able to:
  – Understand the procedure for determining natural modes and frequencies using dynamic analysis
  – Evaluate the natural modes and frequencies of a lumped mass system

Dynamic Analysis Example

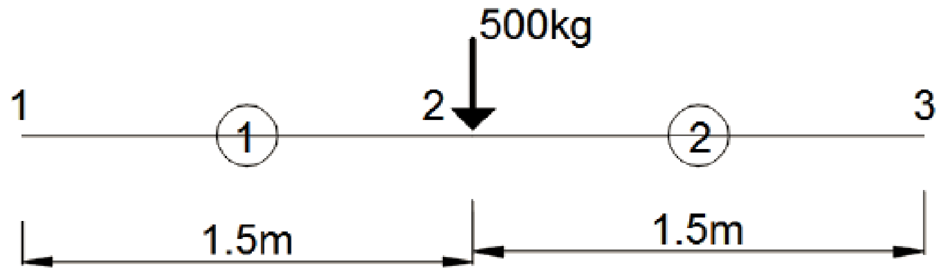
- A beam is fixed at both ends as shown on the next slide. Assume it to be a system with lumped mass of 500 kg located at the mid-span of the beam. The cross-section of the beam consists of a rectangular shape of 4 cm width and 8 cm depth.
- Generate a suitable simplified finite element model for the dynamic analysis of the beam including nodes, element type, element connectivity and an sketch of the model
- Develop the required mass and stiffness matrices and assemble the system of equations for the structure
- Evaluate the first natural frequency of vibration and the relevant time period of the beam

Beam for Dynamic Analysis



Solution

Selecting 2 flexural elements to represent the beam, the discretized model is given as:



For both elements;

$$L = 1.5\ m$$

$$E = 200\ GPa = 2 \times 10^{11}\ N/m^2$$

$$I = \frac{0.04 \times 0.08^3}{12} = 1.7 \times 10^{-6}\ m^4$$

$$\frac{EI}{L^3} = \frac{2 \times 10^{11} \times 1.7 \times 10^{-6}}{1.5^3} = 100741\ N/m$$

Solution (Continued)

$$k^{(1)} = k^{(2)} = \begin{bmatrix} 1208892 & 906669 & -1208892 & 906669 \\ 906669 & 906669 & -906669 & 453335 \\ -1208892 & -906669 & 1208892 & -906669 \\ 906669 & 453335 & -906669 & 906669 \end{bmatrix}$$

Since node 1 and 3 are fixed, after applying the boundary conditions, the assembled stiffness matrix for the structure is given as:

$$K = \begin{bmatrix} 2417784 & 0 \\ 0 & 1813338 \end{bmatrix}$$

Using lumped mass at the only free node 2, the assembled mass matrix for the structure is given as:

$$M = \begin{bmatrix} 500 & 0 \\ 0 & 0 \end{bmatrix}$$

Solution (Continued)

The system of equations for the structure is given as:

$$\left|K - \omega^2 M\right| = 0$$

$$\left| \begin{bmatrix} 2417784 & 0 \\ 0 & 1813338 \end{bmatrix} - \omega^2 \begin{bmatrix} 500 & 0 \\ 0 & 0 \end{bmatrix} \right| = 0$$

Simplying the system of equations obtained in (b) above;

$$\begin{vmatrix} 2417784 - 500\omega^2 & 0 \\ 0 & 181338 \end{vmatrix} = 0$$

$$181338\left(2417784 - 500\omega^2\right) = 0$$

$$\omega^2 = \frac{2417784}{500} = 4836$$

$$\Rightarrow \omega = 69.54\ Hz$$

Solution (Continued)

The natural frequency of the beam is 69.54 Hz. The time period is the reciprocal of the natural frequency:

$$T = \frac{1}{\omega} = \frac{1}{69.54} = 0.0144\ sec$$

Author Information

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