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Finite Element Analysis

3D Space Truss Example

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Lesson Outcomes

- At the end of this lesson, the student should be able to:
  – Apply Bar Element Equations in 3D Space
  – Evaluate 3D space truss

Example

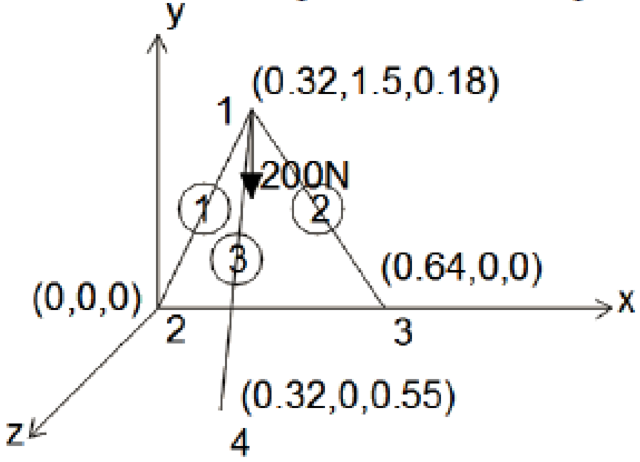
- The tripod shown in **Figure 1** is to be used to support a Total Station weighing 200N. The tripod legs form an equilateral triangle at the base with the side length of the triangle being 0.75 m whereas the height of the top of the tripod from the ground is 1.5 m. It can be assumed that the tripod legs are restricted from any translation at the base. The tripod legs are made of a synthetic linear elastic material with modulus of elasticity being 10GPa. The diameter of the circular cross-section of the tripod legs is 2 cm.
- Generate a suitable finite element model for the supporting structure including number of nodes, element type selected, element connectivity and a sketch of the model
- Compile and assemble the required element stiffness matrices
- c) After applying the boundary conditions to the assembly of the stiffness matrices in (b) above, evaluate the vertical deformation at the top of the tripod

Example

Figure 1

Solution

Using bar elements to model the tripod as a space truss and setting the origin at one of the base points of the tripod:



For all the elements:

$$E = 10\ GPa = 1 \times 10^7\ kN/m^2$$

$$A = \pi r^2 = \pi(2)^2 = 12.5664\ cm^2 = 1.2566 \times 10^{-3}\ m^2$$

Solution (Continued)

Element 1

$$x_1 = 0.32, \quad y_1 = 1.5, \quad z_1 = 0.1848$$

$$x_2 = 0, \quad y_2 = 0, \quad z_2 = 0$$

$$L = \sqrt{(0 - 0.32)^2 + (0 - 1.5)^2 + (0 - 0.1848)^2} = 1.5448\ m$$

$$C_x = \frac{x_2 - x_1}{L} = \frac{0 - 0.32}{1.5448} = -0.2071$$

$$C_y = \frac{y_2 - y_1}{L} = \frac{0 - 1.5}{1.5448} = -0.971$$

$$C_z = \frac{z_2 - z_1}{L} = \frac{0 - 0.1848}{1.5448} = -0.1196$$

$$\frac{AE}{L} = \frac{1.2566 \times 10^{-3} \times 1 \times 10^{7}}{1.5448} = 8134.3863\ kN/m$$

Solution (Continued)

$$k^{(1)} = \begin{bmatrix} 349.0227 & 1636.044 & 201.5606 & -349.023 & -1636.04 & -201.561 \\ 1636.044 & 7668.957 & 944.8155 & -1636.04 & -7668.96 & -944.815 \\ 201.5606 & 944.8155 & 116.4013 & -201.561 & -944.815 & -116.401 \\ -349.023 & -1636.04 & -201.561 & 349.0227 & 1636.044 & 201.5606 \\ -1636.04 & -7668.96 & -944.815 & 1636.044 & 7668.957 & 944.8155 \\ -201.561 & -944.815 & -116.401 & 201.5606 & 944.8155 & 116.4013 \end{bmatrix}$$

