UMP OPEN COURSEWARE

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

CHAPTER 5

BEE3143:POWER SYSTEM ANALYSIS- Power flow solution-Newton-Raphson

Expected Outcomes

Able to solve power flow solution using Newton-Raphson technique

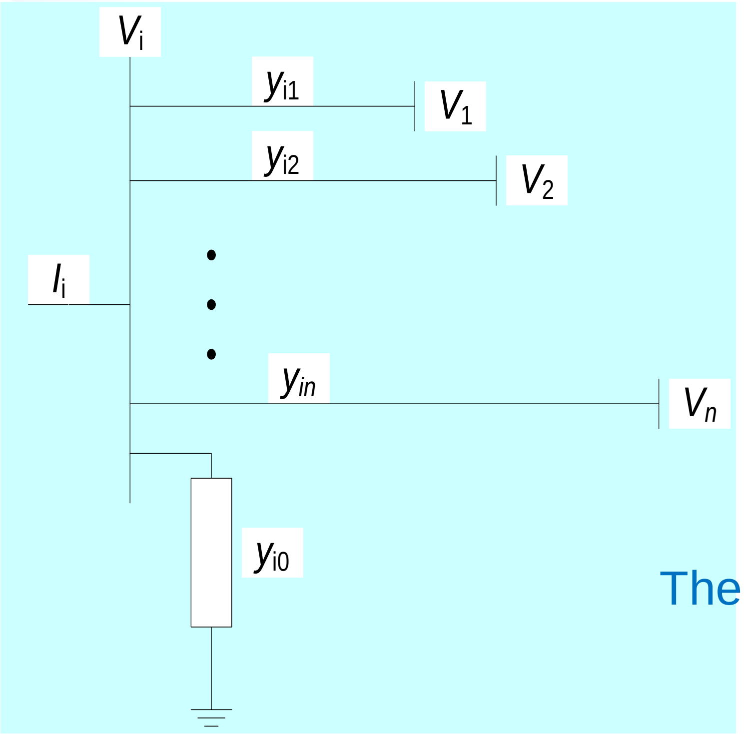
Newton-Raphson Power Flow Solution

- Newton-raphson method is found to be more practical and efficient for large power system.
- The number of iterations required to obtain a solution is independent of the system size, but more functional evaluations are required at each iteration.

... Newton-Raphson Power Flow Solution

Rewrite the current entering bus i in a typical bus power system in figure below in terms of the bus admittance matrix→

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad (1)$$



The typical element Y_{ij} is

$$Y_{ij} = |Y_{ij}|\angle\theta_{ij} = |Y_{ij}|\cos\theta_{ij} + j|Y_{ij}|\sin\theta_{ij} = G_{ij} + jB_{ij}$$

The voltage at a typical bus *i*

$$V_i = |V_i|\angle\delta_i = |V_i|(\cos\delta_i + j\sin\delta_i)$$

Expressing Equation (1) in polar form

$$I_i = \sum_{j=1}^{n} |Y_{ij}||V_j|\angle\theta_{ij} + \delta_j$$

The complex conjugate of the power injected at bus *i* is

$$P_i - jQ_i = V_i^* I_i$$
$$= V_i^* \sum_{j=1}^{n} Y_{ij} V_j$$

...Newton-Raphson Power Flow Solution

The complex conjugate of the power in polar form

$$P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j$$

$$= |V_i| \angle -\delta_i \sum_{j=1}^{n} |Y_{ij}||V_j| \angle \theta_{ij} + \delta_j$$

$$= \sum_{j=1}^{n} |Y_{ij} V_i V_j| \angle \theta_{ij} + \delta_j - \delta_i$$

Separating this equation into real and reactive parts

$$P_i = \sum_{j=1}^{n} |Y_{ij} V_i V_j| \cos(\theta_{ij} + \delta_j - \delta_i) \qquad (2)$$

$$Q_i = -\sum_{j=1}^{n} |Y_{ij} V_i V_j| \sin(\theta_{ij} + \delta_j - \delta_i) \qquad (3)$$

Equation (2) and (3) constitute a polar form of the power flow equations.

