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Finite Element Analysis

Frame Equations
Beam Arbitrarily Oriented in Plane

by

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Lesson Outcomes

- At the end of this lesson, the student should be able to:
  - Transform beam equations in 2 dimensions
  - Develop element equations for a beam arbitrarily orientated in plane

Frames

- For the development of beam equations, it was assumed that the beam is orientated in the positive x-direction
- This is true when only the beams are considered
- But most of the times in real structures, beams are parts of frames
- Frames consist of beams as well as columns
- Columns are the structural members that are mainly loaded axially
- This is unlike beams, which as was discussed in the previous lecture, resist only shear and bending

Frames (Continued)

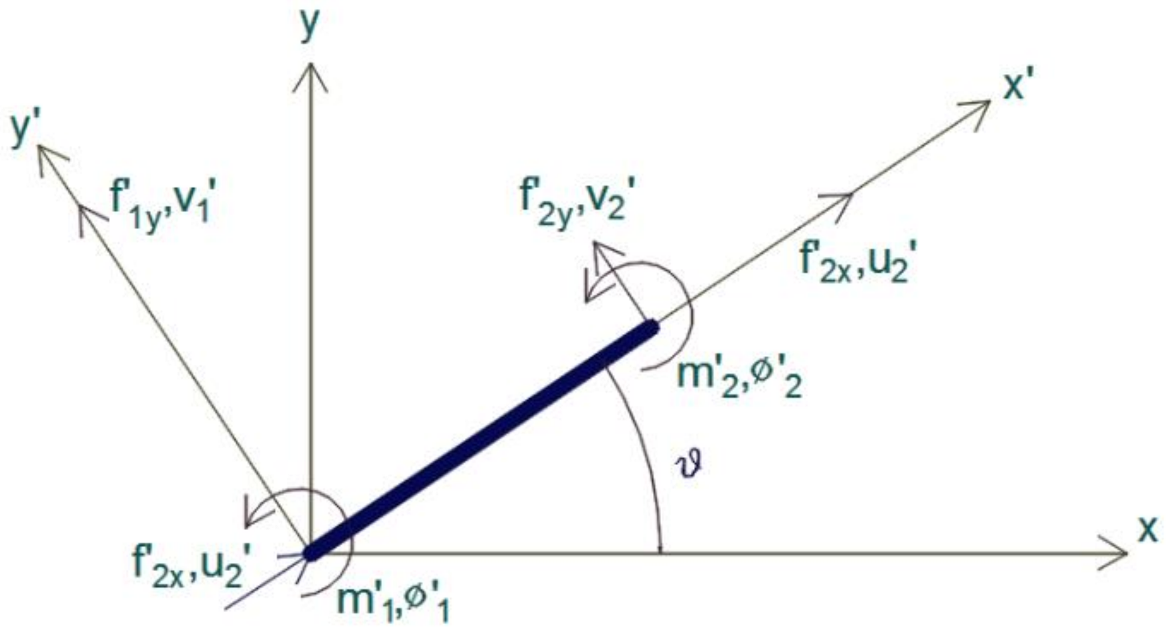
- In order to model frames, we need element equations to include the axial effects as well
- Also, when part of a frame, beam elements can be orientated in any direction
- Therefore, the developed beam equations also need to be transformed
- After transformation and inclusion of axial effects, the beam equations can be used to model any frame member including columns

Frames (Continued)

- Therefore, in summary, we can say that:
- Frames are combinations of beams and columns
- Frames can be 2D plane or 3D space
- To begin with, we will study 2D plane frames
- First we will discuss the transformation of beam equations in 2D plane

Beam Element Arbitrarily Orientated in Plane

- Consider the beam element arbitrarily oriented in 2D plane as shown:



Beam Element Arbitrarily Orientated in Plane (Continued)

- Recalling the transformation of local displacements to global displacements of a bar element:
- $\begin{Bmatrix} u' \\ v' \end{Bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$
- The bar element had 1 local DOF per node, whereas, the beam element has two local DOF per node
- Therefore, transformation of bar equations can be extended for a beam element as
- $$\begin{Bmatrix} v'_1 \\ \phi'_1 \\ v'_2 \\ \phi'_2 \end{Bmatrix} = \begin{bmatrix} -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \phi_1 \\ u_2 \\ v_2 \\ \phi_2 \end{Bmatrix}$$

Beam Element Arbitrarily Orientated in Plane (Continued)

- From above, the transformation matrix for a beam element is defined as
- $[T] = \begin{bmatrix} -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
- We can calculate the global stiffness matrix for the beam element following a similar procedure as used for the bar element
- $[k] = [T]^T[k'][T]$

Beam Element Arbitrarily Orientated in Plane (Continued)

- After performing the necessary matrix operations (multiplication), we get:
- $$[k] = \frac{EI}{L^3}\begin{bmatrix} 12S^2 & -12SC & -6LS & -12S^2 & 12SC & -6LS \\ -12SC & 12C^2 & 6LC & 12SC & -12C^2 & 6LC \\ -6LS & 6LC & 4L^2 & 6LS & -6LC & 2L^2 \\ -12S^2 & 12SC & 6LS & 12S^2 & -12SC & 6LS \\ 12SC & -12C^2 & -6LC & -12SC & 12C^2 & -6LC \\ -6LS & 6LC & 2L^2 & 6LS & -6LC & 4L^2 \end{bmatrix}$$
- This is the stiffness matrix for a beam element arbitrarily orientated in plane
- This element does not yet include axial effects
- It means that while it can model a beam orientated arbitrarily in plane, it can not analyze columns
- To be able to analyze columns, we have to consider axial effects

Author Information

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