CHAPTER 4

Control Charts for Variables

Expected Outcomes

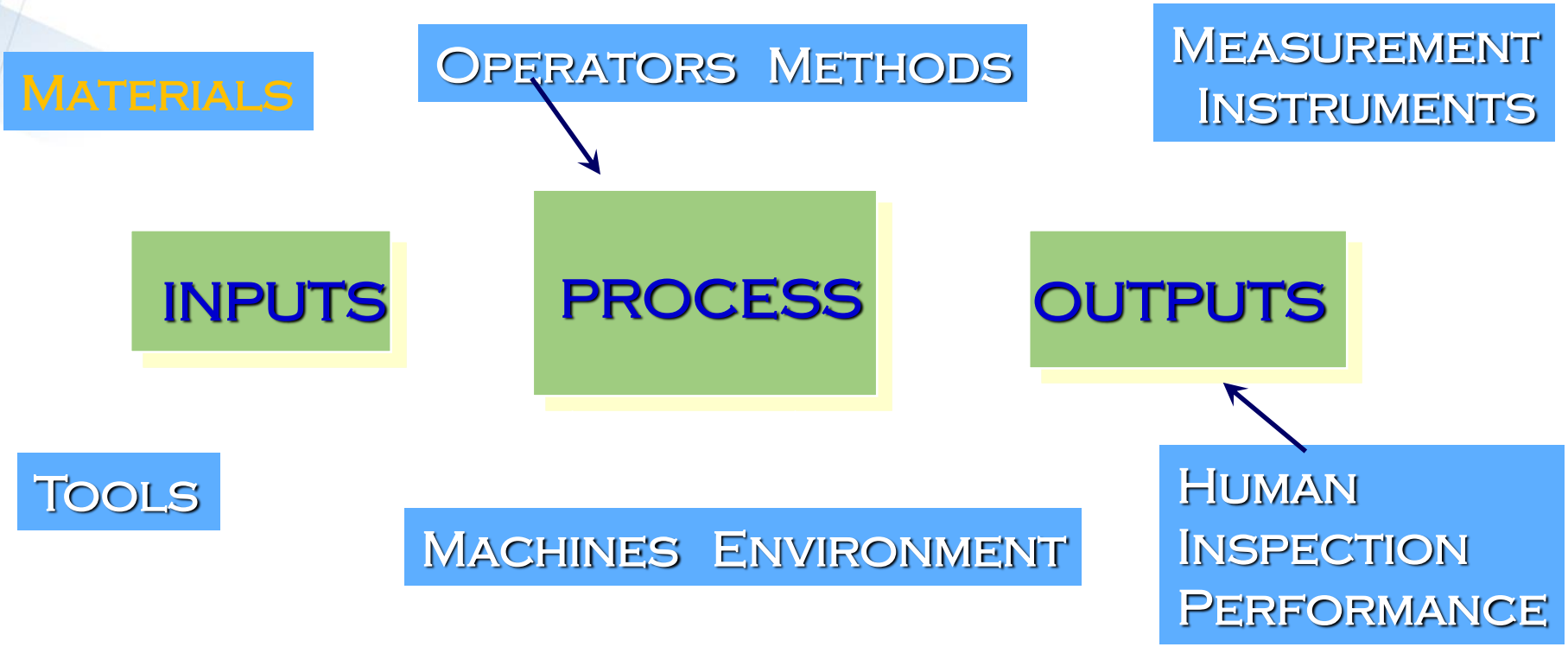
Know the three categories of variation and their sources.
Understand the concept of the control chart method.
Know the purpose of variable control charts.
Know how to select the quality characteristics, the rational subgroup and the method of taking samples
Be able to calculate the central value, trial control limits and the revised control limits for X bar and R chart.
Be able to explain what is meant by a process in control and the various out-of-control patterns.
Know the difference between individual measurements and averages; control limits and specifications.

Variation

- The variation concept is a law of nature in that no two natural items are the same.
- The variation may be quite large and easily noticeable
- The variation may be very small. It may appear that items are identical; however, precision instruments will show difference
- The ability to measure variation is necessary before it can be controlled

Variation

Sources of Variation in production processes:



Variation

Sources of variation are:

1. Equipment:
    i. Toolwear
    ii. Machine vibration
    iii. Electrical fluctuations etc.
2. Material
    i. Tensile strength
    ii. Ductility
    iii. Thickness
    iv. Porosity etc.

