8. Numerical Integration

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8.1 Exercises

Exercises: Trapezoidal Rule

Exercise 8.1 Use the trapezium rule to estimate the value of

$$\int_0^{\frac{\pi}{2}} (5+2\sin(x))dx$$

Exercise 8.2 Given

$$f(x) = \sin(x)$$

and

$$g(x) = \sqrt{9+x^2}.$$

Find $\int_0^{\pi}(1+f(x)g(x))dx$ by using
i. Trapezoidal rule

ii. Trapezoidal rule with $n = 8$.
(*Hint:* $\pi = 3.142$) ■

Exercise 8.3 The arc length of a curve on the interval $a \leq x \leq b$ is given by the integral

$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Find the arc length of the curve if the function $y = \cos(x)$ for $-1 \leq x \leq 1$ by using Trapezoidal rule with 8 intervals. ■

Exercises: Simpson's Rule

Exercise 8.4 Use Simpson's 1/3 rule to evaluate the following functions with $h = 0.25$.
i. $\int_1^3 \frac{1}{x(\sqrt{16+x^2})}dx$
ii. $\int_1^2 \left(\frac{x+1}{x}\right) dx$
iii. $\int_1^2 \sqrt{1+x^2}dx$
■

Exercise 8.5 Approximate the integral of Question 8.2 by using
i. Simpson's 1/3 rule with $n = 8$.
ii. Simpson's 3/8 rule
■

Exercise 8.6 Evaluate the following integral

$$\int_a^b \frac{t^2+4}{\sqrt{t}}dt$$

using Simpson's rule with $a = 2$, $b = 4.5$ and $h = 0.5$. ■

Exercise 8.7 The distance covered by a rocket from $t = 8$ to $t = 30$ is given by

$$x = \int_8^{30} \left(2000\ln\left(\frac{140000}{140000-2100t}\right)\right) dt$$

Use Simpson's $1/3^{rd}$ rule to find the approximate value of x with $n = 4$. ■

Exercise 8.8 A cylindrical fuel tank lying on its side of length $L = 20$ ft. Its ends are elliptically–shaped which is defined by the equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{c}\right)^2 = 1 \qquad (8.1)$$

Let the volume of fuel in the tank can be modelled by

$$V(x) = 2La\int_{-c}^{x} \sqrt[3]{1-\left(\frac{y}{c}\right)^2}dx \qquad (8.2)$$

For $a = 5$, $c = 4$, $x = 4$ and number of strips, $n = 11$, compute the volume of fuel $V(x)$ by using Simpson's rule.
(*Hint:* The integrand in equation (8.2) can be obtained from equation (8.1)) ■

References 1. Chapra, C. S. & Canale, R. P. Numerical Methods for Engineers, Sixth Edition, McGraw–Hill, 2010.