BEE2143 – Signals & Networks

Chapter 9 – Filters

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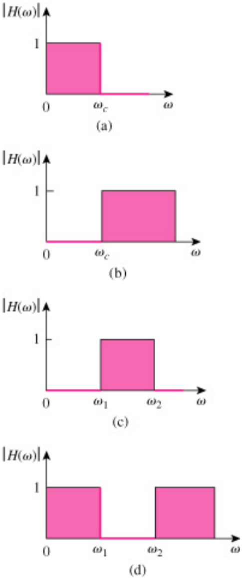
Introduction

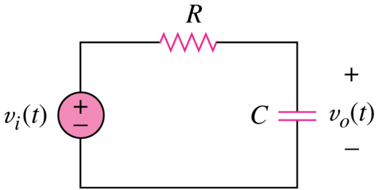
- A filter is a circuit that is designed to pass signals with desired frequency and reject others.
- Passive filters: consists only passive elements (R, L and C)
- Active filters: consists of active elements (transistors, op-amps, etc.)

Types of filters

1. Lowpass filter: passes LF and rejects HF
2. Highpass filter: passes HF and rejects LF
3. Bandpass filter: passes frequencies within a frequency band and blocks frequencies outside the band
4. Bandstop filter: passes frequencies outside a frequency band and blocks frequencies within the band


|H(ω)|
1
0
ω_c
ω
(a)
|H(ω)|
1
0
ω_c
ω
(b)
|H(ω)|
1
0
ω_1
ω_2
ω
(c)
|H(ω)|
1
0
ω_1
ω_2
ω
(d)

Typical lowpass filter



The transfer function is

$$\mathbf{H}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sRC}$$

or

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

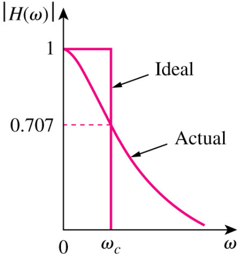
Typical lowpass filter (cont.)

The magnitude of $\mathbf{H}(\omega)$ is

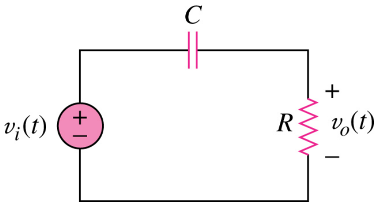
$$H(\omega) = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

The value of $H(\omega)$ at zero and large frequencies are respectively

$$H(0) = 1 \qquad \text{and} \qquad H(\infty) = 0$$



Typical highpass filter



The transfer function is

$$\mathbf{H}(s) = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{1 + sRC}$$

or

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

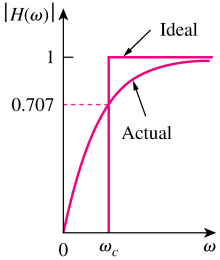
Typical highpass filter (cont.)

The magnitude of $\mathbf{H}(\omega)$ is

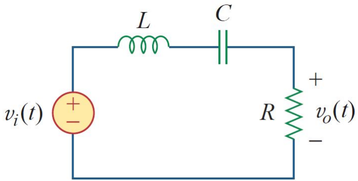
$$H(\omega) = \frac{\omega RC}{\sqrt{1 + \omega^2 R^2 C^2}}$$

The value of $H(\omega)$ at zero and large frequencies are respectively

$$H(0) = 0 \qquad \text{and} \qquad H(\infty) = 1$$



Typical bandpass filter



The transfer function is

$$\mathbf{H}(s) = \frac{R}{R + sL + \frac{1}{sC}}$$

or

$$\mathbf{H}(\omega) = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

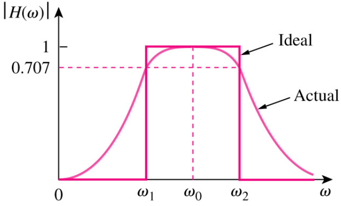
Typical bandpass filter (cont.)

The magnitude of $\mathbf{H}(\omega)$ is

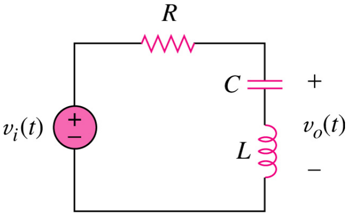
$$H(\omega) = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

The value of $H(\omega)$ at zero and large frequencies are respectively

$$H(0) = 0 \qquad \text{and} \qquad H(\infty) = 0$$



Typical bandstop filter



The transfer function is

$$\mathbf{H}(s) = \frac{sL + \frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

or

$$\mathbf{H}(\omega) = \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

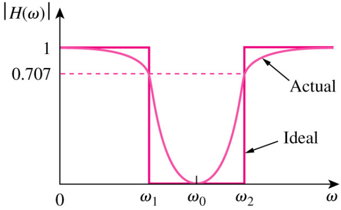
Typical bandstop filter (cont.)

