Universiti Malaysia PAHANG

Process Monitoring

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Chapter 3b
MSPM:Phase I

Process Monitoring

Chapter Description

- Aims
  - Analyze the process performance based on MSPM approach.
- Expected Outcomes
  - Develop a fault detection mechanism as well as perform investigation based on a specified case study by using a specialized software.
- Other related Information

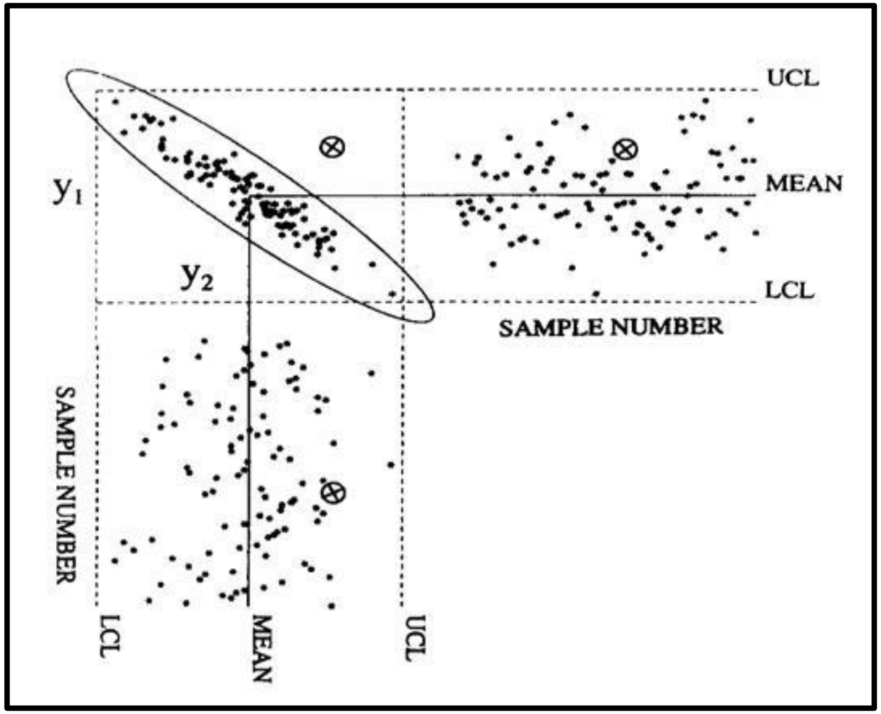
Subtopics

3.10 A Review on SPC

3.11 Overall MSPM Framework

3.12 Phase I Procedures

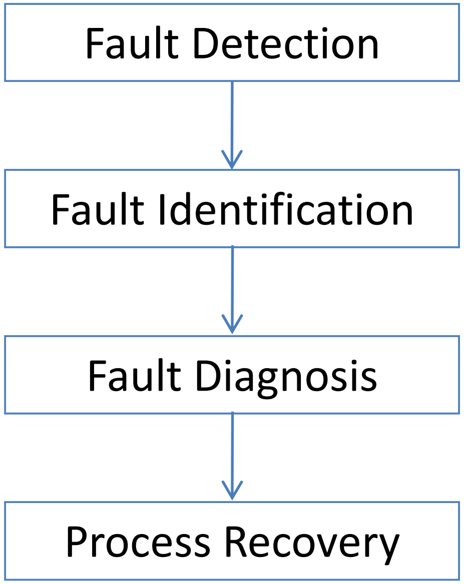
3.10 A Review on SPC



Source: MacGregor and Kourti, (1995)

3.11 Overall MSPM Frameworks

The main steps of MSPM system:



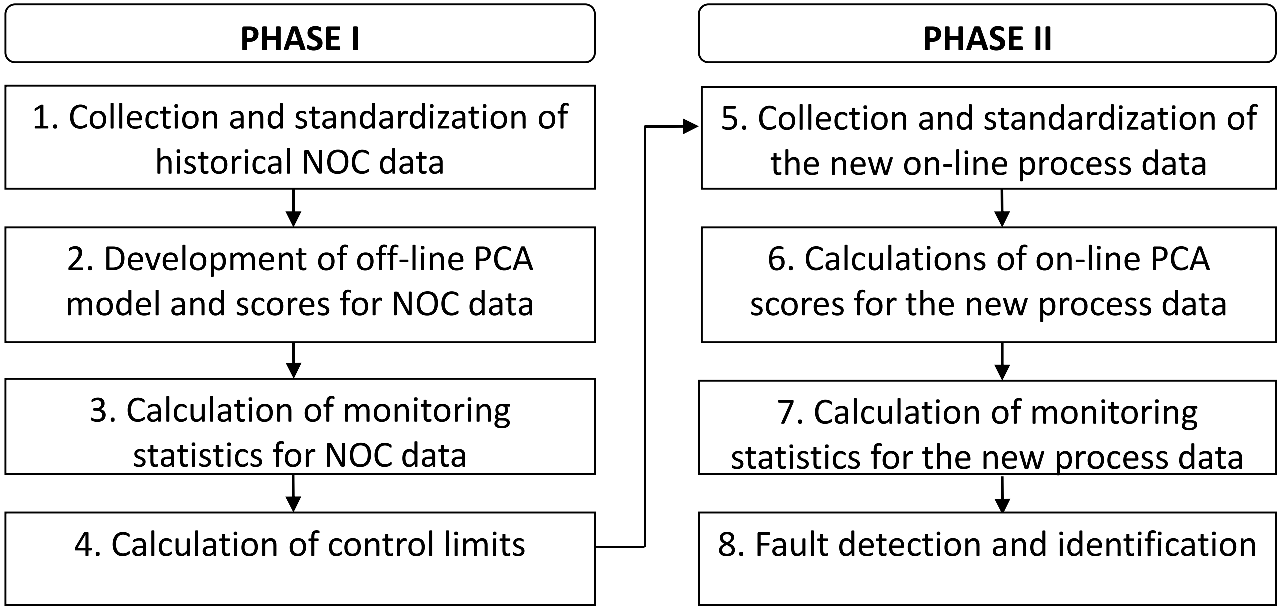
3.11 Overall MSPM Frameworks

The main steps of MSPM system:

- Fault detection: to designate the departure of observed samples from an acceptable range using a set of parameters.
- Fault identification: identifying the observed process variables that are most relevant to the fault which is typically identified by using the contribution plot technique.
- Fault diagnosis: specifically determines the type of fault which has been significantly (and should be also validated) contribute to the signal.
- Process recovery: remove the cause(s) that contribute to the detected fault.

