4. Solving Linear Algebraic of Equations– Direct Methods

Mohd Zuki Salleh,
Applied & Industrial Mathematics Research Group,
Faculty of Industrial Sciences & Technology (FIST),
Universiti Malaysia Pahang,
Lebuhraya Tun Razak,
26300 Gambang, Kuantan,
Pahang, Malaysia
E-mail: zuki@ump.edu.my

4.1 Exercises

Exercises: Gauss Elimination Method

Exercise 4.1 Given the system of linear equations

$$\begin{aligned}
5.3(5x_2 - 13) + 32.13x_3 &= 698.1 - 13.53x_1 + 12.2x_4 \\
7.32x_1 - 24.6x_3 + 5.65x_2 + 4x_4 &= 560 \\
5.32(7x_3 + 12.2x_1) &= 5.75x_2 + 989 - 2.21x_4 \\
12.52x_1 - 3.5x_2 + 8.78x_3 - 6.2x_4 - 485 &= 0
\end{aligned}$$

i. Transform the system of linear equations in matrix form of $\mathbf{Ax} = \mathbf{b}$.
ii. Solve the system of linear equations using Gauss elimination with partial pivoting.

Exercise 4.2 Given the system of linear equations

$$\begin{aligned} 7.14(5x_4+4.1x_1) &= 7.23x_3+832-3.14x_2 \\ 16.67x_1-19.71x_3-26.9x_2+7x_4 &= 658 \\ 3.65(7.1x_3-11)+17.5x_2 &= 682.85+23.53x_4-24.3x_1 \\ 47.2x_1-7.3x_2+9.72x_3-4.9x_4-949 &= 0 \end{aligned}$$

i. Transform the system of linear equations in matrix form of $\mathbf{A}\mathbf{x}=\mathbf{b}$.
ii. Solve the system of linear equations using Gauss elimination with partial pivoting.

■

Exercise 4.3 Given the system of linear equations

$$\begin{aligned} 0.143x_1+0.357x_2+2.01x_3 &= -5.17 \\ -1.31x_1+0.911x_2+1.99x_3 &= -5.46 \\ 11.2x_1-4.30x_2-0.605x_3 &= 4.42 \end{aligned}$$

Solve using Gauss elimination with partial pivoting. ■

Exercise 4.4 The currents i_1, i_2, i_3 and i_4 can be determined by solving the following systems of equations:

$$\begin{aligned} 9i_1-4i_2-2i_3 &= 24 \\ -4i_1+17i_2-6i_3-3i_4 &= -16 \\ -2i_1-6i_2+14i_3-6i_4 &= 0 \\ -3i_2-6i_3+11i_4 &= 18 \end{aligned}$$

Find i_1, i_2, i_3 and i_4 for the system of linear equations by using Naïve Gauss Elimination methods. ■

Exercises: LU Factorization

Exercise 4.5 Given the system of linear equations

$$\begin{aligned} 34x_1+12x_2+15x_3 &= 82 \\ 12x_1+16x_2+17x_3 &= 69 \\ 15x_1+17x_2+22x_3 &= 92 \end{aligned}$$

i. Transform the system of linear equations in matrix form of $\mathbf{A}\mathbf{x}=\mathbf{b}$.
ii. Solve the system of linear equations by using Cholesky factorization.

■

Exercise 4.6 Given the system of linear equations

$$\begin{aligned} 1.25x_1+2.012x_2+5.3x_3 &= 25.81 \\ -2.12x_1+3.52x_2+6.215x_3-11.25 &= 0 \\ 6.21x_1+5.6x_2-2.25x_3 &= -41.2 \end{aligned}$$

i. Transform the system of linear equations in matrix form of $\mathbf{A}\mathbf{x}=\mathbf{b}$.

ii. Solve the system of linear equations by using (a) Crout's method, (b) Naïve Gauss elimination as LU factorization.

Exercise 4.7 Given the system of linear equations

$$\begin{aligned} 20x_1 + 12.5x_2 &= 76.2 - 16.4x_3 \\ 2.5x_1 + 2.2x_3 - 58.4 &= -5x_2 \\ 6x_1 + 3.3x_2 + 8x_3 - 62.11 &= 0 \end{aligned}$$

i. Transform the system of linear equations in matrix form of $\mathbf{Ax} = \mathbf{b}$.

ii. Solve the system of linear equations by using (a) Crout's method, (b) Naïve Gauss elimination as LU factorization.

Exercise 4.8 Given the system of linear equations

$$\begin{aligned} 12x_1 + 4x_2 - 2x_3 + x_4 &= 46.8 \\ 3x_1 + 7x_2 &= 6x_3 - 2x_4 + 62.3 \\ x_1 - 6x_2 + 4x_3 - 76 &= 0 \\ 5x_1 + 2x_2 + x_3 + 3x_4 &= 88.6 \end{aligned}$$

i. Transform the system of linear equations in matrix form of $\mathbf{Ax} = \mathbf{b}$.

ii. Solve the system of linear equations by using (a) Crout's method, (b) Naïve Gauss elimination as LU factorization.

Exercise 4.9 Given the system of linear equations

$$\begin{aligned} 16x_1 + 4x_2 + 4x_3 - 4x_4 &= 32 \\ 4x_1 + 10x_2 + 4x_3 + 2x_4 &= 26 \\ 4x_1 + 4x_2 + 6x_3 - 2x_4 &= 20 \\ -4x_1 + 2x_2 - 2x_3 + 4x_4 &= -6 \end{aligned}$$

Construct a Cholesky decomposition and solve for $\mathbf{Ax} = \mathbf{b}$.

References 1. Chapra, C. S. & Canale, R. P. Numerical Methods for Engineers, Sixth Edition, McGraw–Hill, 2010.