Control Charts

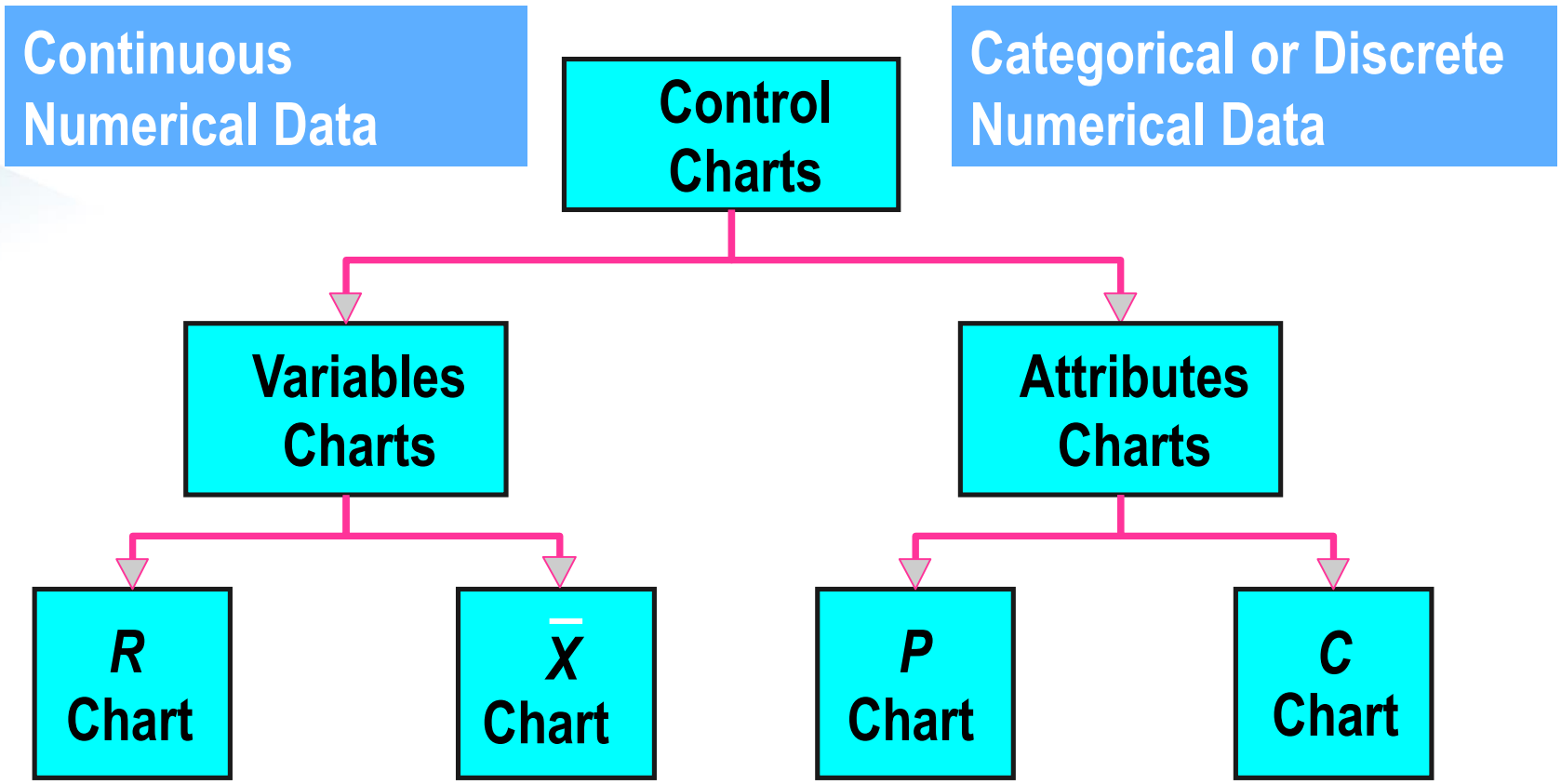
Variable data

- x-bar and R-charts
- x-bar and s-charts
- Charts for individuals (x-charts)