3.11 Overall MSPM Frameworks

The complete procedures of fault detection and identification comprise of two main phases namely as off-line modelling and monitoring (phase I) and on-line monitoring (phase II):



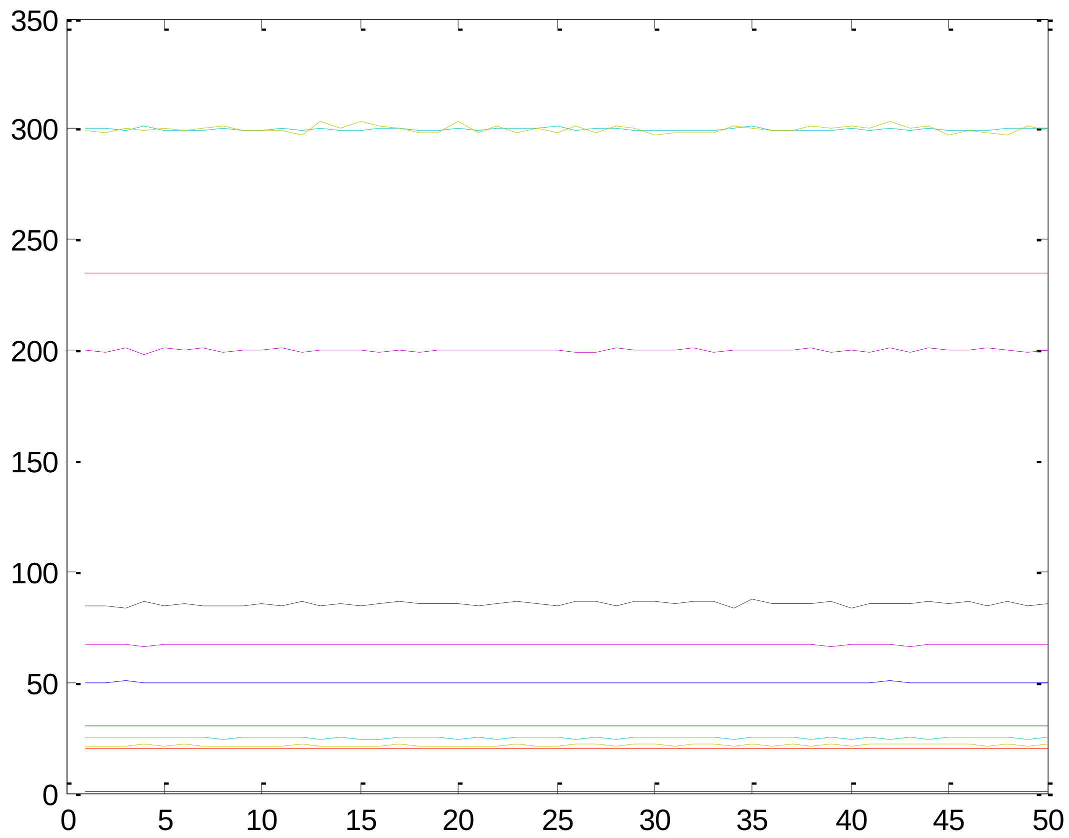
3.12 Phase I

<u>Firstly</u>, a set of normal operation condition (NOC) data, $\mathbf{X}_{n \times m}$ (*n*: samples, *m*: variables), are identified off-line based on the historical process data archive.

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,m} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,m} \end{bmatrix}$$

NOC simply implies that the process is operated at the desired setting condition and produces satisfactory products that meet the qualitative as well as quantitative specified standard (Martin et al., 1996).

3.12 Phase I



3.12 Phase I

Then, the data are standardized to zero mean and unit variance with respective to each of the variables because PCA results depend on data scales.

