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DYNAMICS

Planar Kinematics of a Rigid Body (Relative Motion Analysis)

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Relative Motion Analysis

- Aims
  - To determine the velocity of a point on a rigid body
  - To determine the acceleration of a point on a rigid body

- Expected Outcomes
  - The students are able to calculate the velocity of a point on a rigid body.
  - The students are able to calculate the acceleration of a point on a rigid body.

- References
  - Engineering Mechanics: Dynamics 12th Edition, RC Hibbeler, Prentice Hall

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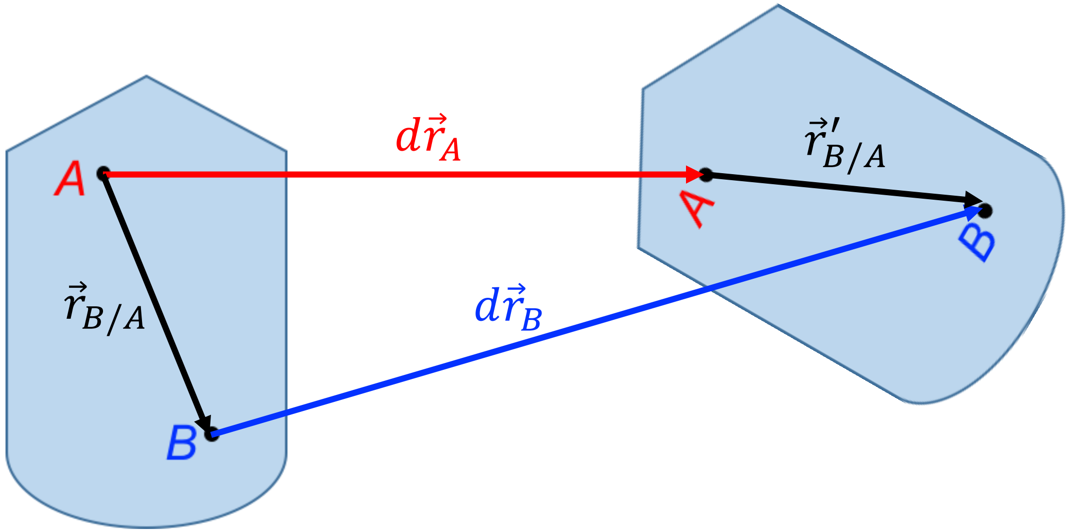
- Conclusion

