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DISCRETE MATHEMATICS AND APPLICATIONS

Relations

Adam Shariff Adli Aminuddin (adamshariff@ump.edu.my)
Mohd Sham Mohamad (mohdsham@ump.edu.my)

Faculty of Industrial Sciences & Technology

Chapter Description

- Chapter outline

  2.8 Relations and Their Properties

  Reflexive

  Symmetric

  Transitive

- Aims
  - Determine whether a relation is reflexive, symmetric or transitive

Adam Shariff Adli Aminuddin
http://ocw.ump.edu.my/course/view.php?id=443

References

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Binary relation (i)

Definition: Let A and B be sets. A **binary relation**, R between A and B is a collection of ordered pairs of element in A and elements in B.

A binary relation is a subset of Cartesian product, $R \subseteq A \times B$.

The ordered pair, (a_n, b_n) is a subset of R , $(a_n, b_n) \subseteq R$.

If a binary relation of set A only, it is a subset of Cartesian product of $R \subseteq A \times A$.

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Binary relation (ii)

Example 1

Let $A=(a,b,c)$ and $B=(1,2)$.

The Cartesian product of $A\times B=\{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$

A binary collection, R can be represented as $R=\{(a,1),(b,2),(c,1),(c,2)\}$.

*Note that $R\subseteq A\times B$, thus it is not compulsory for R to contain all elements of $A\times B$.

*Note that a function can be represented as a relation.

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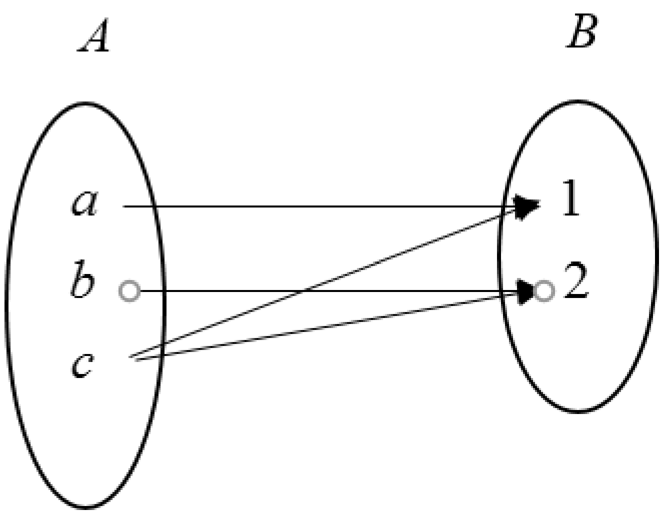
Binary relation (iii)

A relation can also be represented like a '*function*' or graphically. For this example

$R = \{(a,1),(b,2),(c,1),(c,2)\}$.



R	1	2
a	/	
b		/
c	/	/

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Binary relation (iv)

Let $A = \{1, 2, 3\}$. What are the ordered pairs such that $R = \{(a, b) \mid a \geq b\}$?

Answer:

This relation is from A to A.

The number elements in A is 3 and the total ordered pair is $2^3 = 8$.

All of the ordered pairs of $A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$.

However, the ordered pair which satisfy $a \geq b$ is

$R = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$.

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Properties of Relations

There are three basic types of binary relation properties which are:

1. Reflexive
2. Symmetric or antisymmetric
3. Transitive

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Reflexive (i)

Definition: A binary relation, R is **reflexive** if $(a,a) \in R, \forall a \in A$.
It is reflexive is there is an ordered pair of the same elements for all elements.

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Reflexive (ii)

Let $A = \{1, 2, 3\}$. Determine whether these relations are reflexive.

1. $R_1 = \{(1,1), (2,1), (1,2)\}$
2. $R_2 = \{(1,1), (1,2), (2,2), (3,2), (3,3)\}$
3. $R_3 = \{(3,1), (2,3), (2,2), (3,3), (1,1)\}$
4. $R_4 = \{(1,2), (2,1), (3,1), (1,3), (2,3), (3,2)\}$
5. $R_5 = \{(2,2), (3,3)\}$

R_1 is not reflexive as (2,2) and (3,3) do not exist.

R_2 is reflexive as (1,1), (2,2) and (3,3) exist.

R_3 is reflexive as (1,1), (2,2) and (3,3) exist.

Determine R_4 and R_5 on your own.

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Symmetric and Anti-symmetric (i)

Definition: A binary relation, R is **symmetric** if $(b,a) \in R$ if $\exists (a,b) \in R, \forall a,b \in A.$
It is symmetric if there is a symmetric (inverse) of elements for all ordered pairs.

Definition: A binary relation, R is **anti-symmetric** if $(b,a) \in R$ and $\exists (a,b) \in R$ only if $a = b.$ A relation can't be both symmetric and anti-symmetric.

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Symmetric and Anti-symmetric (ii)

Let $A = \{1, 2, 3\}$. Determine whether these relations are symmetric.

1. $R_1 = \{(1,1), (2,1), (1,2)\}$
2. $R_2 = \{(1,1), (1,2), (2,2), (3,2), (3,3)\}$
3. $R_3 = \{(3,1), (2,3), (2,2), (3,3), (1,1)\}$
4. $R_4 = \{(1,2), (2,1), (3,1), (1,3), (2,3), (3,2)\}$
5. $R_5 = \{(2,2), (3,3)\}$

R_1 is symmetric as (1,1) exist for itself and (2,1) exist for (1,2).

R_2 is not symmetric as (2,1) does not exist for (1,2), and (2,3) does not exist for (3,2)

R_3 is not symmetric as (1,3) does not exist for (3,1), and (3,2) does not exist for (2,3)

R_5 is anti-symmetric as (2,2) and (3,3) exist where $a = b$. .

Determine R_4 on your own.

Transitive (i)

Definition: A binary relation, R is **transitive** if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R, \ \forall a,b,c \in A$. It is transitive relations among two ordered pair such that $a \rightarrow b \rightarrow c$.

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Transitive (ii)

Let $A = \{1, 2, 3\}$. Determine whether these relations are transitive.

1. $R_1 = \{(1,1), (2,1), (1,2)\}$
2. $R_2 = \{((1,2), (2,3), (1,3), (3,3)\}$
3. $R_3 = \{(3,1), (2,3), (2,2), (3,3), (1,1)\}$
4. $R_4 = \{(1,2), (2,1), (3,1), (1,3), (2,3), (3,2)\}$
5. $R_5 = \{(2,2), (3,3)\}$

R_1 is not transitive as (2,1) and (1,2) exist, but (2,2) does not.

R_2 is transitive as (1,2) and (2,3) exist, thus (1,3) also exist.

R_3 is not transitive as (2,3) and (3,1) exist, but (2,1) does not.

Determine R_4 and R_5 on your own.

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