Attribute data

- For "defectives" (p-chart, np-chart)
- For "defects" (c-chart, u-chart)

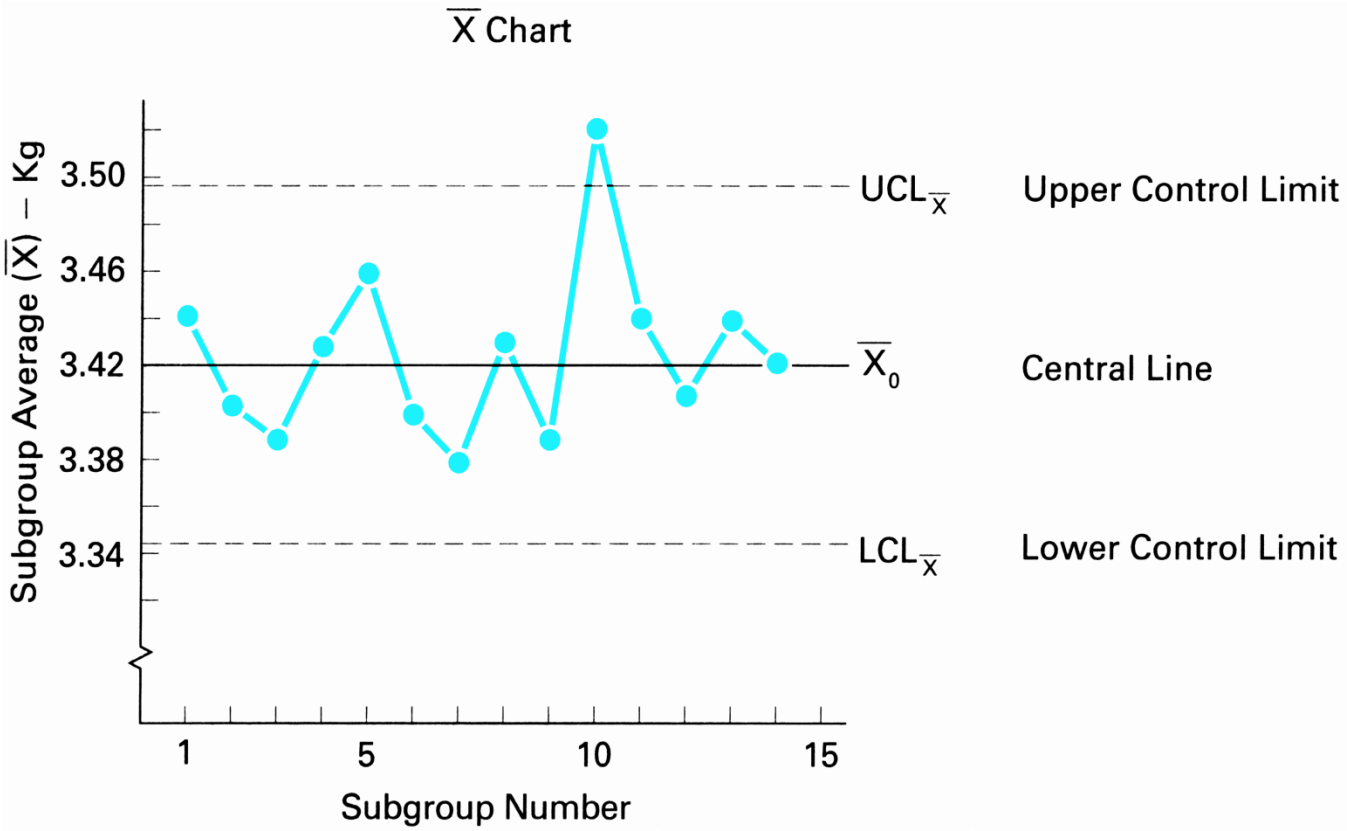
Control Charts



Control Charts for Variables

The control chart for variables is a means of visualizing the variations that occur in the central tendency and the mean of a set of observations. It shows whether or not a process is in a stable state.