General Plane Motion



Time t

Time $t + dt$

The rigid body has undergone both **TRANSLATION** and **ROTATION**

Relative Motion Analysis

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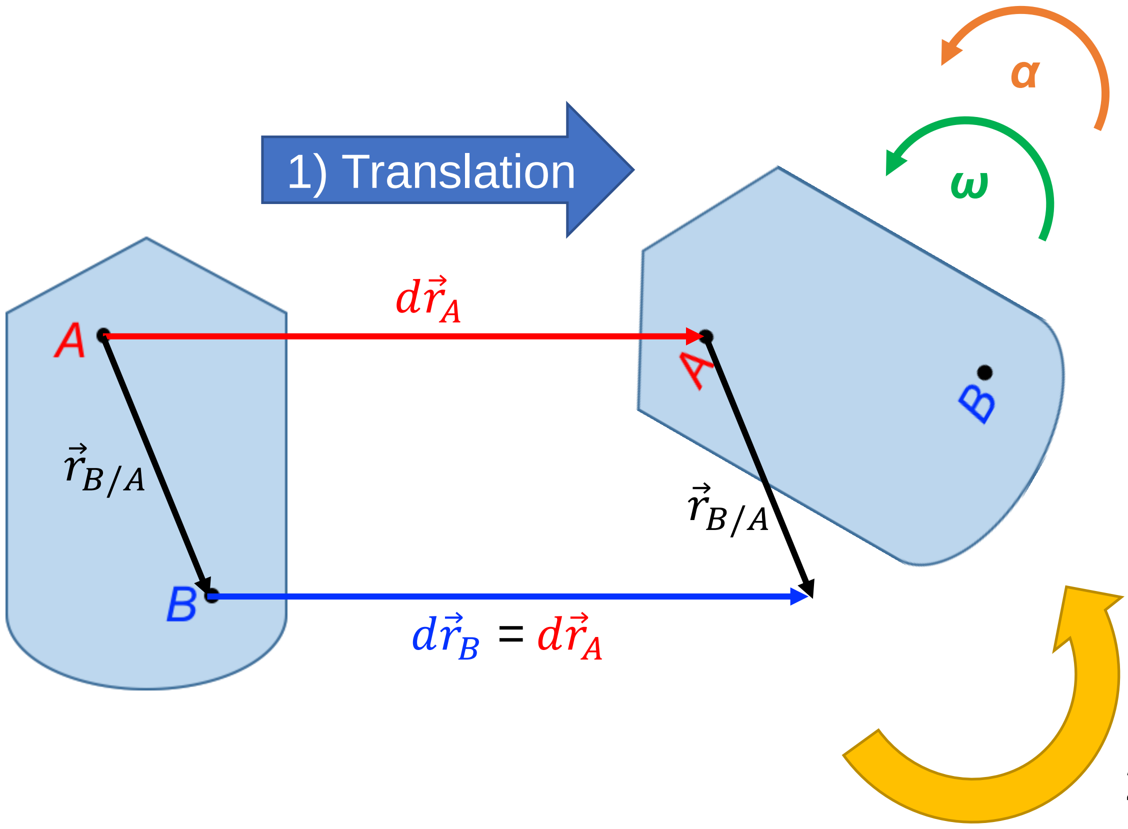


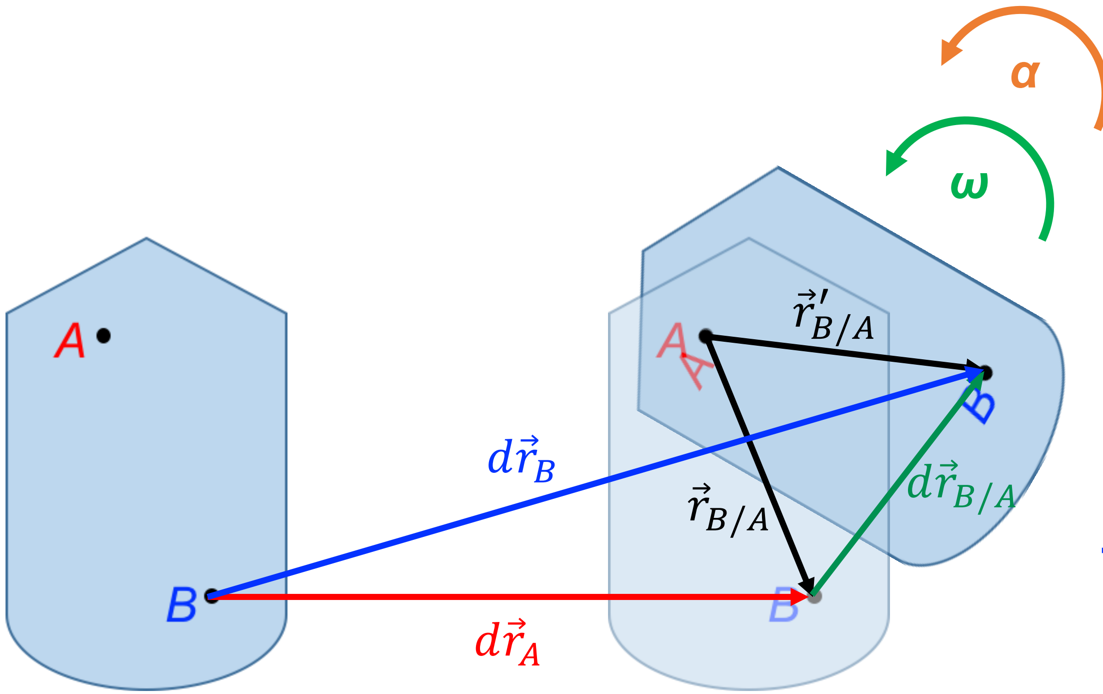
REMEMBER!