$$\breve{x}_{j,i} = \frac{\left(x_{j,i} - \bar{x}_i\right)}{\sigma_i}$$

3.12 Phase I

Variables / Samples	1	2	3	4	5	6	7	8	9	10	11	12	13
1	50.097	29.932	20.032	300.557	199.651	299.359	83.966	0.110	0.802	234.512	24.903	66.641	20.980
2	50.093	30.070	20.052	300.336	199.462	298.682	84.678	0.109	0.800	234.521	25.072	66.597	21.329
3	50.147	29.977	19.890	299.806	200.830	300.558	83.903	0.110	0.804	234.539	24.589	67.054	21.064
4	50.072	30.016	19.834	300.945	198.497	298.966	86.608	0.109	0.799	234.528	25.038	66.338	21.617
5	50.128	29.973	20.129	299.610	200.654	300.142	84.417	0.110	0.797	234.544	24.735	66.959	21.084
6	49.950	29.982	19.775	299.121	199.742	299.274	85.880	0.109	0.801	234.542	24.896	66.684	21.583
7	50.091	30.098	20.247	299.299	200.920	300.085	83.920	0.110	0.796	234.549	24.737	66.990	21.097
8	50.057	30.003	19.807	300.775	199.242	301.747	84.906	0.110	0.796	234.547	24.396	66.561	21.360
9	50.085	29.987	19.855	299.099	200.187	299.454	84.111	0.110	0.799	234.524	24.919	66.875	21.169
10	49.995	29.956	19.797	299.058	199.879	299.289	85.368	0.109	0.800	234.533	24.936	66.779	21.461
11	50.065	29.955	19.825	300.294	200.927	299.336	84.866	0.109	0.801	234.543	24.973	67.013	21.351
12	49.886	30.173	20.107	299.690	199.126	297.588	86.530	0.109	0.804	234.555	25.357	66.553	21.721
13	50.006	29.990	20.008	300.769	200.352	303.558	84.086	0.109	0.799	234.591	23.939	66.918	21.006
14	49.994	30.124	20.250	299.384	199.652	300.449	85.634	0.110	0.799	234.539	24.644	66.655	21.429
15	50.040	30.037	20.149	299.178	199.829	303.021	84.268	0.110	0.801	234.531	23.958	66.698	21.118
16	50.106	30.017	19.996	300.347	199.527	301.462	85.869	0.109	0.796	234.481	24.424	66.687	21.467
17	50.032	29.970	19.816	300.464	200.033	300.439	86.395	0.110	0.801	234.461	24.595	66.714	21.614
18	50.018	29.959	19.788	299.510	199.556	298.756	84.995	0.109	0.801	234.451	25.029	66.603	21.386
19	50.000	30.091	19.845	299.143	200.469	298.401	85.324	0.110	0.802	234.467	25.125	67.003	21.448
20	50.027	29.937	20.058	300.220	200.228	303.051	85.332	0.109	0.798	234.489	24.057	66.766	21.485
21	50.058	30.033	19.938	299.334	200.210	298.427	84.642	0.109	0.800	234.446	25.108	66.789	21.281
22	50.141	29.937	19.914	300.068	200.485	301.644	85.274	0.110	0.796	234.467	24.331	66.947	21.452
23	50.035	29.965	20.113	300.105	199.955	298.239	86.886	0.109	0.802	234.441	25.259	66.746	21.729
24	49.982	29.930	19.901	300.116	200.200	300.447	85.638	0.110	0.802	234.472	24.705	66.808	21.455
25	49.979	30.031	19.930	300.935	199.606	298.029	84.843	0.111	0.799	234.468	25.227	66.562	21.332
26	50.046	29.944	19.991	299.783	199.545	300.935	86.151	0.110	0.796	234.497	24.494	66.654	21.508
27	49.914	30.053	19.783	300.709	199.521	298.131	86.772	0.110	0.804	234.480	25.205	66.670	21.788
28	50.061	30.023	19.874	300.312	200.977	301.554	84.738	0.109	0.801	234.507	24.323	67.119	21.229
29	50.000	29.915	20.060	299.173	199.742	300.244	86.830	0.110	0.803	234.480	24.752	66.686	21.777
30	49.921	29.934	20.089	299.017	199.598	297.569	86.273	0.109	0.799	234.479	25.427	66.602	21.623
31	49.979	29.947	19.801	299.669	199.698	298.670	85.207	0.109	0.802	234.520	25.053	66.728	21.377
32	50.003	29.962	20.053	299.818	200.903	298.286	86.919	0.109	0.801	234.537	25.142	67.001	21.743
33	49.908	30.098	19.933	299.660	199.418	298.419	86.471	0.109	0.797	234.559	25.191	66.561	21.557
34	49.979	30.058	19.797	300.277	199.730	301.885	83.780	0.109	0.800	234.585	24.362	66.752	21.066
35	49.964	29.949	19.788	300.986	200.057	300.749	87.049	0.109	0.801	234.559	24.606	66.818	21.753
36	49.931	29.966	19.965	299.359	199.591	299.579	85.610	0.110	0.800	234.549	24.919	66.596	21.424
37	49.842	30.081	19.951	299.228	200.139	298.891	85.789	0.109	0.803	234.558	25.067	66.749	21.606
38	50.035	30.012	20.136	299.529	200.907	301.876	85.708	0.111	0.799	234.575	24.298	67.041	21.372
39	49.855	30.083	19.835	299.204	198.952	299.874	86.186	0.110	0.796	234.546	24.815	66.366	21.499
40	50.018	30.012	20.216	300.106	200.114	301.868	83.614	0.110	0.796	234.548	24.269	66.853	20.877
41	50.043	30.086	19.865	299.795	199.502	300.565	85.607	0.110	0.801	234.518	24.641	66.650	21.585
42	50.159	30.014	19.970	299.906	201.253	303.348	85.417	0.109	0.805	234.510	23.934	67.144	21.511
43	50.143	30.036	19.833	299.689	198.975	300.315	85.926	0.110	0.801	234.459	24.664	66.366	21.621
44	50.106	29.902	19.853	300.725	200.759	301.112	86.235	0.109	0.801	234.453	24.492	67.077	21.660
45	49.935	29.962	19.953	299.136	200.044	296.850	85.886	0.110	0.797	234.436	25.526	66.868	21.608
46	49.957	29.950	19.787	299.650	200.118	298.936	86.683	0.109	0.802	234.483	25.055	66.737	21.671
47	49.953	29.942	19.913	299.368	200.657	298.205	84.565	0.111	0.805	234.501	25.268	67.018	21.249
48	49.987	30.059	19.974	299.859	199.888	297.766	86.699	0.110	0.800	234.532	25.340	66.730	21.749
49	49.923	30.004	19.821	300.133	199.431	301.061	84.764	0.110	0.801	234.567	24.457	66.590	21.318
50	49.843	30.063	19.966	300.818	200.477	299.373	85.778	0.110	0.799	234.547	24.913	66.938	21.590
Means	**50.012**	**30.004**	**19.945**	**299.881**	**199.984**	**299.929**	**85.460**	**0.110**	**0.800**	**234.514**	**24.783**	**66.765**	**21.436**
Std Dev	**0.080**	**0.061**	**0.132**	**0.589**	**0.604**	**1.630**	**0.959**	**0.001**	**0.003**	**0.041**	**0.404**	**0.196**	**0.233**

