Universiti Malaysia PAHANG
Engineering • Technology • Creativity

Finite Element Analysis

Stresses in a Bar Element in 2D

by

Dr. Gul Ahmed Jokhio
Faculty of Civil Engineering and Earth Resources
jokhio@ump.edu.my

Lesson Outcomes

- At the end of this lesson, the student should be able to:
  – Understand the transformation of global nodal forces to local element forces
  – Evaluate stresses in a bar element orientated at any angle in plane

Element Stresses

- Consider the bar element orientated along positive x-axis
- $\begin{Bmatrix} f'_{1x} \\ f'_{2x} \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix}$
- Here the superscript ‘ refers to the local coordinate system
- Since the equilibrium suggests that these forces at both ends of the element must be equal and opposite, we can evaluate any one
- Taking f'_{2x}:
- $f'_{2x} = \frac{EA}{L} [-1 \quad 1]\{d'\}$
- We know that stress is force divided by area, therefore:
- $\{\sigma\} = \frac{E}{L} [-1 \quad 1]\{d'\}$
- This gives stress in terms of local deformations

Transformation of Element Stresses

- We can transform local deformations using the transformation matrix developed earlier
- $\{\sigma\} = \frac{E}{L}[-1 \quad 1][T^T]\{d\}$
- This can be expressed in the compact form as:
- $\{\sigma\} = [C']\{d\}$
- Where, $[C']$ is the transformation matrix for element stresses. It is written as:
- $[C'] = \frac{E}{L}[-C \quad -S \quad C \quad S]$

Example

- From the truss example evaluated in the previous lecture, we have:
- For Element No. 1:
- $C_1 = 1, S_1 = 0, E = 2 \times 10^8, L = 6m$
- $[C_1'] = \frac{2\times10^8}{6}\begin{bmatrix}-1 & 0 & 1 & 0\end{bmatrix}$
- $[C_1'] = \begin{bmatrix}-3.33 \times 10^7 & 0 & 3.33 \times 10^7 & 0\end{bmatrix}$
- $\{d^{(1)}\} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.0045 \\ 0 \end{Bmatrix}$
- $\{\sigma^{(1)}\} = \begin{bmatrix}-3.33 \times 10^7 & 0 & 3.33 \times 10^7 & 0\end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0.0045 \\ 0 \end{Bmatrix} = 150{,}000 kN/m^2$
- Stress in Element No. 1 is 150MPa (tension)

Example (Continued)

- Similarly, for Element No. 2:
- $C_2 = 0.629, S_2 = 0.777, E = 2 \times 10^8, L = 4.767m$
- $[C_1'] = \frac{2\times10^8}{4.767}[-0.629 \quad -0.777 \quad 0.629 \quad 0.777]$
- $[C_1'] = [-2.64 \times 10^7 \quad -3.26 \times 10^7 \quad 2.64 \times 10^7 \quad 3.26 \times 10^7]$
- $\{d^{(2)}\} = \begin{Bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.011278 \\ -0.00182 \end{Bmatrix}$
- $\{\sigma^{(1)}\} = [-2.64 \times 10^7 \quad -3.26 \times 10^7 \quad 2.64 \times 10^7 \quad 3.26 \times 10^7]\begin{Bmatrix} 0 \\ 0 \\ 0.011278 \\ -0.00182 \end{Bmatrix} = 238{,}407.2$
- Stress in Element No. 1 is 238.4 MPa (tension)

Author Information

Dr. Gul Ahmed Jokhio

is a Senior Lecturer at FKASA, UMP. He completed his PhD from Imperial College London in 2012.