- **Counter-clockwise** rotation is **POSITIVE**
- **Clockwise** rotation is **NEGATIVE**

2) Rotation about a fixed axis (Point A)

The displacement of all **PARTICLES** in a rigid body during **TRANSLATION** is the **SAME**.

Relative Motion Analysis: Velocity



Linear velocities of any **TWO** arbitrary **PARTICLES** on a **rigid body** undergoing **General Plane Motion**

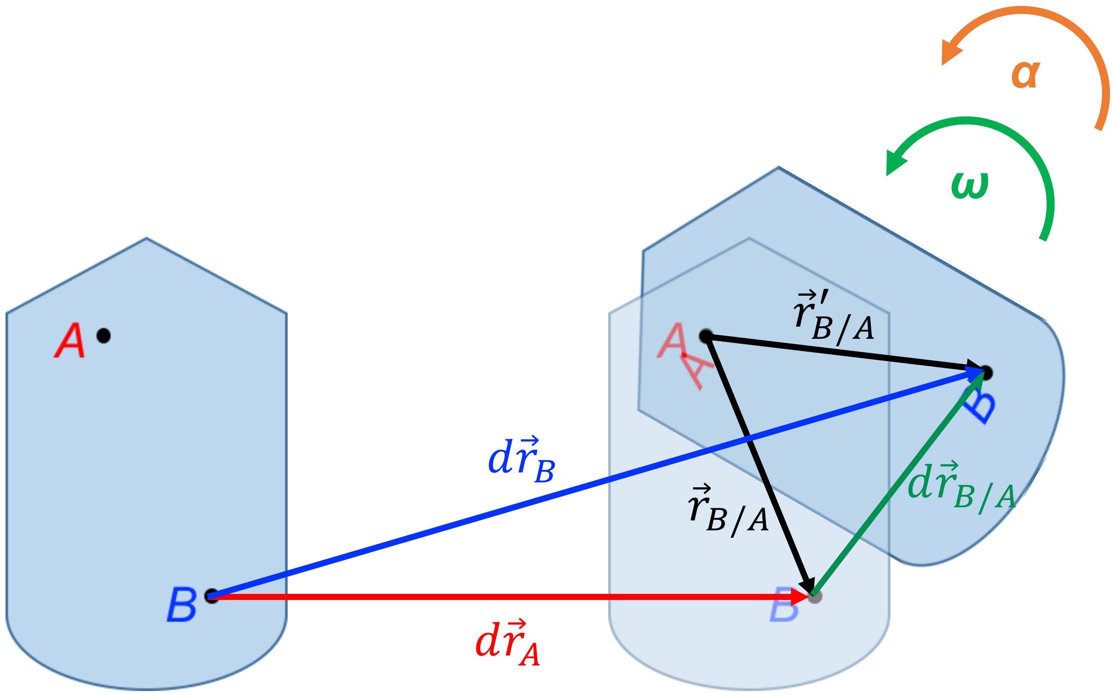
$$d\vec{r}_B = d\vec{r}_A + d\vec{r}_{B/A}$$

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_{B/A}}{dt}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

translation ($\vec{v}_A$) rotation ($\vec{v}_{B/A}$)

