BEE2143 – Signals & Networks

Chapter 8 – Applications of the Laplace Transform

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Integrodifferential equations

- The Laplace transform is useful in solving linear integrodifferential equations
- Using the differentiation and integration properties of Laplace transforms, each term in the integrodifferential equation is transformed
- Initial conditions are automatically taken into account
- We solve the resulting algebraic equation in the s-domain
- We then convert the solution back to the time domain by using the inverse transform

Practice Problem 15.15, pg. 707 (Alexander & Sadiku, 2009)

Solve the following differential equation using the Laplace transform method.

$$\frac{d^2v(t)}{dt^2} + 4\frac{dv(t)}{dt} + 4v(t) = e^{-t}u(t)$$

if $v(0) = v'(0) = 2$.

Answer:

$$v(t) = (2e^{-t} + 4te^{-2t})u(t)$$

Practice Problem 15.16, pg. 707 (Alexander & Sadiku, 2009)

Use the Laplace transform to solve the integrodifferential equation

$$\frac{dy(t)}{dt} + 3y(t) + 2\int_0^t y(\tau)d\tau = 2e^{-3t}u(t), \quad y(0) = 0.$$

Answer:

$$y(t) = (-e^{-t} + 4e^{-2t} - 3e^{-3t})u(t)$$

Circuit analysis

- Steps in Applying the Laplace Transform:
  1. Transform the circuit from the time domain to the s-domain
  2. Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any circuit analysis technique with which we are familiar
  3. Take the inverse transform of the solution and thus obtain the solution in the time domain

- Circuit element models:
  - For a resistor,

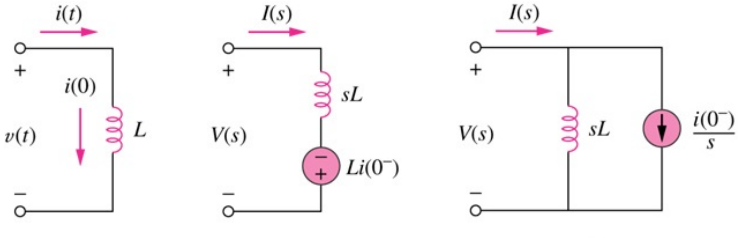
$$V(s) = RI(s)$$

  - For an inductor,

$$V(s) = LsI(s) - Li(0^-)$$

or

$$I(s) = \frac{1}{Ls}V(s) + \frac{i(0^-)}{s}$$



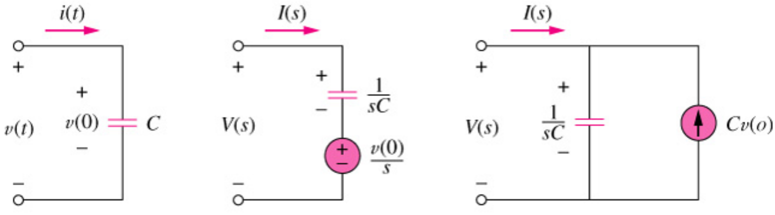
(a) (b) (c)

- Circuit element models (cont.):
  - For a capacitor,

$$I(s) = CsV(s) - Cv(0^-)$$

or

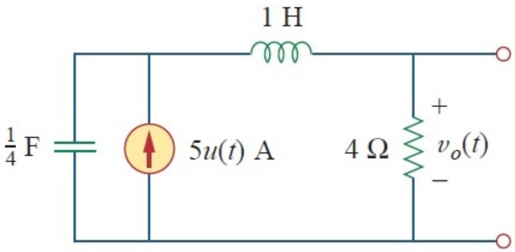
$$V(s) = \frac{1}{Cs}I(s) + \frac{v(0^-)}{s}$$



(a) (b) (c)

Practice Problem 16.1, pg. 719 (Alexander & Sadiku, 2009)

Determine $v_o(t)$ in the circuit of Fig. 16.6, assuming zero initial conditions.

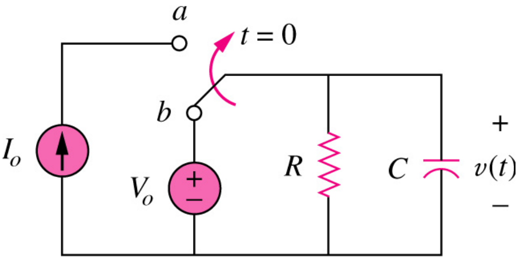


Answer:

$$v_o(t) = 20(1 - e^{-2t} - 2te^{-2t})u(t) \text{ V}$$

Practice Problem 16.3, pg. 722 (Alexander & Sadiku, 2009)

The switch in Fig. 16.11 has been in position b for a long time. It is moved to position a at $t = 0$. Determine $v(t)$ for $t > 0$.