Control Charts



Variable Control Charts

The objectives of the variable control charts are:

- For quality improvement
- To determine the process capability
- For decisions regarding product specifications
- For current decisions on the production process
- For current decisions on recently produced items

Control Chart Techniques

Procedure for establishing a pair of control charts for the average Xbar and the range R:

- Select the quality characteristic
- Choose the rational subgroup
- Collect the data
- Determine the trial center line and control limits
- Establish the revised central line and control limits
- Achieve the objective

Subgroup Size

- As the subgroup size increases, the control limits become closer to the central value, which make the control chart more sensitive to small variations in the process average
- As the subgroup size increases, the inspection cost per subgroup increases
- When destructive testing is used and the item is expensive, a small subgroup size is required

Subgroup Size

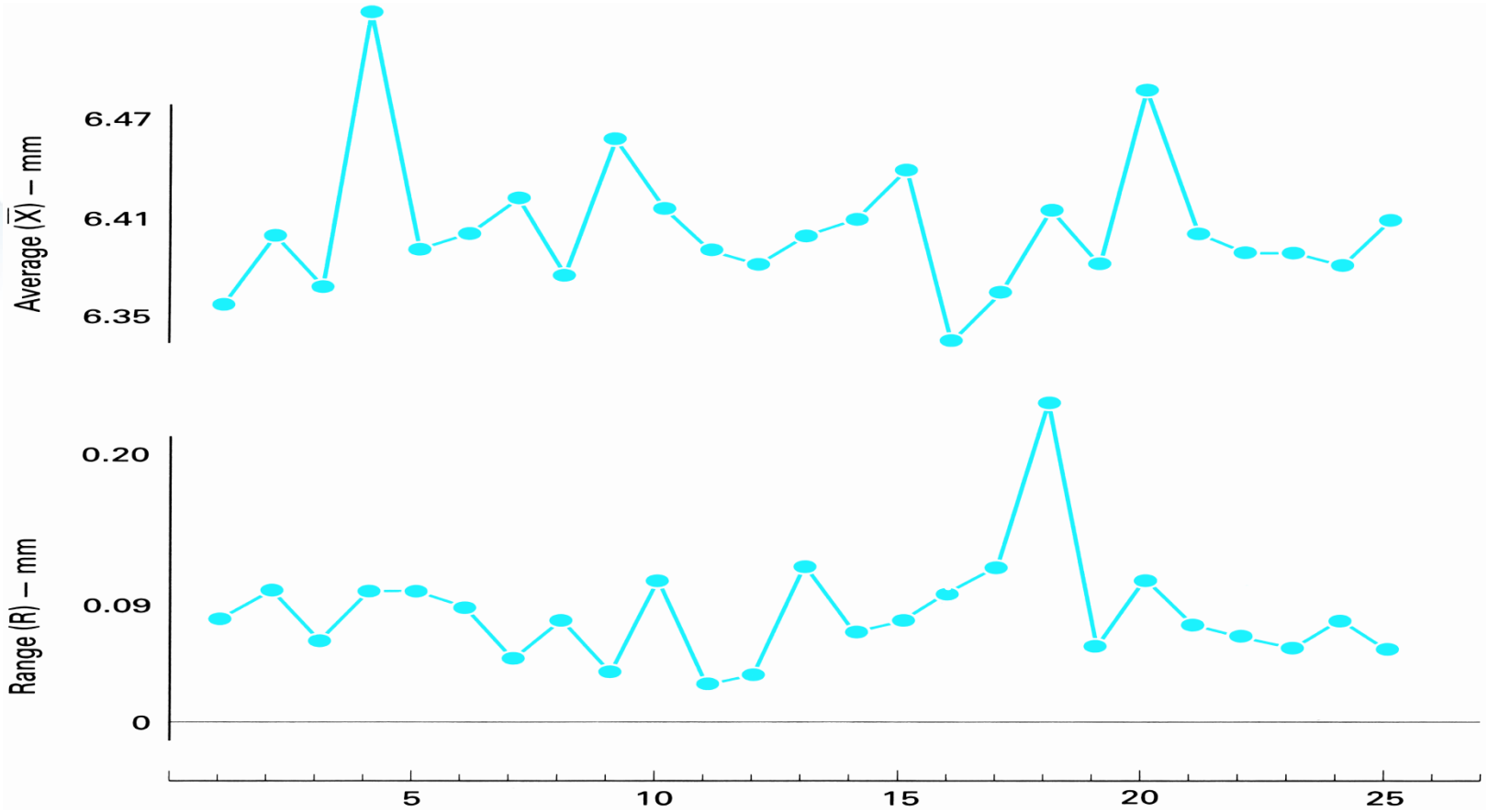
- From a statistical basis a distribution of subgroup averages are nearly normal for groups of 4 or more even when samples are taken from a non-normal distribution
- When a subgroup size of 10 or more is used, the s chart should be used instead of the R chart.