Relative Motion Analysis: Velocity

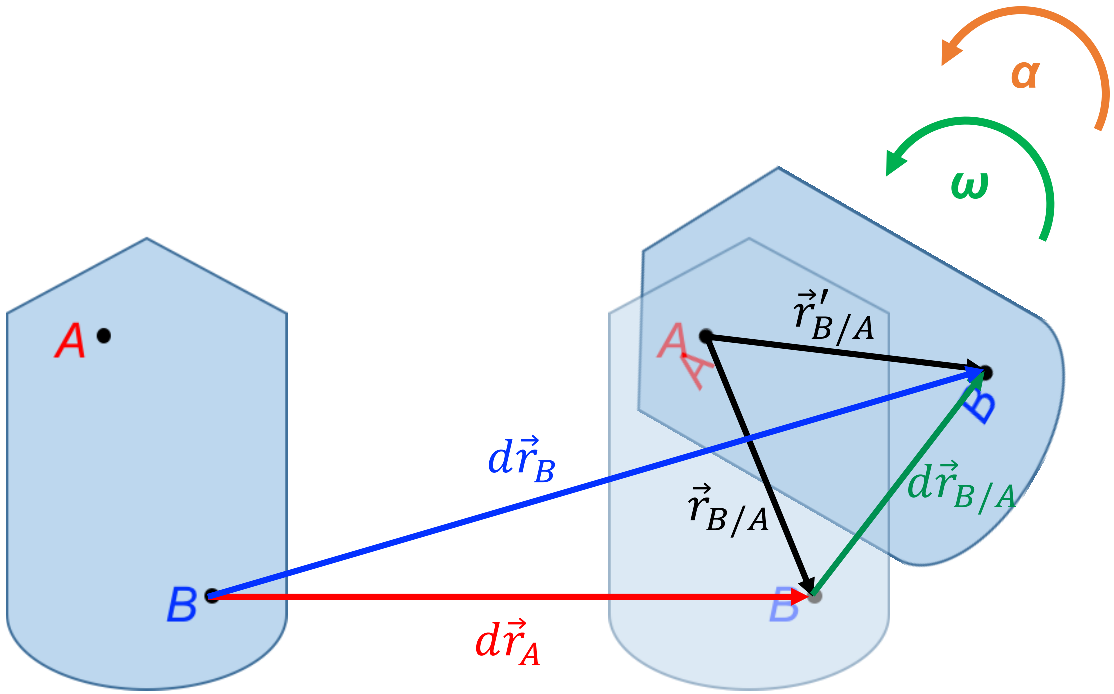


$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

Relative Motion Analysis: Acceleration



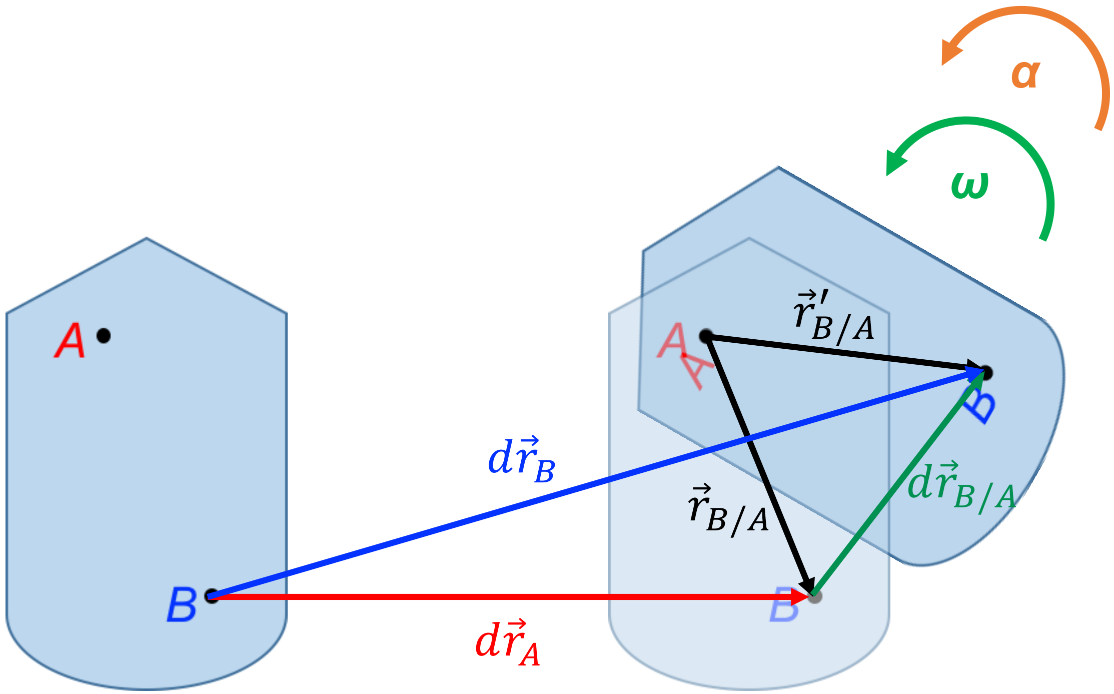
$$d\vec{r}_B = d\vec{r}_A + d\vec{r}_{B/A}$$