3.12 Phase I

Samples	Variables												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1.063991	-1.18125	0.659289	1.145328	-0.55115	-0.34964	-1.55856	0.241663	0.618662	-0.05001	0.295111	-0.63356	-1.95698
2	1.01521	1.071188	0.806202	0.771266	-0.86374	-0.7654	-0.81577	-0.39429	0.066989	0.150226	0.714676	-0.85737	-0.45921
3	1.686896	-0.44077	-0.42135	-0.12869	1.398691	0.385996	-1.62408	0.718631	1.485576	0.60198	-0.48016	1.477375	-1.59554
4	0.755041	0.1937	-0.84089	1.80388	-2.46145	-0.59074	1.197913	-1.34823	-0.52409	0.326044	0.630268	-2.18137	0.779396
5	1.447991	-0.50927	1.389309	-0.45983	1.108937	0.130295	-1.08764	0.559642	-1.3516	0.724076	-0.11851	0.991928	-1.51259
6	-0.77595	-0.35106	-1.28844	-1.28955	-0.40156	-0.40184	0.437697	-0.71227	0.539851	0.66547	0.279764	-0.41485	0.631553
7	0.991444	1.53766	2.282901	-0.98861	1.547953	0.09551	-1.60634	1.036609	-1.66684	0.851055	-0.11381	1.150337	-1.45672
8	0.563666	-0.01018	-1.04838	1.516336	-1.22813	1.115186	-0.57759	0.718631	-1.70625	0.794891	-0.95839	-1.04542	-0.32684
9	0.913894	-0.26951	-0.68186	-1.32704	0.335155	-0.29148	-1.40698	0.87762	-0.6029	0.228368	0.336449	0.560135	-1.14513
10	-0.20433	-0.77676	-1.12335	-1.39643	-0.17386	-0.39276	-0.09561	-1.66621	0.066989	0.450582	0.377292	0.070088	0.110667
11	0.66123	-0.80285	-0.9098	0.700016	1.559206	-0.36399	-0.62005	-1.66621	0.22461	0.694773	0.469621	1.267356	-0.36251
12	-1.57272	2.759293	1.225736	-0.32462	-1.42058	-1.43606	1.116749	-0.2353	1.643197	0.992686	1.42113	-1.08374	1.225072
13	-0.0755	-0.22873	0.473755	1.505648	0.609189	2.225904	-1.43347	-1.34823	-0.6029	1.859565	-2.08985	0.781908	-1.84781
14	-0.2281	1.95357	2.306377	-0.8439	-0.55032	0.318574	0.181162	0.87762	-0.36647	0.60198	-0.34525	-0.56048	-0.02772
15	0.351028	0.536214	1.546066	-1.19387	-0.25643	1.897014	-1.24308	0.400653	0.461041	0.409069	-2.04356	-0.34178	-1.36389
16	1.176565	0.21001	0.382882	0.790436	-0.75701	0.940465	0.42643	-0.2353	-1.50922	-0.807	-0.89007	-0.39696	0.135594
17	0.257217	-0.54678	-0.9772	0.987899	0.081144	0.312562	0.975493	1.195598	0.421636	-1.31492	-0.4653	-0.26002	0.766932
18	0.0771	-0.73435	-1.19151	-0.631	-0.70803	-0.71969	-0.48464	-0.39429	0.264015	-1.56155	0.608981	-0.83029	-0.2151
19	-0.15055	1.413702	-0.75683	-1.2524	0.802469	-0.93779	-0.14172	0.87762	0.579257	-1.15864	0.84661	1.214723	0.053077
20	0.195927	-1.08665	0.850882	0.573632	0.402505	1.914927	-0.13369	-0.71227	-0.76052	-0.61409	-1.79851	0.002637	0.212523
21	0.581178	0.480759	-0.05407	-0.92923	0.37305	-0.92153	-0.85291	-1.50722	0.066989	-1.67144	0.804283	0.121188	-0.66336
22	1.616851	-1.08828	-0.23355	0.316624	0.828118	1.051874	-0.19409	0.718631	-1.78506	-1.14887	-1.11978	0.929586	0.071557
23	0.293491	-0.62996	1.267386	0.378713	-0.04859	-1.03674	1.487937	-1.03025	0.855093	-1.79354	1.177064	-0.0965	1.261173
24	-0.36819	-1.21224	-0.33654	0.398392	0.357329	0.31747	0.185961	0.082674	0.697472	-1.04387	-0.19277	0.217767	0.083591
25	-0.41447	0.43346	-0.11162	1.786915	-0.62645	-1.16563	-0.64415	1.513576	-0.48468	-1.12934	1.097112	-1.03878	-0.44331
26	0.427328	-0.9839	0.349561	-0.16652	-0.72722	0.617096	0.72094	-0.07631	-1.62744	-0.43583	-0.7163	-0.57019	0.309652
27	-1.21999	0.806963	-1.23013	1.404541	-0.76611	-1.10337	1.368798	0.87762	1.524982	-0.84363	1.04315	-0.48639	1.51474
28	0.6162	0.304609	-0.54252	0.730891	1.642276	0.996783	-0.75307	-0.87126	0.382231	-0.18432	-1.14007	1.810546	-0.88942
29	-0.14805	-1.452	0.866784	-1.20235	-0.40073	0.193177	1.429515	1.195598	1.012714	-0.84852	-0.07767	-0.40616	1.465745
30	-1.13119	-1.13884	1.09094	-1.46581	-0.63902	-1.4479	0.848529	-1.34823	-0.36647	-0.86561	1.593411	-0.83233	0.804323
31	-0.41072	-0.92192	-1.09457	-0.36008	-0.47404	-0.77263	-0.26441	-1.34823	0.855093	0.123365	0.666655	-0.19052	-0.25249
32	-0.10677	-0.6789	0.816047	-0.10714	1.52098	-1.00797	1.522155	-0.39429	0.30342	0.545816	0.886958	1.205014	1.320052
33	-1.3013	1.529505	-0.09572	-0.37654	-0.93755	-0.92668	1.054362	-0.55328	-1.27279	1.095246	1.010228	-1.04082	0.52368
34	-0.40947	0.877097	-1.12335	0.671516	-0.42109	1.199909	-1.75229	-1.18924	-0.05123	1.715492	-1.04378	-0.06481	-1.58737
35	-0.59959	-0.89256	-1.19075	1.873263	0.120694	0.503172	1.657986	-1.34823	0.461041	1.090362	-0.43882	0.269377	1.365178
36	-1.01111	-0.6267	0.153425	-0.88648	-0.65011	-0.21491	0.156959	0.241663	-0.20885	0.836404	0.335211	-0.86197	-0.04878
37	-2.12433	1.262017	0.045891	-1.10888	0.255891	-0.63681	0.34297	-0.71227	1.20974	1.051292	0.702547	-0.08372	0.73298
38	0.293491	0.133352	1.44459	-0.59826	1.527103	1.19451	0.259197	1.513576	-0.28766	1.485952	-1.20072	1.40788	-0.27355
39	-1.96673	1.284852	-0.83558	-1.14926	-1.70785	-0.03412	0.757558	0.718631	-1.46981	0.76803	0.079017	-2.0388	0.272692
40	0.083354	0.133352	2.054202	0.381258	0.215348	1.189173	-1.9262	0.87762	-1.74565	0.829078	-1.27201	0.44976	-2.40265
41	0.391054	1.335413	-0.6084	-0.14599	-0.79739	0.39029	0.153621	1.354587	0.342825	0.074527	-0.35193	-0.59063	0.642298
42	1.843248	0.159448	0.187503	0.041973	2.098834	2.097133	-0.0447	-1.34823	1.840223	-0.11838	-2.10272	1.936251	0.325984
43	1.644369	0.52969	-0.85224	-0.32632	-1.67062	0.236428	0.486313	1.195598	0.539851	-1.36376	-0.29451	-2.03778	0.798306
44	1.179066	-1.66892	-0.70155	1.430666	1.282524	0.725622	0.80826	-0.87126	0.461041	-1.50783	-0.72224	1.592861	0.965058
45	-0.95607	-0.6789	0.055736	-1.26393	0.098354	-1.88936	0.444479	0.87762	-1.391	-1.90831	1.838467	0.524365	0.742435
46	-0.67964	-0.87462	-1.19681	-0.39299	0.220809	-0.60963	1.275844	-0.39429	0.933903	-0.75816	0.672349	-0.14146	1.012763
47	-0.74093	-1.01652	-0.24339	-0.87121	1.112412	-1.05791	-0.93355	1.672565	1.879628	-0.32595	1.199837	1.293416	-0.80346
48	-0.3144	0.893407	0.221581	-0.03759	-0.15996	-1.32735	1.29264	-0.07631	-0.09063	0.423721	1.378307	-0.17877	1.345838
49	-1.10867	0.004502	-0.94085	0.426892	-0.91504	0.69415	-0.72636	1.354587	0.461041	1.271064	-0.80888	-0.89314	-0.50348
50	-2.10807	0.958648	0.154183	1.588434	0.815045	-0.34129	0.331495	0.082674	-0.32706	0.792449	0.321102	0.884107	0.665076
Means	-2E-07	-1.8E-07	-4.4E-07	-2E-08	-2.8E-07	0	2.6E-07	1.4E-07	-2E-07	-1.4E-07	-2E-07	-1E-07	5E-07
Std Dev	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