Element 2

$$x_1 = 0.32, \quad y_1 = 1.5, \quad z_1 = 0.1848$$

$$x_2 = 0.64, \quad y_2 = 0, \quad z_2 = 0$$

$$L = \sqrt{(0.64 - 0.32)^2 + (0 - 1.5)^2 + (0 - 0.18)^2} = 1.5448 \ m$$

$$C_x = \frac{x_2 - x_1}{L} = \frac{0.64 - 0.32}{1.5448} = 0.2071$$

Solution (Continued)

$$C_y = \frac{y_2 - y_1}{L} = \frac{0 - 1.5}{1.5448} = -0.971$$

$$C_z = \frac{z_2 - z_1}{L} = \frac{0 - 0.1848}{1.5448} = -0.1196$$

$$\frac{AE}{L} = \frac{1.2566 \times 10^{-3} \times 1 \times 10^{7}}{1.5448} = 8134.3863\ kN/m$$

$$k^{(2)} = \begin{bmatrix} 349.0227 & -1636.044 & -201.5606 & -349.023 & 1636.04 & 201.561 \\ -1636.044 & 7668.957 & 944.8155 & 1636.04 & -7668.96 & -944.815 \\ -201.5606 & 944.8155 & 116.4013 & 201.561 & -944.815 & -116.401 \\ -349.023 & 1636.04 & 201.561 & 349.0227 & -1636.044 & -201.5606 \\ 1636.04 & -7668.96 & -944.815 & -1636.044 & 7668.957 & 944.8155 \\ 201.561 & -944.815 & -116.401 & -201.5606 & 944.8155 & 116.4013 \end{bmatrix}$$

Element 3

$$x_1 = 0.32, \quad y_1 = 1.5, \quad z_1 = 0.1848$$

Solution (Continued)

$$x_2 = 0.32, \quad y_2 = 0, \quad z_2 = 0.5543$$

$$L = \sqrt{(0.32 - 0.32)^2 + (0 - 1.5)^2 + (0.55 - 0.18)^2} = 1.5448\ m$$

$$C_x = \frac{x_2 - x_1}{L} = \frac{0.32 - 0.32}{1.5448} = 0$$

$$C_y = \frac{y_2 - y_1}{L} = \frac{0 - 1.5}{1.5448} = -0.971$$

$$C_z = \frac{z_2 - z_1}{L} = \frac{0.5543 - 0.1848}{1.5448} = 0.2392$$

$$\frac{AE}{L} = \frac{1.2566 \times 10^{-3} \times 1 \times 10^{7}}{1.5448} = 8134.3863\ kN/m$$

Solution (Continued)

$$k^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7668.957 & -1889.14 & 0 & -7668.96 & 1889.144 \\ 0 & -1889.14 & 465.3592 & 0 & 1889.144 & -465.359 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -7668.96 & 1889.144 & 0 & 7668.957 & -1889.14 \\ 0 & 1889.144 & -465.359 & 0 & -1889.14 & 465.3592 \end{bmatrix}$$

Only the first node is free for all three elements with the second nodes being restricted. After application of the boundary conditions, the element stiffness matrices become:

$$k^{(1)} = \begin{bmatrix} 349.0227 & 1636.044 & 201.5606 \\ 1636.044 & 7668.957 & 944.8155 \\ 201.5606 & 944.8155 & 116.4013 \end{bmatrix}$$

Solution (Continued)

$$k^{(2)} = \begin{bmatrix} 349.0227 & -1636.044 & -201.5606 \\ -1636.044 & 7668.957 & 944.8155 \\ -201.5606 & 944.8155 & 116.4013 \end{bmatrix}$$

$$k^{(3)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7668.957 & -1889.14 \\ 0 & -1889.14 & 465.3592 \end{bmatrix}$$

Assembling the matrices to formulate the system of equations:

$$\begin{Bmatrix} 0 \\ -0.2 \\ 0 \end{Bmatrix} = \begin{bmatrix} 698 & 0 & 0 \\ 0 & 23007 & 0 \\ 0 & 0 & 698 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \end{Bmatrix}$$

The solution of the system of equations can be directly obtained as:

$$u_1 = 0$$

Solution (Continued)

$$v_1 = \frac{-0.2}{23007} = -8.7 \times 10^{-6}\ m$$

$$w_1 = 0$$

The total displacement at the top of the tripod is 0.0087 mm straight downwards.

Author Information

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