$$\frac{d^2\vec{r}_B}{dt^2} = \frac{d^2\vec{r}_A}{dt^2} + \frac{d^2\vec{r}_{B/A}}{dt^2}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

translation — rotation

Relative Motion Analysis: Acceleration



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_{B/A} = \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

tangential acceleration (for $\vec{\alpha} \times \vec{r}_{B/A}$) normal acceleration (for $\omega^2 \vec{r}_{B/A}$)

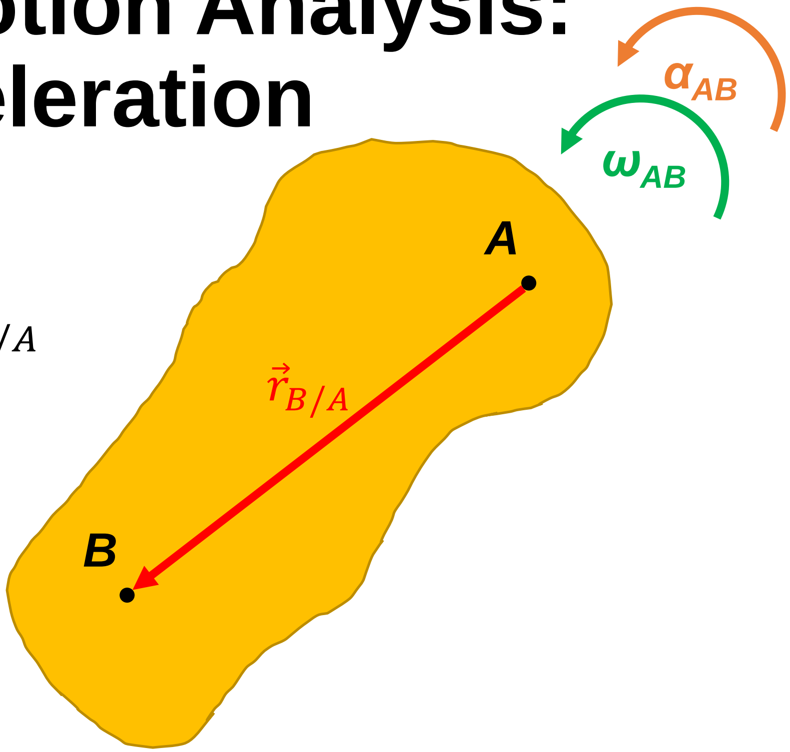
$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

