DISCRETE MATHEMATICS AND APPLICATIONS

Number Theory 2

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Chapter Description

- Chapter outline

  1.4 Greatest Common Divisors (GCD)

  1.5 Least Common Multiples (LCM)

- Aims
  - Able to determine whether the set of integers are pairwise or relatively prime
  - Able to find GCD using prime factorization
  - Able to find the least common multiple of two integers

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References

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Greatest common divisor (GCD)

Definition 1.4 : Greatest common divisor

Let a, b, k and d be positive integers

- If $k \mid a$ and $k \mid b$, then k is the **common divisor** of a and b.
- If d is the largest k, d is the **greatest common divisor** (GCD) of a and b

$$d = \gcd(a, b).$$

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Relative prime

Definition 1.5 : Relative prime

Let a and b be positive integers.

If GCD(a,b)=1, then a and b are relatively prime

GCD using prime factorization

Theorem 1.4

If the prime factorizations of the integers *a* and *b* are:

$$a = p_1^{a_1} p_2^{a_2} \ldots p_n^{a_n} \text{ and } b = p_1^{b_1} p_2^{b_2} \ldots p_n^{b_n}$$

Then GCD (*a*, *b*) is given by:

$$\mathrm{GCD}(a,b) = p_1^{\min(a_1,b_1)} p_2^{\min(a_2,b_2)} \ldots p_n^{\min(a_n,b_n)}$$

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GCD using prime factorization (Example)

Determine GCD(8,20)

$8 = 2^3$

$20 = 2^2 \times 5$

GCD(8,24)$= 2^{\min(3,2)} \times 5^{\min(0,1)}$

$= 2^2 \times 5^0$

$= 4 \times 1$

$= 4$

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Least common multiple (LCM)

Definition 1.5 : Least common multiple

Let a, b, k and d be positive integers

- If $a \mid k$ and $b \mid k$, then k is the **common multiple** of a and b.
- If d is the smallest k, then d is the **least common divisor** (LCM) of a and b

$$d = \text{LCM}(a, b).$$

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LCM using prime factorization

Theorem 1.4

If the prime factorizations of the integers *a* and *b* are:

$$a = p_1^{a_1} p_2^{a_2} \ldots p_n^{a_n} \text{ and } b = p_1^{b_1} p_2^{b_2} \ldots p_n^{b_n}$$

Then LCM(*a*, *b*) is given by:

$$\mathrm{LCM}(a,b) = p_1^{\max(a_1,b_1)} p_2^{\max(a_2,b_2)} \ldots p_n^{\max(a_n,b_n)}$$

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LCM using prime factorization (Example)

Determine LCM(8,20)

$8 = 2^3$

$24 = 2^2 \times 5$

LCM(8,24)$= 2^{\max(3,2)} \times 5^{\max(0,1)}$

$= 2^3 \times 5^1$

$= 8 \times 5$

$= 40$

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Find LCM from the GCD

Theorem 1.5

If a and b are two positive integers, then

$$ab = GCD(a, b) \cdot LCM(a, b)$$

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Find LCM from the GCD: Example

Given GCD(8,20)=4, then determine LCM(8,20)

$$\begin{aligned} ab &= \mathrm{GCD}(a, b) \cdot \mathrm{LCM}(a, b) \\ 8 \cdot 20 &= 4 \cdot \mathrm{LCM}(a, b) \\ \mathrm{LCM}(a, b) &= 160/4 \\ &= 40 \end{aligned}$$

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