Data Collection

It is necessary to collect a minimum of 25 subgroups of data.

A run chart can be used to analyze the data in the development stage of a product or prior to a state of statistical control

Run Chart



Trial Central Lines

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{g} \overline{X_i}}{g} \quad \text{and} \quad \overline{R} = \frac{\sum_{i=1}^{g} R_i}{g}$$

where

$\overline{\overline{X}}$ = average of subgroup averages

$\overline{X_i}$ = average of the ith subgroup

g = number of subgroups

$\overline{R}$ = average of subgroup ranges

R_i = range of the ith subgroup

Trial Control Limits

Trial control limits are established at ±3 standard deviatons from the central value

$$UCL_{\overline{X}} = \overline{\overline{X}} + 3\sigma_{\overline{X}} \qquad UCL_R = \overline{R} + 3\sigma_R$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - 3\sigma_{\overline{X}} \qquad LCL_R = \overline{R} - 3\sigma_R$$

where

UCL=upper control limit

LCL=lower control limit

$\sigma_{\overline{X}}$ = *population standard deviation of the subgroup averages*

σ_R = *population standard deviation of the range*

Trial Control Limits

In practice calculations are simplified by using the following equations where A_2, D_3 and D_4 are factors that vary with the subgroupsize and are found in Table B of the Appendix.

$$UCL_{\bar{X}} = \overline{\overline{X}} + A_2\overline{R} \qquad UCL_R = D_4\overline{R}$$

$$LCL_{\bar{X}} = \overline{\overline{X}} - A_2\overline{R} \qquad LCL_R = D_3\overline{R}$$

Revised Central Lines

$$\overline{\overline{X}}_{new} = \frac{\sum \overline{X} - \overline{X}_d}{g - g_d} \quad and \quad \overline{R}_{new} = \frac{\sum R - R_d}{g - g_d}$$

where

$\overline{X}_d$ = *discarded subgroup averages*

g_d = *number of discarded subgroups*

R_d = *discarded subgroup ranges*

Standard Values

$$\overline{X}_0 = \overline{\overline{X}}_{new} \qquad R_0 = \overline{R}_{new} \qquad and \qquad \sigma_0 = \frac{R_0}{d_2}$$

$$UCL_{\overline{X}} = \overline{X}_0 + A\sigma_0 \qquad UCL_R = D_2\sigma_0$$

$$LCL_{\overline{X}} = \overline{X}_0 - A\sigma_0 \qquad LCL_R = D_1\sigma_0$$

Revised Central Lines

$$\overline{\overline{X}}_{new} = \frac{\sum \overline{X} - \overline{X}_d}{g - g_d} \quad and \quad \overline{R}_{new} = \frac{\sum R - R_d}{g - g_d}$$

where

$\overline{\overline{X}}_d$ = *discarded subgroup averages*

g_d = *number of discarded subgroups*

R_d = *discarded subgroup ranges*

Sample Standard Deviation Control Chart

For subgroup sizes >=10, an s chart is more accurate than an R Chart. Trial control limits are given by:

$$\bar{s} = \frac{\sum_{i=1}^{g} \bar{s}_i}{g} \qquad \bar{\bar{X}} = \frac{\sum_{i=1}^{g} \bar{X}_i}{g}$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_3 \bar{s} \qquad UCL_s = B_4 \bar{s}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_3 \bar{s} \qquad LCL_s = B_3 \bar{s}$$

