PART A: Determine whether each of these statements is **T** (True) or **F**(False)

(a) Let $A = \{x, y\}$. Given that $R_1 = \{(x, y), (y, x)\}$, then R_1 is transitive.......... ☐

(b) $R_2 = \{(a, b)|a - b = 0\}$ is reflexive. .... ☐

(c) If $f(x) = 3x$ and $g(x) = x^2$, then $(g + f)(x) = x(3 + x)$ .... ☐

(d) A bijective function is not onto .... ☐

(e) Given that if x, then y is false. Then x is false... ☐

(f) Let A be a statement. Thus, $A = \neg(\neg A)$ .... ☐

(g) $\exists x \in \mathbb{R}, x^2 > 0$ is true.. ☐

(h) Modus tollens is a syllogism of affirming .... ☐

(i) A binary relation R, is transitive if $(a, c) \in R$ and $(b, c) \in R$, then $(a, c) \in R$ ☐

(j) Theorem is a statement that can be shown to be false................................. ☐

(10 marks)

PART B
Question 2

(a) Let $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = \dfrac{5x}{7}$ and $g(x) = 7x - 5$.

(i) Determine $(g \circ f)(x)$.

(ii) Prove that $(g \circ f)(x)$ is one-to-one.

(iii) Then, determine $(g \circ f)^{-1}(x)$

(7 marks)

(b) Determine whether $(p \leftrightarrow q) \vee r \ \equiv (p \to q) \wedge (r \to p)$

(8 marks)

Question 3

(a) Let a binary relation $R = \{(a, b) \mid b - a = 0\}$, where $a, b \in \mathbb{Z}$. Determine whether R is symmetric and justify your answer.

(3 marks)

(b) Let $x, y \in \mathbb{Z}$. Determine the truth value of $\forall x, \exists y$, such that $x + y \leq x - y$. Give reason to support your answer by determining the value of y.

(3 marks)

(c) Given a series of integers.

(i) By using the principle of mathematical induction, prove that $\forall k \geq 1$,

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(ii) Then, determine the sum of $1^2 + 2^2 + 3^2 + ... + 50^2$.

(9 marks)

END OF QUESTION