DISCRETE MATHEMATICS AND APPLICATIONS

Functions

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Chapter Description

- Chapter outline
  2.6 Introduction to Functions
  2.7 One-to-One and Onto Functions
- Aims
  - Identify a function and find the domain and range of a function and define a function as relation, find a binary relation from A to B and relations on a set
  - Identify a one-to-one & an onto function, bijection and find the inverse of a function

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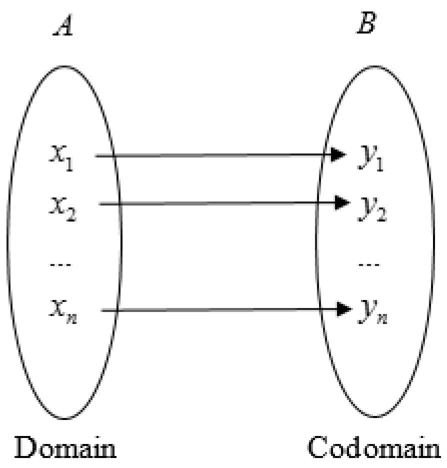
References

- Rosen K.H., Discrete Mathematics & Its Applications, (Seventh Edition), McGraw-Hill, 2011
- Epp S.S, Discrete Mathematics with Applications, (Fourth Edition), Thomson Learning, 2011
- Ram Rabu, Discrete Mathematics, Pearson, 2012
- Walls W.D., A beginner's guide to Discrete Mathematics, Springer, 2013
- Chandrasekaren, N. & Umaparvathi, M., Discrete Mathematics, PHI Learning Private Limited, Delhi, 2015

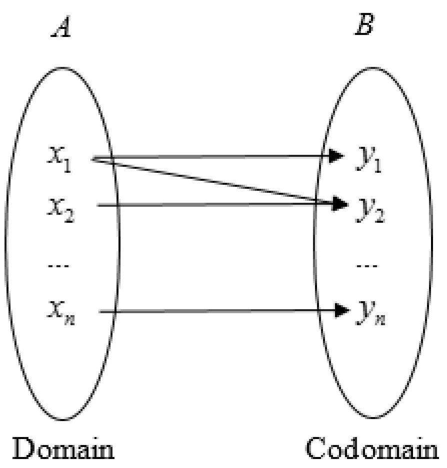
Functions (i)

Definition: Let *A* and *B* be sets. A **function**, f from *A* to *B*, $f : A \to B$ is the **assignment** of all elements in *A* to **exactly one element** in *B*.

The assignment of element $x \in A$ to $y \in B$, is denoted by $f(x) = y$.



A function as $x_1, x_2, ..., x_n$ is assigned exactly to one elements in *B*.



Not a function as $\exists x \in A$ which is assigned to more than one elements in *B*.

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Functions (ii)

Example 1

Let $f(x) = 2x + 1$. Determine the value of $f(1)$.

$f(1) = 2(1) + 1$

$f(1) = 3$

* Notice that $f(1)$ has only one value. It cannot be assigned to more than one values.

Example 2

Let $f(x) = x^2$. Determine the value of $f(2)$ and $f(-2)$

$f(2) = (2)^2$ $\qquad$ $f(2) = (-2)^2$

$f(2) = 4$ $\qquad$ $f(2) = 4$

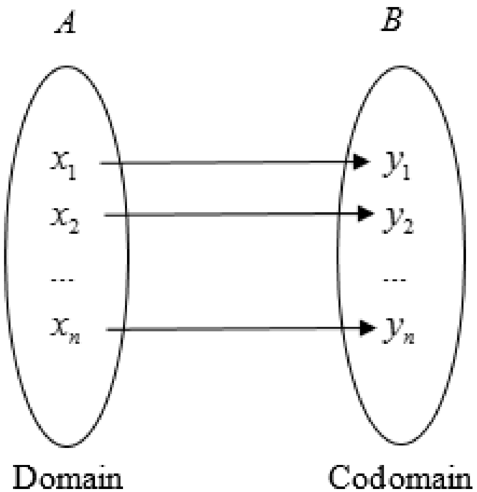
This function is valid, as $f(2)$ and $f(-2)$ is assigned to one value, although the value is the same.

Functions (iii)

Definition: If f is a function from A to B, $f: A \rightarrow B$, then A is the **domain** and B is the **codomain**

Definition: If $f(x) = y$, then x is the **preimage** and y is **the image**. f maps x to y.

Definition: The **range** if function, f is the set of all images of elements in A.



The $x_1, x_2, ..., x_n$ are the pre-images and $y_1, y_2, ..., y_n$ are the images.

The range is the set of assigned images which are $y_1, y_2, ..., y_n$.