Relative Motion Analysis: Acceleration

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

The **position vector** is **always** drawn **FROM** the **Reference Point**

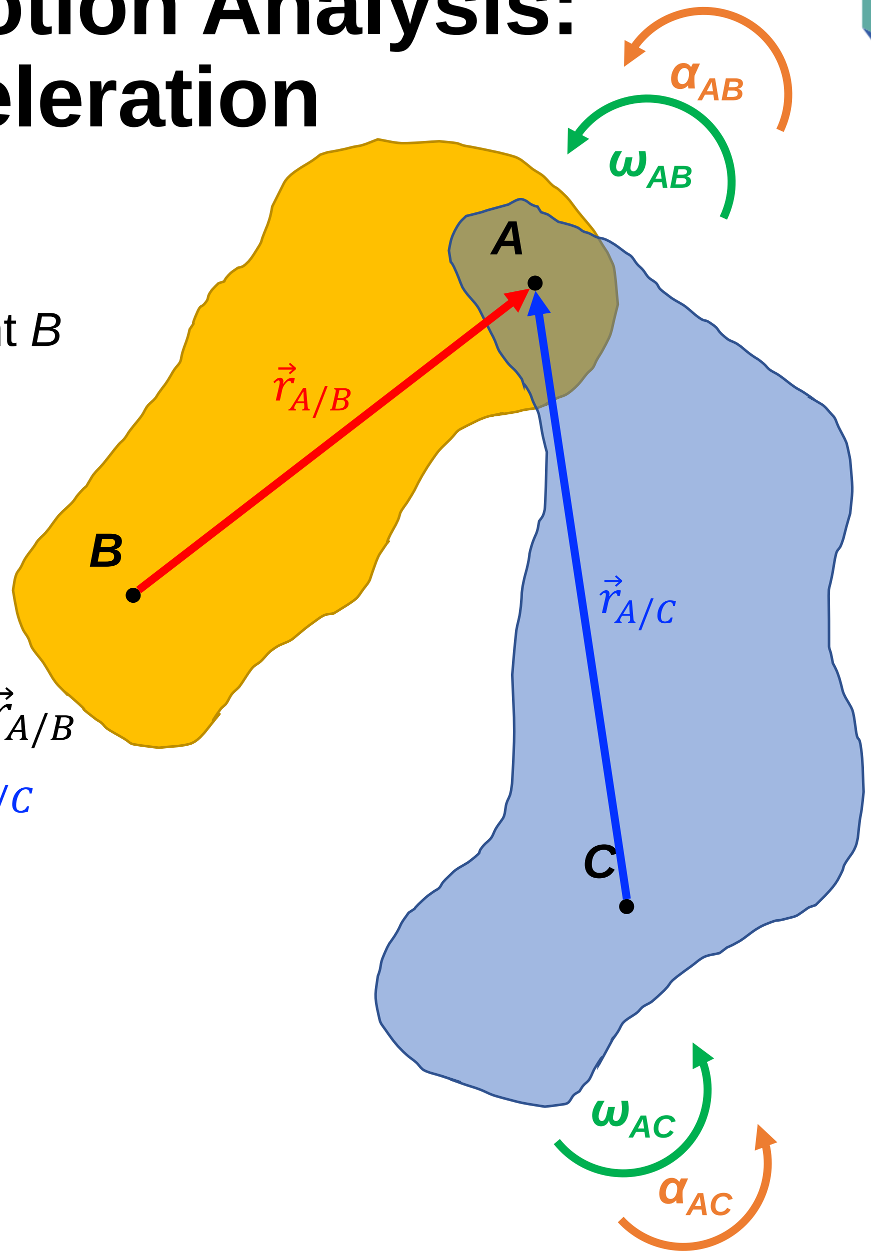


Relative Motion Analysis: Acceleration

To determine the velocity and accelaration of Point *A*, set Point *B* as the reference point.

$$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$$
$$= \vec{v}_C + \vec{\omega}_{AC} \times \vec{r}_{A/C}$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \times \vec{r}_{A/B} - \omega_{AB}^2 \vec{r}_{A/B}$$
$$= \vec{a}_C + \vec{\alpha}_{AC} \times \vec{r}_{A/C} - \omega_{AC}^2 \vec{r}_{A/C}$$



In the case of joints, **Point *A*** not only belongs to the Rigid Body *AB* but also to the Rigid Body *AC*

Conclusions

- General plane motion is the combination of translation and rotation.
- Counter-clockwise rotation is always taken as positive.
- Relative velocity and acceleration of a point on a rigid body undergoing general plane motion comprise of translation term and rotation term.

Planar Kinematics of a Rigid Body (Relative Motion Analysis)

"What we know is a drop, what we don't know is an ocean."

– *Sir Isaac Newton*