The magnitude of $\mathbf{H}(\omega)$ is

$$H(\omega) = \frac{\left|\omega L - \frac{1}{\omega C}\right|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

The value of $H(\omega)$ at zero and large frequencies are respectively

$$H(0) = 1 \qquad \text{and} \qquad H(\infty) = 1$$



Decibel scale

- Power gain is defined as

$$\text{Gain} = \frac{P_2}{P_1} = \frac{V_2^2}{V_1^2} = \left(\frac{V_2}{V_1}\right)^2 = H^2$$

- In bels,

$$\begin{aligned}\text{Gain} =& \log \frac{P_2}{P_1} \text{ bel} \\ =& 10 \log \frac{P_2}{P_1} \text{ decibel (dB)}\end{aligned}$$

- Substituting $P_1 = V_1^2$ and $P_2 = V_2^2$ in the last equation gives

$$
\begin{aligned}
\text{Gain } =& 10\log\left(\frac{V_2}{V_1}\right)^2 \text{ dB} \\
=& 20\log\frac{V_2}{V_1} \text{ dB} \\
=& 20\log H \text{ dB}
\end{aligned}
$$

The cutoff frequency

- The cutoff frequency (also known as the half power frequency) is the frequency which the power gain is half of the maximum gain
- The cutoff frequency occurs when

$$\text{Gain} = \frac{\text{Gain}_{\text{max}}}{2}$$

$$H^2 = \frac{H_{\text{max}}^2}{2}$$

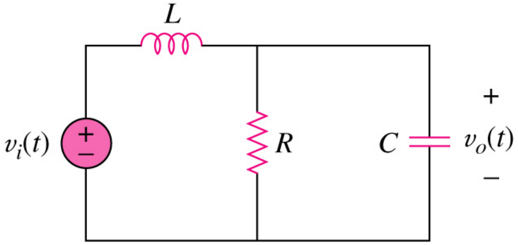
$$H = \frac{H_{\text{max}}}{\sqrt{2}}$$

- Or in dB, the cutoff frequency occurs when

$$
\begin{aligned}
20\log H =& 20\log\left(\frac{H_{\max}}{\sqrt{2}}\right) \\
=& 20\log H_{\max} - 20\log\sqrt{2} \\
H_{\mathrm{dB}} \approx& H_{\max(\mathrm{dB})} - 3\ \mathrm{dB}
\end{aligned}
$$

Example 14.10, pg. 640 (Alexander & Sadiku, 2009)

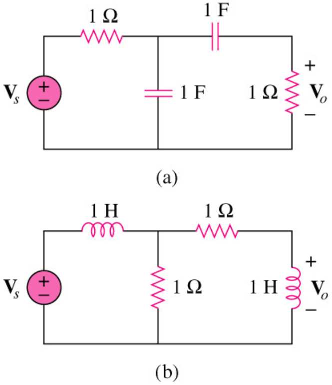
Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take $R = 2\ \text{k}\Omega$, $L = 2\ \text{H}$ and $C = 2\ \mu\text{F}$.



Answer: Lowpass filter, $\omega_c = 742$ rad/s.

Problem 14.57, pg. 669 (Alexander & Sadiku, 2009)

Determine the center frequency and bandwidth of the bandpass filters in Fig. 14.88.



Answer: 1 rad/s, 3 rad/s

Frequency response: Bode plot

- A transfer function can be expressed as the combination of factors such that

$$\mathbf{H}(s) = \frac{As(s+z_1)(s+z_2)\ldots(s^2+2\zeta_1\omega_k s+\omega_k^2)\ldots}{(s+p_1)(s+p_2)\ldots(s^2+2\zeta_2\omega_n s+\omega_n^2)\ldots}$$

$$\mathbf{H}(\omega) = \frac{Aj\omega(j\omega+z_1)(j\omega+z_2)\ldots((j\omega)^2+j2\zeta_1\omega_k\omega+\omega_k^2)\ldots}{(j\omega+p_1)(j\omega+p_2)\ldots((j\omega)^2+j2\zeta_2\omega_n\omega+\omega_n^2)\ldots}$$

- The magnitude of $\mathbf{H}(\omega)$ is

$$H(\omega) = \frac{A|\omega||j\omega+z_1||j\omega+z_2|\ldots|(j\omega)^2+j2\zeta_1\omega_k\omega+\omega_k^2|\ldots}{|j\omega+p_1||j\omega+p_2|\ldots|(j\omega)^2+j2\zeta_2\omega_n\omega+\omega_n^2|\ldots}$$