Revised Limits for s chart

$$\overline{\overline{X}}_0 = \overline{\overline{X}}_{new} = \frac{\sum \overline{X} - \overline{X}_d}{g - g_d}$$

$$s_0 = \overline{s}_{new} = \frac{\sum s - s_d}{g - g_d} \qquad \sigma_0 = \frac{s_0}{c_4}$$

$$UCL_{\overline{X}} = \overline{\overline{X}}_0 + A\sigma_0 \qquad UCL_s = B_6\sigma_0$$

$$LCL_{\overline{X}} = \overline{\overline{X}}_0 - A\sigma_0 \qquad LCL_s = B_5\sigma_0$$

where

s_d = *discarded subgroup averages*

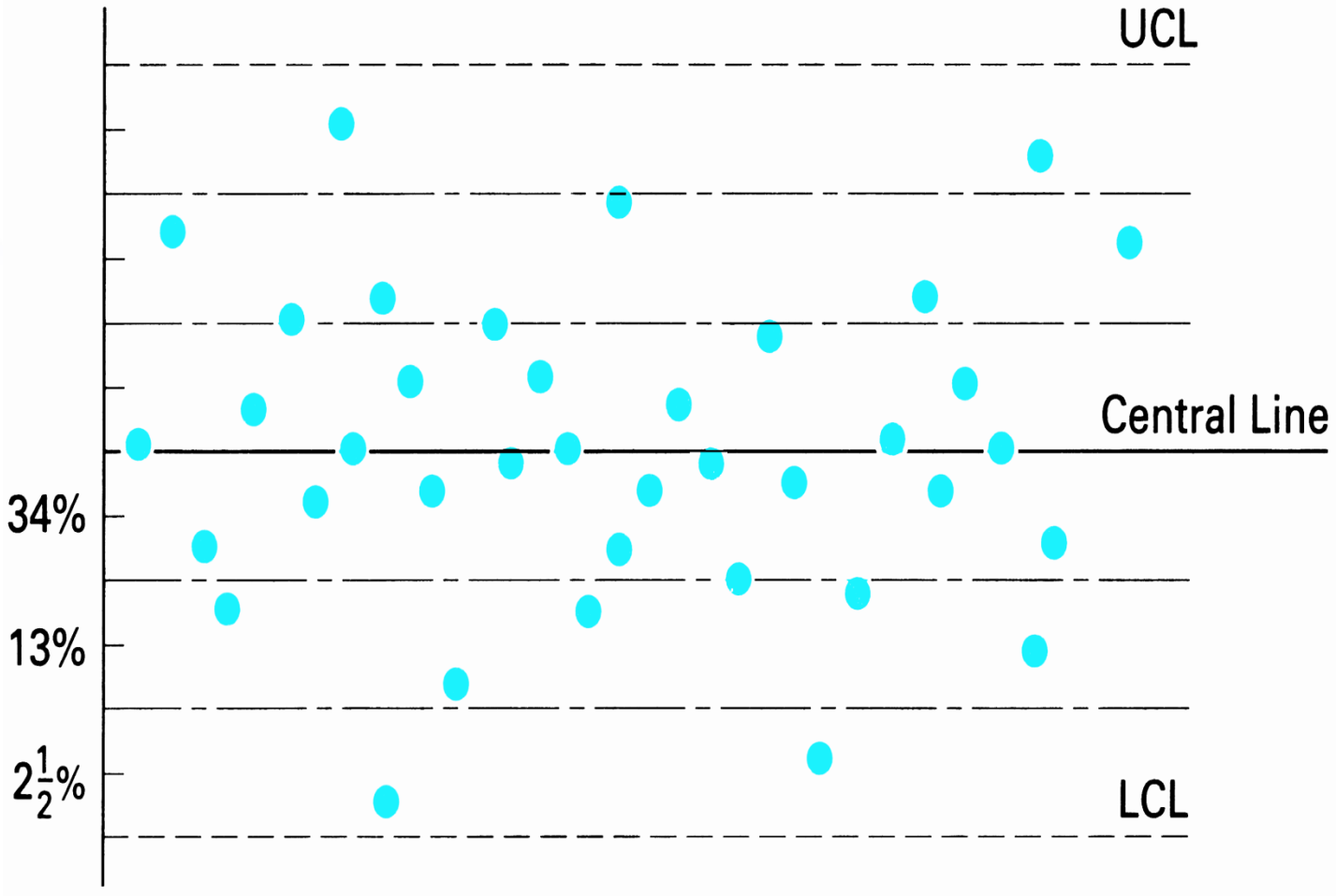
c_4, A, B_5, B_6 = *factors found in Table B*