...Newton-Raphson Power Flow Solution

These Eqs. constitute a set of non-linear algebraic equations in terms of the independent variables, voltage magnitude in per-unit, and phase angle in radians.

Expanding (2) and (3) in Taylor’s series about the initial estimate and neglecting all higher order terms results in the following set of linear equations.

...Newton-Raphson Power Flow Solution

Newton-Raphson power flow equations:

$$\underbrace{\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_2}{\partial \delta_n}^{(k)} & \frac{\partial P_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_2}{\partial |V_n|}^{(k)} \\ \vdots & J_1 & \vdots & \vdots & J_2 & \vdots \\ \frac{\partial P_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_n}{\partial \delta_n}^{(k)} & \frac{\partial P_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_n}{\partial |V_n|}^{(k)} \\ \hline \frac{\partial Q_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_2}{\partial \delta_n}^{(k)} & \frac{\partial Q_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_2}{\partial |V_n|}^{(k)} \\ \vdots & J_3 & \vdots & \vdots & J_4 & \vdots \\ \frac{\partial Q_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_n}{\partial \delta_n}^{(k)} & \frac{\partial Q_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_n}{\partial |V_n|}^{(k)} \end{bmatrix}}_{\textit{JACOBIAN MATRIX}} \underbrace{\begin{bmatrix} \Delta\delta_2^{(k)} \\ \vdots \\ \Delta\delta_n^{(k)} \\ \hline \Delta|V_2|^{(k)} \\ \vdots \\ \Delta|V_n|^{(k)} \end{bmatrix}}_{\textit{CORRECTIONS}} = \underbrace{\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix}}_{\textit{MISMATCHES}}$$

...Newton-Raphson Power Flow Solution

- From the previous equation, bus 1 is assumed to be slack bus.
- The Jacobian matrix gives the linearized relationship between small changes in voltage angle and voltage magnitude with the small changes in real and reactive power.
- Elements of Jacobian matrix are the partial derivatives of (2) and (3), evaluated at small changes in voltage angle and voltage magnitude

...Newton-Raphson Power Flow Solution

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- Elements of Jacobian matrix are the partial derivatives of (2) and (3), evaluated at small changes in voltage angle and voltage magnitude
- In short form, it can be written as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad (4)$$

Procedure of NR

- For load buses, $P_i^{(k)}$ is calculated from (2) and $Q_i^{(k)}$ calculated from (3);
- $\Delta P_i^{(k)}$ is calculated from: $\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)}$
- $\Delta Q_i^{(k)}$ is calculated from: $\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)}$
- For PV buses, $P_i^{(k)}$ is calculated from (2) and $\Delta P_i^{(k)}$ calculated from
- The element of Jacobian matrix (J1, J2, J3 and J4) are calculated as follows: $\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)}$

...Procedure of NR

The diagonal and off-diagonal elements of J_1 are

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i V_j Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|Y_{ij} V_i V_j| \sin\left(\theta_{ij} + \delta_j - \delta_i\right)$$

The diagonal and off-diagonal elements of J_2 are

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i Y_{ii}| \cos\theta_{ii} + \sum_{j \neq i} |V_j Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial |V_j|} = |Y_{ij} V_i| \cos\left(\theta_{ij} + \delta_j - \delta_i\right)$$

...Procedure of NR

The diagonal and off-diagonal elements of J_3 are

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} |V_i V_j Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i V_j Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

The diagonal and off-diagonal elements of J_4 are

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i Y_{ii}| \sin\theta_{ii} - \sum_{j \neq i} |V_j Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

...Procedure of NR

- The equation (4) is solved directly by optimally ordered triangular factorization of Gaussian elimination
- The new voltage and phase angle are computed from:

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}$$

$$\left|V_i^{(k+1)}\right| = V_i^{(k)} + \Delta V_i^{(k)}$$

- The process is continued until the residuals $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are less than specified accuracy


والله أعلم