3.12 Phase I

In the second step, the development of PCA model for the NOC data requires the establishment of a set of correlation (or variance-covariance) matrix, $\mathbf{C}_{mxm}$.

$$C = \frac{1}{n-1}\breve{X}'\breve{X} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,m} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,m} \\ \vdots & \vdots & \vdots & \vdots \\ c_{m,1} & c_{m,2} & \cdots & c_{m,m} \end{bmatrix}$$

3.12 Phase I

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13
V1	1.00	-0.26	0.11	0.17	0.29	0.42	-0.36	0.01	-0.12	-0.24	-0.44	0.30	-0.35
V2	-0.26	1.00	0.21	-0.09	-0.26	-0.12	-0.04	0.18	-0.04	0.36	0.11	-0.25	-0.01
V3	0.11	0.21	1.00	-0.20	0.19	0.13	-0.15	0.18	-0.25	0.15	-0.12	0.14	-0.24
V4	0.17	-0.09	-0.20	1.00	-0.05	0.27	0.03	-0.15	-0.05	0.05	-0.26	-0.01	0.00
V5	0.29	-0.26	0.19	-0.05	1.00	0.25	-0.27	-0.06	0.15	0.02	-0.25	0.97	-0.25
V6	0.42	-0.12	0.13	0.27	0.25	1.00	-0.36	0.00	-0.19	0.29	-1.00	0.25	-0.40
V7	-0.36	-0.04	-0.15	0.03	-0.27	-0.36	1.00	-0.14	0.18	-0.26	0.38	-0.29	0.96
V8	0.01	0.18	0.18	-0.15	-0.06	0.00	-0.14	1.00	-0.12	-0.09	-0.02	-0.07	-0.13
V9	-0.12	-0.04	-0.25	-0.05	0.15	-0.19	0.18	-0.12	1.00	-0.12	0.19	0.14	0.26
V10	-0.24	0.36	0.15	0.05	0.02	0.29	-0.26	-0.09	-0.12	1.00	-0.27	0.03	-0.34
V11	-0.44	0.11	-0.12	-0.26	-0.25	-1.00	0.38	-0.02	0.19	-0.27	1.00	-0.26	0.41
V12	0.30	-0.25	0.14	-0.01	0.97	0.25	-0.29	-0.07	0.14	0.03	-0.26	1.00	-0.26
V13	-0.35	-0.01	-0.24	0.00	-0.25	-0.40	0.96	-0.13	0.26	-0.34	0.41	-0.26	1.00