Sum of Functions

Definition: Let f_1 and f_2 be functions. The sum of f_1 and f_2 is given by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x)$$

Example 1

Let $f_1 = 3x$ and $f_1 = -2x$

$$(f_1 + f_2)(x) = f_1(x) + f_2(x)$$

$$= 3x + (-2x)$$

$$= x$$

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Product of Functions

Definition: Let f_1 and f_2 be functions. The product of f_1 and f_2 is given by
$$(f_1 \cdot f_2)(x) = f_1(x) \cdot f_2(x)$$

Example 1

Let $f_1 = 3x$ and $f_1 = -2x$

$$(f_1 \cdot f_2)(x) = f_1(x) \cdot f_2(x)$$
$$= (3x)(-2x)$$
$$= -6x^2$$

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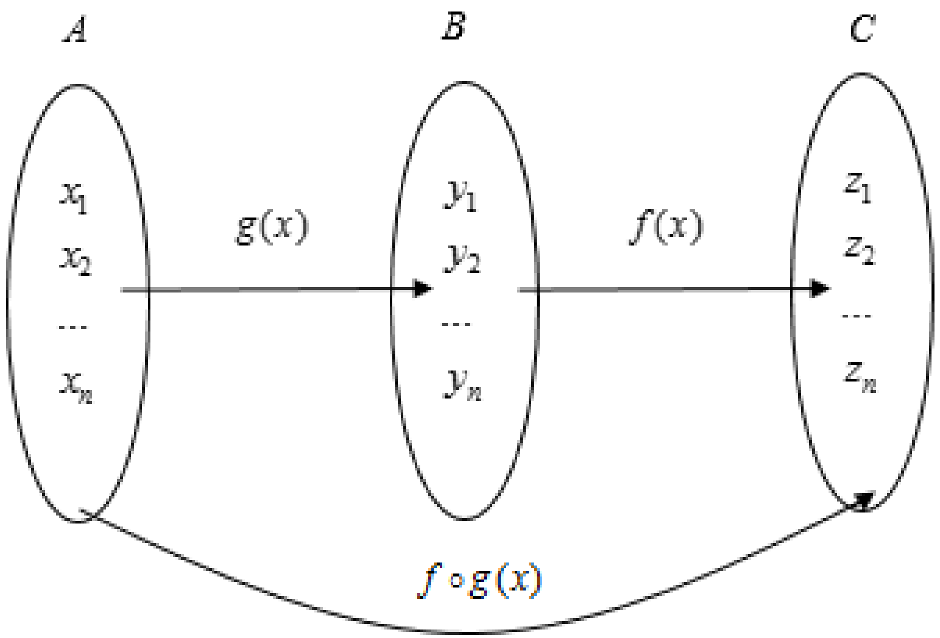
Composite Function (i)

Definition: Let f and g be functions. The composite function, $f \circ g$ is given by $(f \circ g)(x) = f(g(x))$

If $g(x) = y$, then

$$f \circ g = f(g(x))$$
$$f \circ g = f(y)$$
$$f \circ g = z$$



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Composite function (ii)

Example 1

Let $f(x) = 5x$ and $g(x) = 2x^2$. Determine $f \circ g(3)$.

$$(f \circ g)(x) = f(g(x))$$

$$= f(2x^2)$$

$$= 5(2x^2)$$

$$= 10x^2$$

$$(f \circ g)(3) = 10(3)^2$$

$$= 90$$

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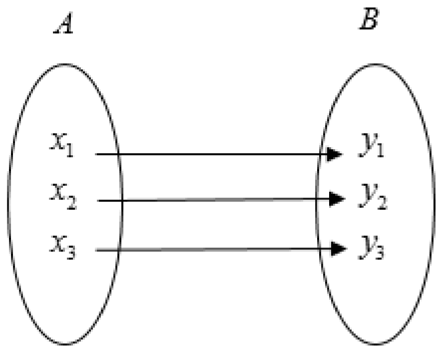
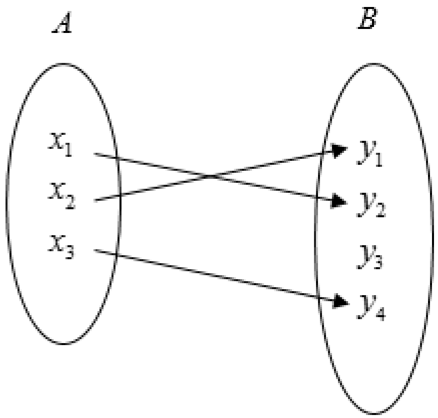
One to one function (i)

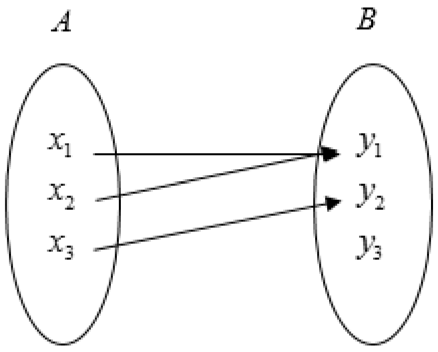
Definition: A function, f is one to one (injective) if and only if $f(x) = f(y)$ implies $x = y$, $\forall x, y$.



One to one



One to one



NOT One to one

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One to one function (ii)

Example 1

Suppose there are two sets $A = \{1,2,3\}$ and $B = \{a,b,c,d\}$. Let $f : A \rightarrow B$ such that $f(1) = a,\ f(2) = c,\ f(3) = b$. Is *f* one to one?

f is one to one as all element in *A* is assigned uniquely to one element in *B*, although element *d* is not assigned.