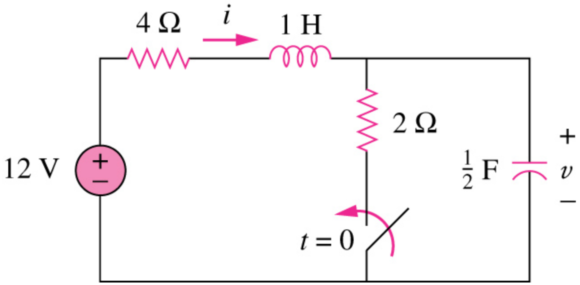


Answer:

$$v(t) = (V_o - I_oR)1 - e^{-\frac{t}{RC}} + I_oR \text{ V}, \quad t > 0$$

Example 8.9, pg. 339 (Alexander & Sadiku, 2009)

Find the complete response $v(t)$ and then $i(t)$ for in the circuit of Fig. 8.25.



Answer:

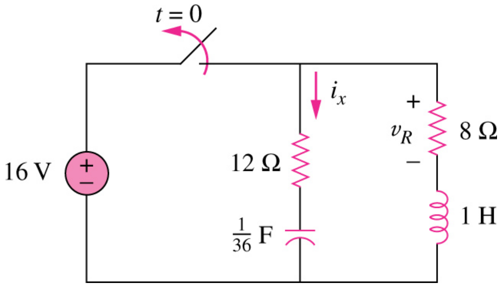
$$v(t) =(4 + 12e^{-2t} - 4e^{-3t})u(t) \text{ V}$$
$$i(t) =(2 - 6e{-}2t + 4e^{-3t})u(t) \text{ A}$$

Problem 8.57, pg. 364 (Alexander & Sadiku, 2009)

If the switch in Fig. 8.103 has been closed for a long time before $t = 0$ but is opened at $t = 0$, determine:

(a) the characteristic equation of the circuit,

(b) $i_x(t)$ and $v_R(t)$ for $t > 0$.



Problem 8.57, pg. 364 (cont.)

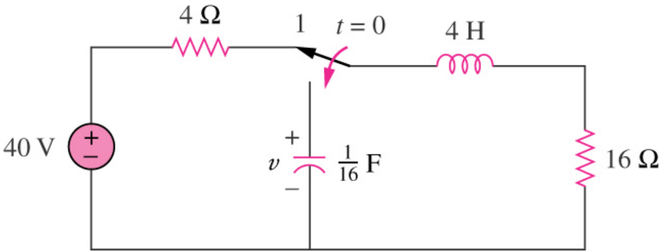
Answer:

(a) $s^2 + 20s + 36 = 0$

(b) $i_x(t) = -(0.75e^{-2t} + 1.25e^{-18t})u(t)$ A,
$v_R(t) = (6e^{-2t} + 10e^{-18t})u(t)$ V

Problem 8.59, pg. 365 (Alexander & Sadiku, 2009)

The make before break switch in Fig. 8.105 has been in position 1 for $t < 0$. At $t = 0$, it is moved instantaneously to position 2. Determine $v(t)$.

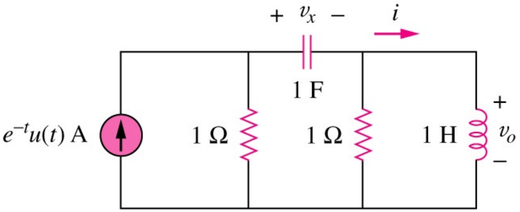


Answer:

$$v(t) = -32te^{-t}u(t) \text{ V}$$

Problem 16.20, pg. 749 (Alexander & Sadiku, 2009)

Find $v_o(t)$ in the circuit of Fig. 16.54 if $v_x(0) = 2$ V and $i(0) = 1$ A.



