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Finite Element Analysis

Formulation of a Beam Element

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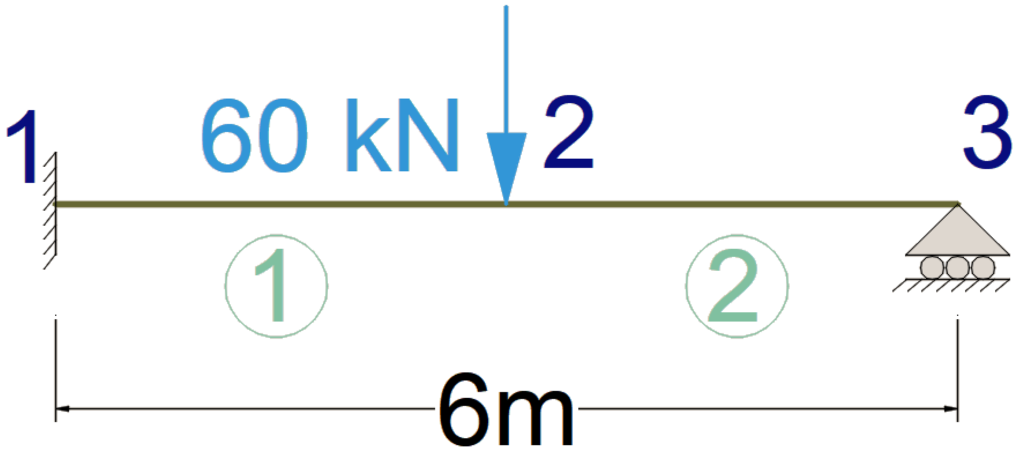
Lesson Outcomes

- At the end of this lesson, the student should be able to:
  – Apply the formulation of beam element to analyse a structure
  – Interpret the results of analysis using beam elements

Example Structure

- Calculate the support reactions for the beam structure shown below using Finite Element Analysis
- Use $b = 7cm, d = 16cm, E = 200GPa$



Discretization

- The discretization of the example beam is shown in the Figure on the previous slide
- Because the force vector can only take nodal values, a node has been added at mid-span where the load is applied
- The structure, therefore, consists of 2 elements and 3 nodes with element 1 connected to nodes 1 and 2 and element 2 connected to nodes 2 and 3
- Node 1 is fixed, which means there will be neither rotation nor translation at this node
- Node 3 has a roller support, which means there will be no translation at this node, however, rotation is possible
- Length of each of the elements is 3m

Element Stiffness Matrices

- Since both the elements in the example structure are the same, we need to write only one stiffness matrix
- The same stiffness matrix will be used for both the elements
- The general formula for the stiffness matrix was developed in the previous lecture
- $[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$

Element Stiffness Matrices (Continued)

- For both the elements:
- $E = 200GPa = 2 \times 10^{11} N/m^2$
- $I = \frac{bd^3}{12} = \frac{0.07 \times 0.16^3}{12} = 2.39 \times 10^{-5} m^4$
- $L = 3m$
- $\frac{EI}{L^3} = \frac{2 \times 10^{11} \times 2.39 \times 10^{-5}}{3^3} = 177{,}037 N/m$
- $[k] = 177{,}037 \begin{bmatrix} 12 & -18 & -12 & -18 \\ -18 & 36 & 18 & 18 \\ -12 & 18 & 12 & 18 \\ -18 & 18 & 18 & 36 \end{bmatrix}$

Element Equations for Element No. 1

- Element Equations for Element No. 1 are written according to its connectivity as:
- $$\begin{Bmatrix} f_1 \\ m_1 \\ f_2 \\ m_2 \end{Bmatrix} = 177{,}037 \begin{bmatrix} 12 & -18 & -12 & -18 \\ -18 & 36 & 18 & 18 \\ -12 & 18 & 12 & 18 \\ -18 & 18 & 18 & 36 \end{bmatrix} \begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix}$$

Element Equations for Element No. 2

- Element Equations for Element No. 1 are written according to its connectivity as:

- $$\begin{Bmatrix} f_2 \\ m_2 \\ f_3 \\ m_3 \end{Bmatrix} = 177{,}037 \begin{bmatrix} 12 & -18 & -12 & -18 \\ -18 & 36 & 18 & 18 \\ -12 & 18 & 12 & 18 \\ -18 & 18 & 18 & 36 \end{bmatrix} \begin{Bmatrix} v_2 \\ \phi_2 \\ v_3 \\ \phi_3 \end{Bmatrix}$$

Assembly of Structure Stiffness Equations

- The system of equations for the entire structure can be assembled using the direct stiffness approach
- $$\begin{Bmatrix} f_1 \\ m_1 \\ f_2 \\ m_2 \\ f_3 \\ m_3 \end{Bmatrix} = 177{,}037 \begin{bmatrix} 12 & -18 & -12 & -18 & 0 & 0 \\ -18 & 36 & 18 & 18 & 0 & 0 \\ -12 & 18 & 24 & 0 & -12 & -18 \\ -18 & 18 & 0 & 72 & 18 & 18 \\ 0 & 0 & -12 & 18 & 12 & 18 \\ 0 & 0 & -18 & 18 & 18 & 36 \end{bmatrix} \begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \\ v_3 \\ \phi_3 \end{Bmatrix}$$

Boundary Conditions

- We know that:
- $v_1 = 0, \phi_1 = 0, v_3 = 0$ and
- $f_2 = -60000N, m_2 = 0, m_3 = 0$
- Therefore, the reduced system of equations becomes:
- $\begin{Bmatrix} -60000 \\ 0 \\ 0 \end{Bmatrix} = 177037 \begin{bmatrix} 24 & 0 & -18 \\ 0 & 72 & 18 \\ -18 & 18 & 36 \end{bmatrix} \begin{Bmatrix} v_2 \\ \phi_2 \\ \phi_3 \end{Bmatrix}$

Solution of the System of Equations

- The solution of the linear system of equations yields:
- $v_2 = -0.02471m = -2.47cm$
- $\phi_2 = 0.00353\, rad = 0.2°$
- $\phi_3 = -0.01412\, rad = -0.81°$
- Substituting these values in the equations that we removed earlier:
- $f_1 = 177037(-12v_2 - 18\phi_2) = 41{,}224.84N \cong 41.22kN$
- $m_1 = 177037(18v_2 + 18\phi_2) = -67{,}493.6Nm \cong -67.5kNm$
- $f_3 = 177037(-12v_2 + 18\phi_2 + 18\phi_3) = 18{,}726.97N \cong 18.73kN$
- Since the beams are orientated in positive x-direction, there is no need to transform these forces

Author Information

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