Example 2

Determine whether $f(x) = 2x + 2$ is one to one.

To show that *f* is one to one, we must use proving method to prove that all *x* is assigned uniquely.

Let $f(x) = y$

$$2x + 2 = y$$

$$x = \frac{y-2}{2}, \forall x \in X$$

Thus, *f* is one to one.

Relate it with CHAPTER 4: Proof Methods !!!

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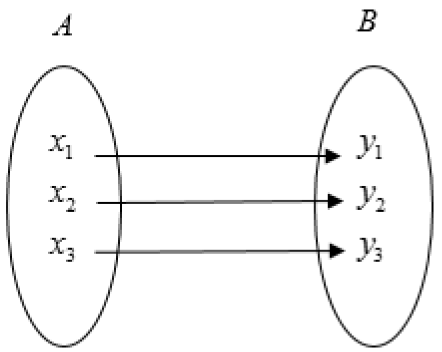
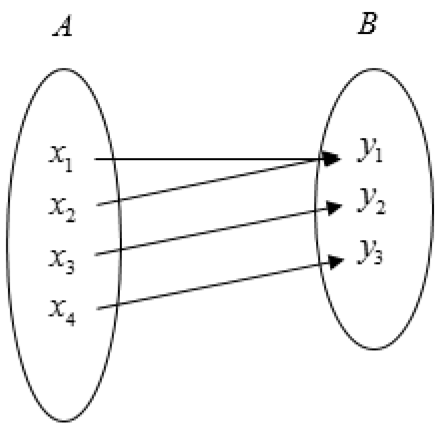
Onto function (i)

Definition: A function, f is **onto** (surjective) if and only if $\forall b \in B, \exists a \in A$ such that $f(a) = b$. All element of B is assigned by element from A .



Onto



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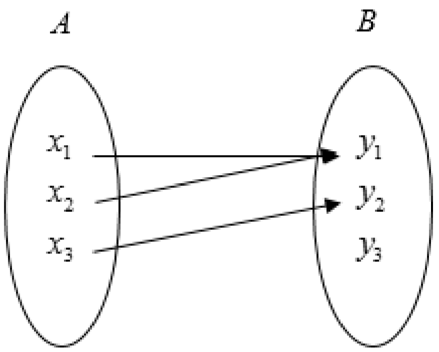

NOT Onto

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Onto function (ii)

Example 1

Suppose there are two sets $A=\{1,2,3\}$ and $B=\{a,b,c,d\}$. Let $f:A\rightarrow B$ such that $f(1)=a,\ f(2)=c,\ f(3)=b$. Is *f* onto?

Although *f* is one to one, but in this example, *f* is not onto because element $d\in B$ is not assigned by elements from *A*.

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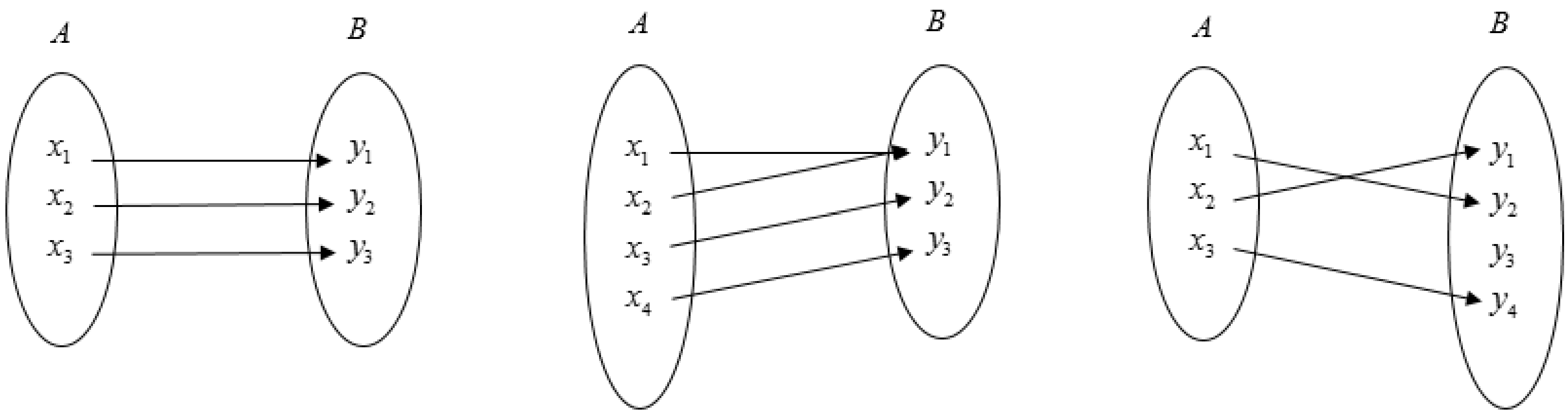
One to one and onto

Definition: A function, *f* is **one to one and onto** (bijective) iff *f* is both one to one and onto.

A function must satisfy both one to one and onto condition in order to be called bijective.



One to one and onto

NOT One to one
but Onto

One to one
but **NOT** Onto

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