- or

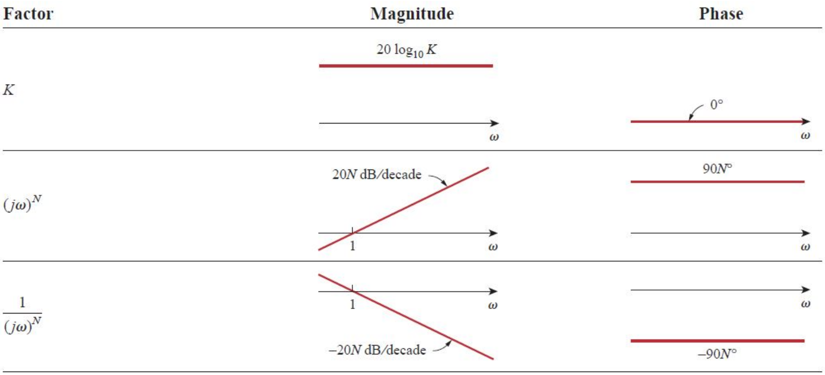
$$H(\omega) = \frac{K|\omega|\left|1+\frac{j\omega}{z_1}\right|\left|1+\frac{j\omega}{z_2}\right|\dots\left|1+\frac{j2\zeta_1\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2\right|\dots}{\left|1+\frac{j\omega}{p_1}\right|\left|1+\frac{j\omega}{p_2}\right|\dots\left|1+\frac{j2\zeta_2\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right|\dots}$$

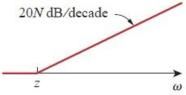
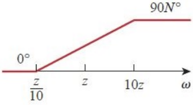
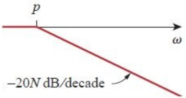
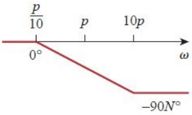
- or in dB

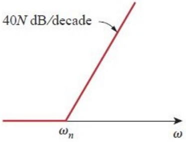
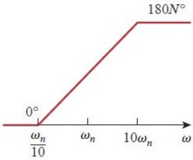
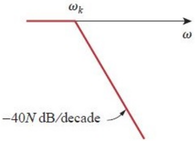
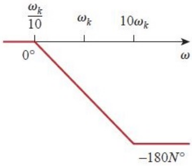
$$\begin{aligned} H_{\mathrm{dB}}(\omega) = {} & 20\log K + 20\log|\omega| + 20\log\left|1+\frac{j\omega}{z_1}\right| + \dots \\ & + 20\log\left|1+\frac{j2\zeta_1\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2\right|\dots - 20\log\left|1+\frac{j\omega}{p_1}\right| - \\ & - \dots - 20\log\left|1+\frac{j2\zeta_2\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right|\dots \end{aligned}$$

- and the phase of $\mathbf{H}(\omega)$ is

$$\underline{/H(\omega)} = 90^\circ + \tan^{-1}\left(\frac{\omega}{z_1}\right) + \tan^{-1}\left(\frac{2\zeta_1\omega/\omega_k}{1-(\omega/\omega_k)^2}\right) + \ldots$$
$$- \tan^{-1}\left(\frac{\omega}{p_1}\right) - \tan^{-1}\left(\frac{2\zeta_2\omega/\omega_n}{1-(\omega/\omega_n)^2}\right) + \ldots$$


Factor
Magnitude
Phase
K
$20 \log_{10} K$
$0°$
ω
ω
$(j\omega)^N$
$20N$ dB/decade
$90N°$
1
ω
ω
$\frac{1}{(j\omega)^N}$
1
ω
ω
$-20N$ dB/decade
$-90N°$

Factor	Magnitude	Phase
$\left(1 + \frac{j\omega}{z}\right)^N$		
$\frac{1}{(1 + j\omega/p)^N}$		

Factor	Magnitude	Phase
$\left[1 + \frac{2j\omega\zeta}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right]^N$		
$\frac{1}{[1 + 2j\omega\zeta/\omega_k + (j\omega/\omega_k)^2]^N}$		

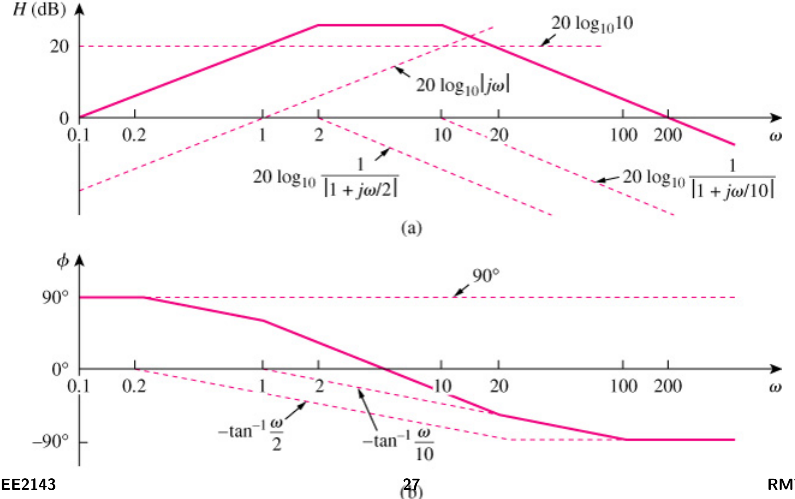
Example 14.3, pg. 624 (Alexander & Sadiku, 2009)

Construct the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{200j\omega}{(j\omega + 2)(j\omega + 10)}$$

Example 14.3, pg. 624 (cont.)

Answer:



List of References

1. C.K. Alexander and M.N.O. Sadiku (2009), Fundamentals of Electric Circuits 4th ed., New York: McGraw-Hill.