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Finite Element Analysis

Plane Strain and Plane Stress Stiffness Equations

by

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Lesson Outcomes

- At the end of this lesson, the student should be able to:
 - Understand plain strain condition
 - Understand plain stress condition
 - Develop plain strain and plain stress stiffness equations



2D Planar Elements

- Both the plane stress and the plane strain conditions can be modelled using 2D plane elements
- 2D Planar Elements are defined by at least 3 nodes in a two-dimensional plane (x-y plane)
- These elements can be connected at common nodes and/or along common edges
- Some of the example structures that can be modelled using 2D elements include long structures with constant cross-section such as a dam or a pipeline, or deep beams or plates with holes loaded in plane



2D Planar Elements (Continued)

- It is made sure during the formulation that the compatibility between nodal displacements is maintained – this is achieved through ensuring the selection of a suitable shape function
- Compatibility along common edges is maintained by choosing proper displacement functions
- These elements are used for:
 - Plane Stress Analysis
 - Performed for structures loaded in their plane such as plates with holes etc.
 - Plane Strain Analysis
 - Performed generally for long structures subjected to constant load along their length such as dams, culverts, etc.



Plane Stress Condition

- This is a state of stress when all the stresses act in a single plane i.e. the direct as well as shear stresses perpendicular to the plane are zero
- $\sigma_z = \tau_{xz} = \tau_{yz} = 0$
- Generally thin structural members loaded in their plane are considered to be in plane stress condition



Plane Strain Condition

- If there are no strains out of the plane i.e. the strains perpendicular to the plane being considered, or any components therefore, are all zero, then this condition is called the plane strain condition
- $\epsilon_z = \gamma_{xz} = \gamma_{yz} = 0$
- This happens in long bodies with constant cross-sectional area and loads acting in x or y directions only with a constant value along the z-direction
- Only a unit thickness of these structures is considered for analysis because all unit thicknesses except near the ends of the structure behave identically



2 Dimensional States of Stress and Strain

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

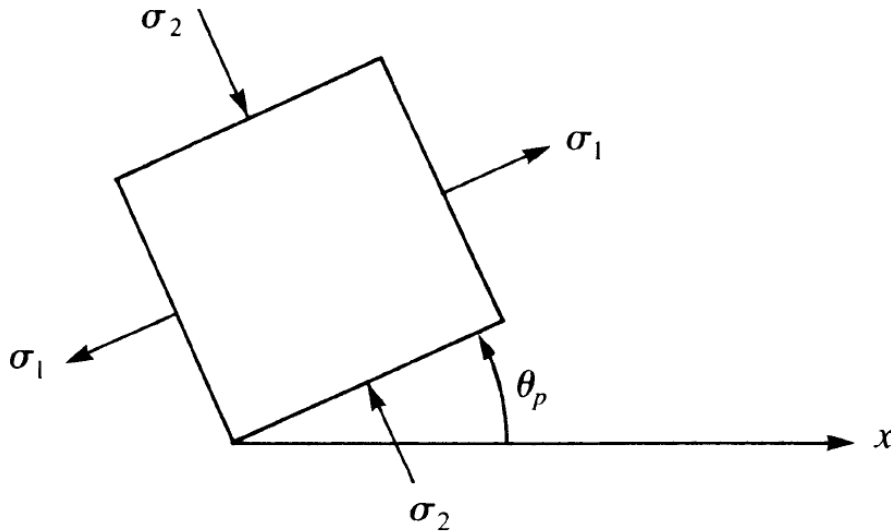
$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$



Principal Stress and Principal Plane



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\max}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\min}$$



Stress-strain Relationship for Isotropic Materials (in Plane Stress condition)

- $\{\sigma\} = [D]\{\varepsilon\}$
- The matrix $[D]$ is called the *stress-strain matrix* or the *constitutive matrix*

$$\bullet [D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

- E is the modulus of Elasticity
- ν is the Poisson's ratio



Stress-strain Relationship for Isotropic Materials (in Plane Stress condition)

- For plane strain condition, the D matrix changes to:

- $$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

Constant Strain Triangular Elements (CST)

- This is a basic 2D finite element
- Its derivation is the simplest among all the 2D elements
- The strain remains constant throughout the element; hence the name CST
- The formulation for the CST can most feasibly be achieved through the principle of minimum potential energy
- An example employing the CST elements will be demonstrated in the next lecture



Author Information

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