FACULTY OF INDUSTRIAL SCIENCES & TECHNOLOGY
TEST 2

COURSE	**:**	**ORDINARY DIFFERENTIAL EQUATIONS**
COURSE CODE	**:**	**BUM2133**
LECTURER	**:**	**SAMSUDIN BIN ABDULLAH** **NOR AIDA ZURAIMI BINTI MD NOAR** **NOR ALISA BINTI MOHD DAMANHURI** **LAILA AMERA BINTI AZIZ** **NURFATIHAH BTE MOHAMAD HANAFI**
DATE	**:**	
DURATION	**:**	**1 HOUR & 30 MINUTES**

NAME : ______________________	**QUESTION**	**MARKS**
______________________	**1**	
I.D. NUMBER: ______________________	**2**	
	3	
	4	
	TOTAL MARKS	**50**

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of **FOUR** questions. Answer all questions.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of **TEN (10)** printed pages including the front page

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QUESTION 1

Use the method of undetermined coefficients to solve the Euler equation

$$x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - 8y = \ln^3 x - \ln x .$$

(12 Marks)

Solution

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QUESTION 2

(a) Use the second shift theorem to find the Laplace transforms of

$$u(t)e^{2t}\sinh 3t$$

(6 Marks)

(b) Use the Laplace transforms of integral to find the Laplace transforms of

$$\int_0^t u\cos 2u \; du$$

(7 Marks)

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QUESTION 3

(a) Find the inverse of the Laplace transforms of

$$\frac{s^2+9s+22}{(s-1)(s+3)^2}$$

(8 Marks)

(b) Use the convolution theorem to find the inverse of Laplace transforms of

$$\frac{4}{s^2-9}$$

(7 Marks)

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QUESTION 4

Use Laplace transforms to solve the equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 1, \qquad x(0) = 1, \quad x'(0) = 0$$

(10 Marks)

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END OF QUESTION PAPER

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APPENDIX

Euler Equation

$$ax^2\frac{d^2y}{dx^2}+bx\frac{dy}{dx}+cy=f(x)$$

$$x=e^t,\qquad t=\ln x,\qquad a\frac{d^2y}{dt^2}+(b-a)\frac{dy}{dt}+cy=f(e^t)$$

Laplace Transforms

$f(t)$	$F(s)=\int_0^\infty f(t)e^{-st}dt$
a	$\frac{a}{s}$
t^n	$\frac{n!}{s^{n+1}},\ n=1,2,3,\ldots$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$

Given : $\mathcal{L}\{f(t)\}=F(s)$ and $\mathcal{L}\{g(t)\}=G(s)$

Linearity : $\mathcal{L}\{\alpha f(t)+\beta g(t)\}=\alpha F(s)+\beta G(s)$

First Shift Theorem : $\mathcal{L}\{e^{-at}f(t)\}=F(s+a)$

Second Shift Theorem : $\mathcal{L}\{u(t-a)f(t-a)\}=e^{-as}F(s)$

Derivative of *t*-transform : $\mathcal{L}\{t^n f(t)\}=(-1)^n\frac{d^nF(s)}{ds^n};\quad n=1,2,3,\ldots$

Laplace of Integral : $\mathcal{L}\left\{\int_0^t f(u)\,du\right\}=\frac{F(s)}{s}$

Convolution Theorem : $\mathcal{L}^{-1}\{F(s)G(s)\}=\int_0^t f(t-u)\,g(u)\,du$

Laplace Transforms of Derivatives

$$\mathcal{L}\left\{\frac{dy}{dt}\right\}=sY(s)-y(0)$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\}=s^2Y(s)-sy(0)-y'(0)$$

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