3.12 Phase I

– **C** is then transformed into a set of basic structures of eigen-based formula =>

$$\mathbf{C} = \mathbf{V}\Lambda\mathbf{V}^{T}$$

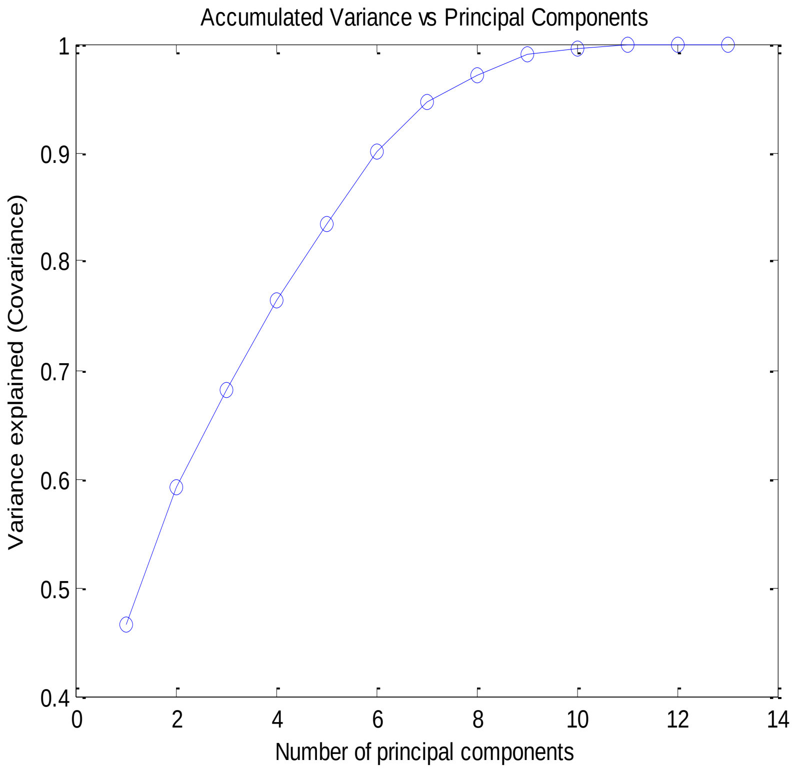
-0.31	-0.18	-0.14	0.40	0.01	0.00	0.13	-0.50	-0.27	0.59	0.00	0.00	0.01
0.08	0.45	0.19	-0.26	-0.14	0.25	0.32	-0.38	-0.55	-0.23	0.05	-0.06	0.02
-0.15	0.21	0.35	0.11	-0.54	-0.28	0.14	-0.33	0.54	-0.07	-0.04	0.07	-0.01
-0.08	-0.05	-0.52	-0.12	0.04	0.05	0.79	0.09	0.26	-0.07	-0.03	-0.01	-0.01
-0.30	-0.41	0.33	-0.18	-0.14	-0.01	0.14	0.17	-0.14	-0.03	-0.36	-0.61	0.01
-0.41	0.06	-0.34	-0.09	-0.26	0.16	-0.28	0.00	-0.01	-0.17	-0.03	0.03	0.71
0.39	-0.19	-0.18	-0.09	-0.51	-0.02	-0.03	0.05	-0.04	0.20	0.58	-0.35	0.04
-0.03	0.22	0.22	0.42	-0.17	0.67	0.12	0.42	0.10	0.20	0.00	0.00	0.01
0.12	-0.33	0.10	-0.32	0.19	0.58	-0.12	-0.46	0.40	0.02	0.04	-0.01	0.01
-0.15	0.36	0.03	-0.60	0.02	-0.07	-0.06	0.18	0.10	0.66	-0.07	0.03	-0.02
0.41	-0.06	0.34	0.07	0.25	-0.18	0.28	0.02	0.04	0.18	-0.04	0.02	0.71
-0.31	-0.41	0.31	-0.20	-0.11	0.00	0.18	0.18	-0.18	-0.03	0.39	0.58	0.00
0.40	-0.23	-0.15	-0.08	-0.45	0.07	-0.03	0.01	-0.17	0.11	-0.60	0.38	-0.05

183	0	0	0	0	0	0	0	0	0	0	0	0
0	102	0	0	0	0	0	0	0	0	0	0	0
0	0	82	0	0	0	0	0	0	0	0	0	0
0	0	0	64	0	0	0	0	0	0	0	0	0
0	0	0	0	52	0	0	0	0	0	0	0	0
0	0	0	0	0	43	0	0	0	0	0	0	0
0	0	0	0	0	0	36	0	0	0	0	0	0
0	0	0	0	0	0	0	33	0	0	0	0	0
0	0	0	0	0	0	0	0	24	0	0	0	0
0	0	0	0	0	0	0	0	0	15	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0

3.12 Phase I

The following equation presents a measure of data variations captured by the first '*k*' principal components (Jolliffe, 2002).

$$k = \frac{\lambda_1 + \lambda_2 + \ldots + \lambda_a}{\lambda_1 + \lambda_2 + \ldots + \lambda_a + \ldots + \lambda_m}$$



3.12 Phase I

Finally, the PCA model can be simply developed by:

$$\mathbf{P} = \breve{\mathbf{X}}\mathbf{V}$$

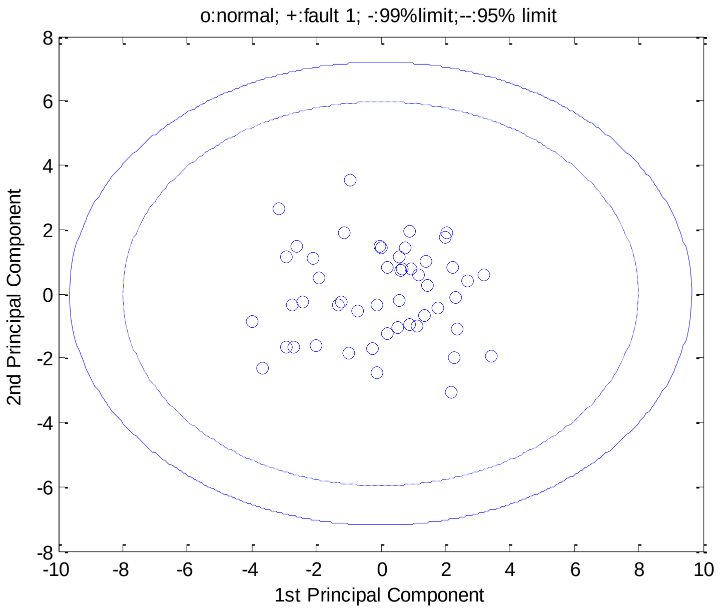
$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 & \cdots & \mathbf{p}_m \end{bmatrix}$$