State of Control

Process in Control

- When special causes have been eliminated from the process to the extent that the points plotted on the control chart remain within the control limits, the process is in a state of control
- When a process is in control, there occurs a natural pattern of variation

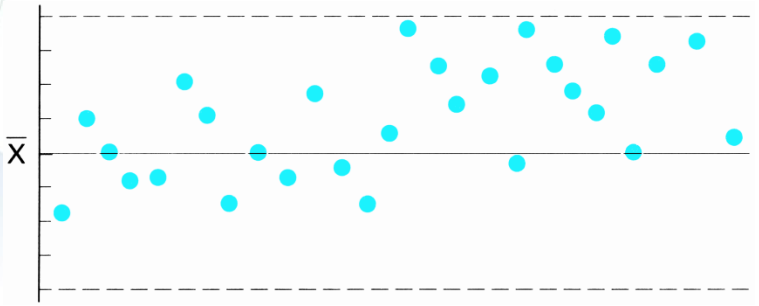
State of Control



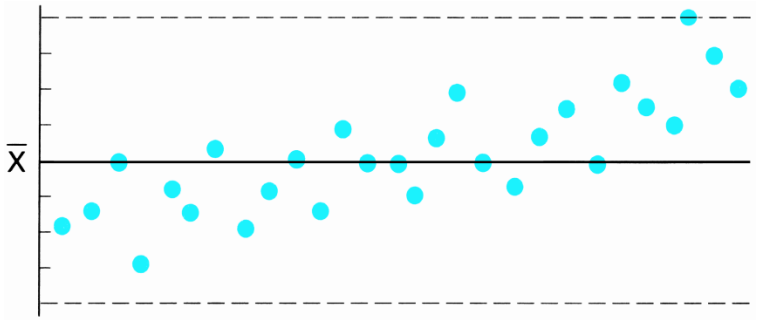
Out-of-Control Condition

1. Change or jump in level.
2. Trend or steady change in level
3. Recurring cycles
4. Two populations (also called mixture)
5. Mistakes

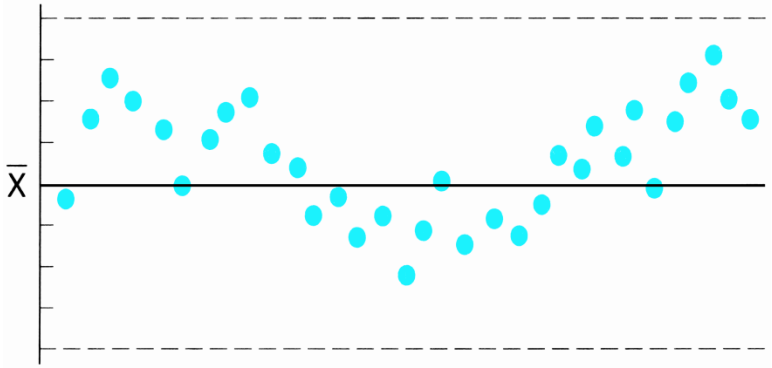
Out-of-Control Patterns



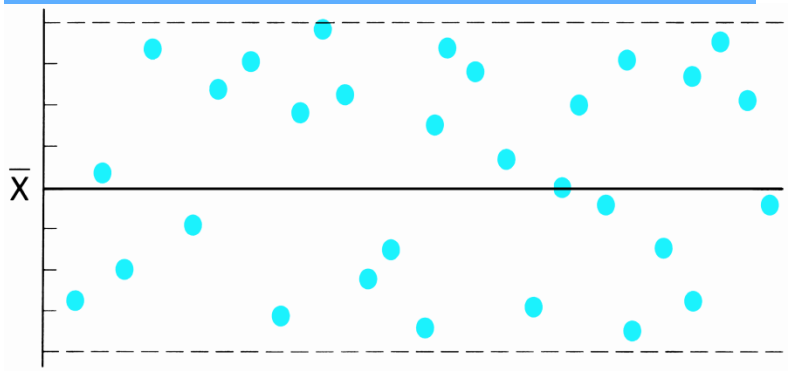
Change or jump in level



Trend or steady change in level



Recurring cycles



Two populations