2. Roots of Equations

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2.1 Exercises

Exercises: Graphical and Incremental Search Methods

Exercise 2.1 Given $f(x) = x^2 - 6.45x + 9.15$ for $1 \leq x \leq 5$.
i. Plot and determine those subintervals which contain root of the function, $f(x)$.
ii. Using incremental search method, divide the interval into eight subintervals and find those subintervals that contain root of the function, $f(x)$.

Exercise 2.2 Given $f(x) = x^2 - \sin(x) - 1.4$ for $0 \leq x \leq 3.5$.
i. Plot and determine those subintervals which contain root of the function, $f(x)$.
ii. Using incremental search method, divide the interval into seven subintervals and find those subintervals that contain root of the function, $f(x)$.

Exercises: Bisection Method

Exercise 2.3 Determine the real root of $f(x) = x^3 - 3x^2 + x + 6$ using three iterations of the bisection method with initial guesses of $x_l = -2.0$ and $x_u = -0.9$. If the true root is $x = -1.0946$, calculate true percent relative error and approximate percent relative error for each iteration.

Exercise 2.4 Determine the first positive root of $x^2 \sin(x) = 1.4$ by using bisection method with initial guesses of $x_l = 1.0$ and $x_u = 1.5$. Perform the calculation until the stopping criterion, $\varepsilon_s = 5\%$.
(Use radian mode in your calculator)

Exercise 2.5 Determine the root of $2 = 0.5x^3 - \sqrt[3]{x}$ by using bisection method. Given the initial guesses are $1, 2, 3$ and 4. Decide the best lower and upper bound that bracket the root. Hence, carry out the computation until $\varepsilon_a < 10\%$.

Exercises: False Position Method

Exercise 2.6 Determine the first real root of $f(x) = -x^2 - 5x + 4$ using false position method with initial guesses of $x_l = -7.5$, $x_u = -5$ and stopping criterion, $\varepsilon_s = 0.5\%$. If the true root is $x = -5.7016$, calculate the true percent relative error and approximate percent relative error for each iteration.

Exercise 2.7 Find the positive root of $f(x) = \exp(-x)(3.2\sin(x) - 0.5\cos(x))$ using false position method with initial guesses of $x_l = 3$ and $x_u = 4$. Perform your calculation until four iterations.
(Use radian mode in your calculator)

Exercise 2.8 The concentration of pollutant bacteria, c in a lake decreases can be formulate as

$$c = 75\exp(-1.5t) + 20\exp(-0.075t)$$

Determine the time required for the bacteria concentration to be reduced to 15 using false position method with an initial guess of $t_l = 2.5s$ and $t_u = 5.5s$. Calculate until $\varepsilon_a < 4\%$.

Exercise 2.9 Water is discharged from a reservoir through a long pipe. By neglecting the change in the level of the reservoir, the transient velocity, $v(t)$ of the water flowing from the pipe at time, t is given by

$$v(t) = \sqrt{2gh} + \frac{t}{2L}\cos(2gh)$$

where h is the height of the fluid in the reservoir, L is the length of the pipe and $g = 9.81\text{ms}^{-2}$ is the gravity. Find the value of h that is required to achieve a velocity of $v = 4\text{ms}^{-1}$ at time $t = 4$s, when $L = 5$m. Use false position method for the calculation with the initial height is $h_l = 0.55$m and $h_u = 1.15$m. Perform the computation until three iterations and calculate approximate percent relative error in each iteration.
(Use radian in your calculator)

Exercises: Newton Raphson Method

Exercise 2.10 Determine the root of

$$f(x) = 10.5x^2 - 1.5x - 5$$

by using Newton Raphson method with $x_0 = 0$ and perform the iterations until $\varepsilon_a < 1.00\%$. Compute ε_t for each approximation if given the true root is $x = 0.7652$. ■

Exercise 2.11 Determine the root of $f(x) = 10\exp(-x)\cos(x) + 9$ by using the Newton Raphson method with three iterations and $x_0 = -0.5$. ■

Exercise 2.12 Compute three iterations of Newton Raphson method to find the root of the following equations

i. $f(x) = x^3 - x - 1$ with $x_0 = 2.5$.

ii. $f(x) = \sin(2x) - \cos(x) - x^2 - 1$ with $x_0 = 2.0$.

iii. $x\exp(x) = 2$ with $x_0 = 0.55$.

■

Exercise 2.13 Suppose a company must supply N units/month at a uniform rate. Assume the storage cost/unit is S_1 dollars/month and that setup cost is S_2 dollars. Further assume that production is at a uniform rate of m units/month and x be the number of items produced each run. The total average cost per month is expressed by

$$C = \frac{S_1}{2}\left(1 - \frac{N}{m}\right)x + \frac{S_2 N}{x}$$

Assume that the storage cost/unit is $S_1 = 25$ dollars/month, setup cost is $S_2 = 520$ dollars, the production $m = 100$ units/month and a company must supply $N = 10$ units/month. If the total average cost per month is minimize, that is $C = 1625\sin(x)$, find the number of items being produced for each run by using three iterations of Newton Raphson method. Let initial guess, $x_0 = 10$.

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Exercises: Secant Method

Exercise 2.14 Use secant method to estimate the root of

$$f(x) = -x^2 - 6.45x + 9.15$$

Start with initial estimates $x_{-1} = -10$ and $x_0 = -9$. Perform the computation until $\varepsilon_a < 1\%$. Calculate true percent relative error in each iteration if given true root is $x = -7.6466$. ■

Exercise 2.15 Use secant method to estimate the root of

$$f(x) = \exp(-x) - x^2$$

Start with initial estimates $x_{-1} = 1.25$ and $x_0 = 1.4$. Perform the computation until $\varepsilon_a < 5\%$. ■

Exercise 2.16 Use secant method to estimate the root of

$$\ln\left(\frac{x}{2}\right) + \frac{1}{5}x^2 = 2$$

Perform the three iterations with initial estimates $x_{-1} = 4.0$ and $x_0 = 4.5$. ■

References 1. Chapra, C. S. & Canale, R. P. Numerical Methods for Engineers, Sixth Edition, McGraw–Hill, 2010.