$$= \begin{bmatrix} \breve{x}_{1,1}v_{1,1} + \cdots + \breve{x}_{1,m}v_{m,1} & \cdots & \breve{x}_{1,1}v_{1,m} + \cdots + \breve{x}_{1,m}v_{m,m} \\ \vdots & \cdots & \vdots \\ \breve{x}_{n,1}v_{1,1} + \cdots + \breve{x}_{n,m}v_{m,1} & \cdots & \breve{x}_{n,1}v_{1,m} + \cdots + \breve{x}_{n,m}v_{m,m} \end{bmatrix}$$

Samples	PC				
	1	2	3	4	5
1	-1.3	-0.37	-0.21	-1.17	-1.92
2	0.21	-1.24	0.02	-0.39	-0.76
3	-2.91	1.15	1.08	0.11	-1.43
4	2.37	-1.11	-3.06	-0.15	-0.59
5	-2.72	-0.35	1.46	-0.97	0.11
6	1.45	0.23	-0.02	0.95	-0.69
7	-2.93	-1.65	2.86	-0.57	0.87
8	-1	-1.85	-2.5	-0.64	-0.4
9	-1.24	-0.24	1.34	-1.05	-1.39
10	0.63	0.71	0.1	0.74	-1.18
11	-1.12	1.91	0.12	1.28	-1.31
12	3.46	-1.93	1.25	1.91	0.77
13	-3.99	-0.88	-1.42	1.93	-0.27
14	-0.1	-2.44	1.2	-0.02	1.87
15	-2.68	-1.67	0.26	-0.11	0.61
16	-0.71	-0.54	-1.72	-1.25	1.05
17	0.58	1.13	-1.34	-1	0.54
18	1.17	0.6	-0.27	-1.31	-1.68
19	0.68	0.77	2.16	-0.28	-0.35
20	-1.91	0.48	-1.62	-0.32	1.35
21	0.21	0.82	1.1	-0.73	-1.28
22	-2.11	1.09	-0.88	-1.95	0.88
23	2.07	1.9	0.13	-0.71	1.01
24	0	1.42	-0.48	-0.35	-0.12
25	1.11	-1	-0.15	-1.55	-1
26	-0.13	-0.33	-1.28	-1.3	1.04
27	3.2	0.57	-0.67	0.47	-0.04
28	-2.61	1.46	-0.1	1.05	-0.31
29	1.38	1	0.17	-1.14	1.7
30	2.68	0.42	1.02	-0.6	0.18
31	0.96	0.79	-0.16	0.63	-1.8
32	0.89	1.92	1.24	0.94	1.44
33	2.28	-1.99	0.18	0.97	0.34
34	-2	-1.6	-1.09	1.93	-1.83
35	0.76	1.41	-2.39	2.15	0.57
36	0.88	-0.98	0.28	0.12	-0.16
37	1.78	-0.46	1.47	2.38	0.08
38	-2.44	-0.26	1.28	0.58	2.1
39	2.2	-3.07	-0.48	0.13	0.2
40	-3.64	-2.29	0.68	-0.65	0.22
41	0.51	-1.07	-0.44	-0.31	0.41
42	-3.13	2.64	-0.48	1.5	1.24
43	1.34	-0.66	-1.58	-2.17	0.03
44	-0.96	3.54	-1.52	-0.24	0.69
45	2.23	0.83	1.95	-2	0.12
46	2	1.74	-0.22	0.24	-0.18
47	0	1.45	2.65	-0.22	-1.4
48	2.31	-0.1	0.67	0.53	0.77
49	-0.27	-1.69	-0.89	0.62	-0.82
50	0.56	-0.22	0.34	1.97	0.75

3.12 Phase I

- PCA scores by the 1st and 2nd PC.



3.12 Phase I

The third step basically involves calculation of the Hotelling's T^2 and SPE monitoring statistics.

$$T_i^{\,2} = \sum_{j=1}^{a} \frac{p_{i,j}^{2}}{\lambda_j}$$

$$\begin{aligned}\tilde{\mathbf{E}} &= \breve{\mathbf{X}} - \hat{\mathbf{X}} \\ &= \breve{\mathbf{X}} - \mathbf{P}_a \mathbf{V}_a^{\mathrm{T}} \\ &= \breve{\mathbf{X}} - \breve{\mathbf{X}} \mathbf{V}_a \mathbf{V}_a^{\mathbf{T}} \\ &= \breve{\mathbf{X}}\left(\mathbf{I} - \mathbf{V}_a \mathbf{V}_a^{\mathbf{T}}\right)\end{aligned}$$

$$SPE_i = \tilde{\mathbf{e}}_{\mathbf{i}} \tilde{\mathbf{e}}_{\mathbf{i}}^{\,T}$$