Answer:

$$V_o(s) = -\frac{2s^2+4s+1}{(s+1)(2s^2+2s+1)} = \frac{1}{s+1} - \frac{4s+2}{2s^2+2s+1}$$

$$v_o(t) = (e^{-t} - 2e^{-0.5t}\cos 0.5t)u(t) \text{ V}$$

Transfer functions

- The transfer function is a key concept in signal processing because it indicates how a signal is processed as it passes through a network
- It is a fitting tool for finding the network response, determining (or designing for) network stability, and network synthesis
- The transfer function of a network describes how the output behaves with respect to the input
- It specifies the transfer from the input to the output in the s-domain, assuming no initial energy

- The transfer function $H(s)$ is the ratio of the output response $Y(s)$ to the input excitation $X(s)$, assuming all initial conditions are zero:

$$H(s) = \frac{Y(s)}{X(s)}$$

- The transfer function depends on what we define as input and output
- Since the input and output can be either current or voltage at any place in the circuit, there are four possible transfer functions:

$$\text{Voltage gain} = \frac{V_o(s)}{V_i(s)}, \quad \text{Impedance} = \frac{V_o(s)}{I_i(s)},$$
$$\text{Current gain} = \frac{I_o(s)}{I_i(s)}, \quad \text{Admittance} = \frac{I_o(s)}{V_i(s)}$$

- Sometimes, we know the input $X(s)$ and the transfer function $H(s)$
- We find the output $Y(s)$ as

$$Y(s) = H(s)X(s)$$

  and take the inverse transform to get $y(t)$
- A special case is when the input is the unit impulse (delta) function, $x(t) = \delta(t)$, so that $X(s) = 1$
- For this case,

$$Y(s) = H(s) \quad \text{or} \quad y(t) = h(t)$$

- The term $h(t)$ represents the unit impulse response
- Once we know the impulse response $h(t)$ of a network, we can obtain the response of the network to any input signal using in the s-domain or using the convolution integral in the time domain

Example 16.7, pg. 727 (Alexander & Sadiku, 2009)

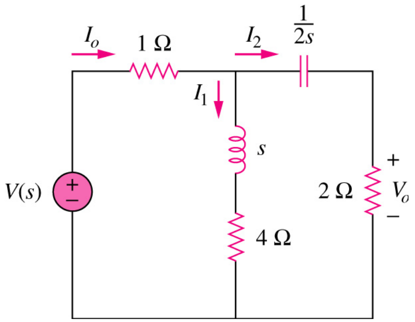
The output of a linear system is $y(t) = 10e^{-t}\cos 4tu(t)$ when the input is $x(t) = e^{-t}u(t)$. Find the transfer function of the system and its impulse response.

Answer:

$$H(s) = \frac{10(s^2 + 2s + 1)}{s^2 + 2s + 17},$$
$$h(t) = 10\delta(t) - 40e^{-t}\sin 4tu(t)$$

Example 16.8 & Practice Problem 16.8, pg. 728 (Alexander & Sadiku, 2009)

Determine the transfer function $V_o(s)/I_o(s)$ and $I_1(s)/I_o(s)$ of the circuit in Fig. 16.18.



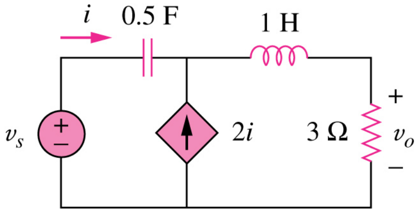
Example 16.8 & Practice Problem 16.8, pg. 728 (cont.)

Answer:

$$\frac{V_o(s)}{I_o(s)} = \frac{4s(s+4)}{2s^2+12s+1},$$
$$\frac{I_1(s)}{I_o(s)} = \frac{4s+1}{2s^2+12s+1}$$

Problem 16.35, pg. 751 (Alexander & Sadiku, 2009)

Obtain the transfer function $H(s) = V_o/V_s$ for the circuit of Fig. 16.65.



Answer:

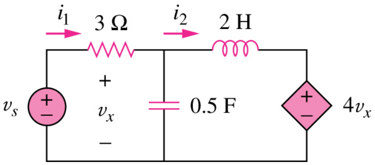
$$H(s) = \frac{9s}{3s^2 + 9s + 2}$$

Problem 16.37, pg. 751 (Alexander & Sadiku, 2009)

For the circuit in Fig. 16.66, find:

(a) I_1/V_s

(b) I_2/V_x



Answer:

(a) $$\frac{I_1}{V_s} = \frac{s^2 - 3}{3s^2 + 2s - 9}$$

(b) $$\frac{I_2}{V_x} = -\frac{3}{2s}$$

List of References

1. C.K. Alexander and M.N.O. Sadiku (2009), Fundamentals of Electric Circuits 4th ed., New York: McGraw-Hill.