3.12 Phase I

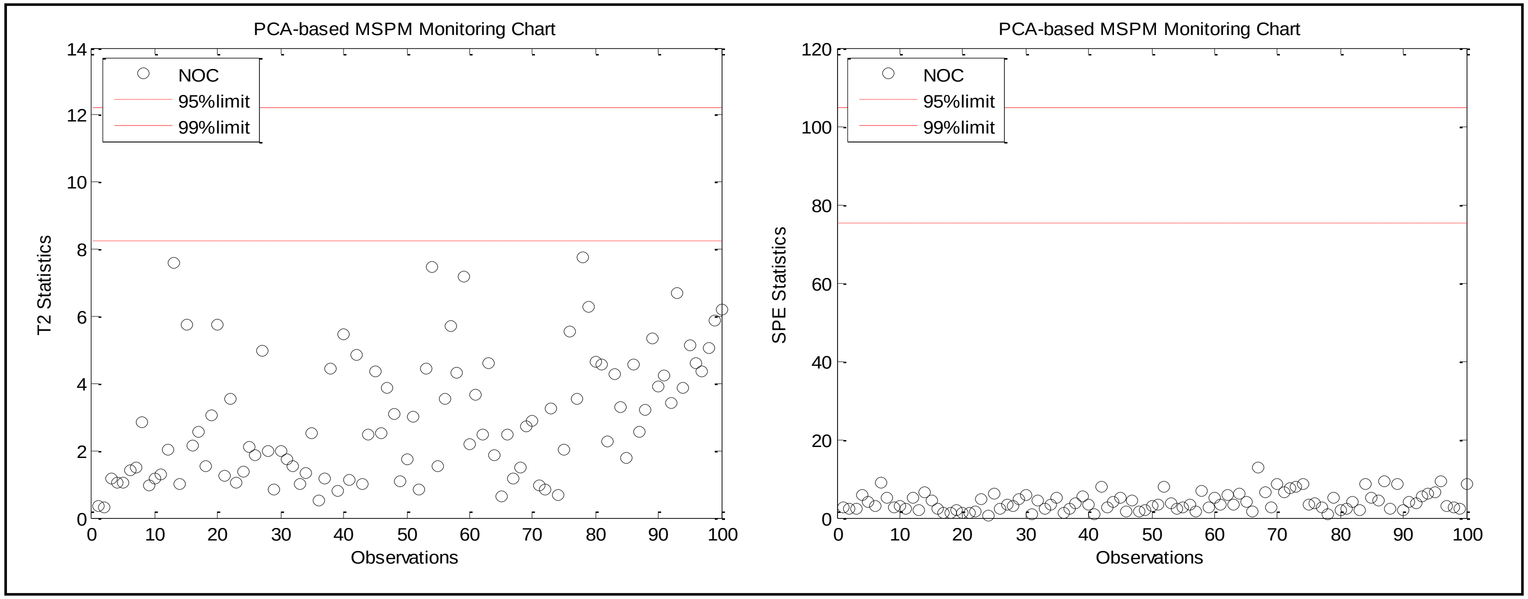
The final task in phase I (4th step) deals with developing the control limits for both of the statistics.

$$T_{\alpha} = \frac{A(n-1)}{(n-A)} F_{A,n-A,\alpha}$$

$$SPE_{\alpha} = \theta_1 \left(\frac{z_{\alpha}\sqrt{2\theta_2 h_0^2}}{\theta_1} + \frac{\theta_1 h_0 (h_0 - 1)}{\theta_1^2} + 1 \right)^{\frac{1}{h_0}}$$

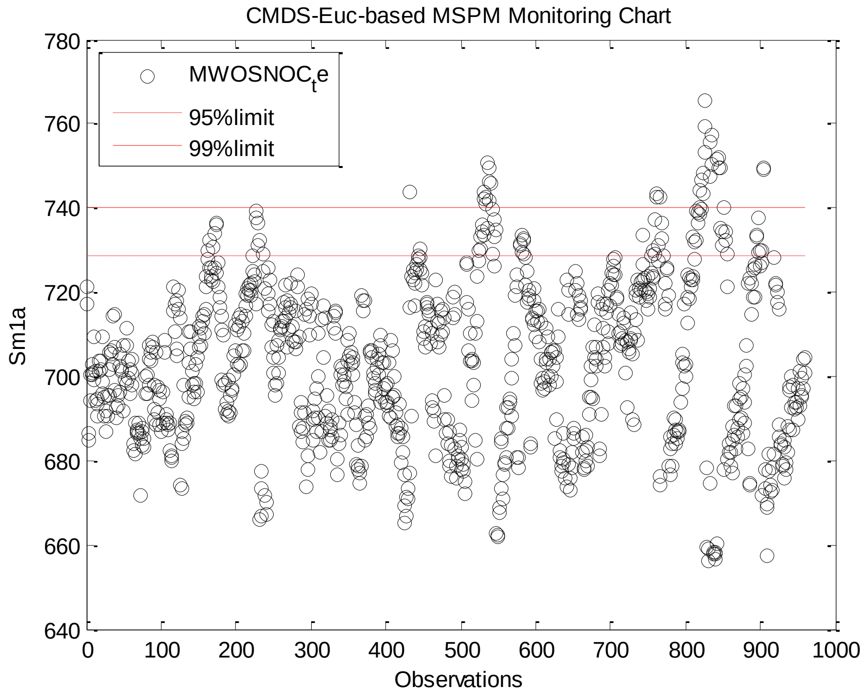
3.12 Phase I

T^2 and SPE progressions.



3.12 Phase I

- False alarm rate (FAR) analysis is conducted to evaluate the robustness of the monitoring statistics
- High FAR indicates that the developed limit settings are unsuitable to be used for monitoring



$$FAR = \frac{\text{Total amount of statistics in the NOC set located beyond the 99\% limit}}{\text{Total amount of NOC samples}}$$

References

- Green, P.E., and Carroll, J.D., (1976). *Mathematical Tools for Applied Multivariate Analysis.* New York, USA: Academic Press.
- Jackson, J.E., (1991). *A User's Guide To Principal Components.* John Wiley and Sons. USA.
- Martin., E.B., Morris, A.J., and Zhang, J. (1996). Process Performance Monitoring Using Multivariate Statistical Process Control. *Systems Engineering for Automation,* IEEE Proceedings.

Authors Information

Credit to the authors: