BEE2143 – Signals & Networks

Chapter 4 – Applications of the Fourier Series

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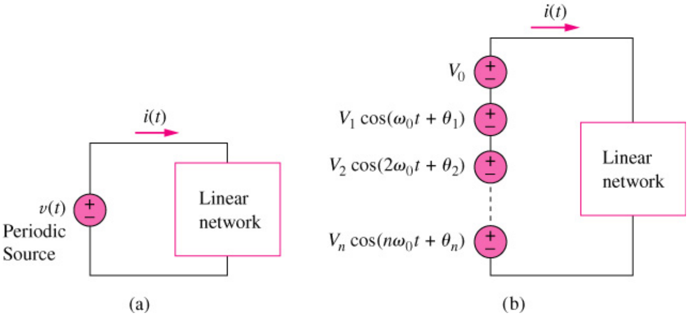
Circuit analysis

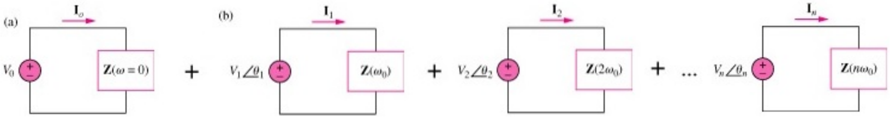
- To find the steady-state response of a circuit to a nonsinusoidal periodic excitation requires the application of a Fourier series, ac phasor analysis, and the superposition principle
- The procedure usually involves four steps:
  1. Express the excitation as a Fourier series
  2. Transform the circuit from the time domain to the frequency domain
  3. Find the response of each term in the Fourier series
  4. Add the individual responses using the superposition principle

- The first step is to determine the Fourier series expansion of the excitation, for example

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

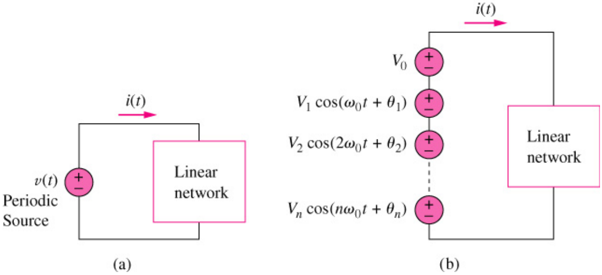


- The third step is finding the response to each term in the Fourier series
- The response to the dc component can be determined in the frequency domain by setting $n = 0$ or $\omega = 0$, or in the time domain by replacing all inductors with short circuits and all capacitors with open circuits
- The response to the ac component is obtained by the phasor techniques



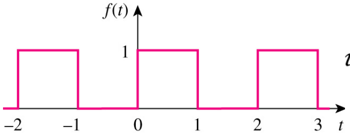
- Finally, following the principle of superposition, we add all the individual responses
- For example, if the desired output response is the current

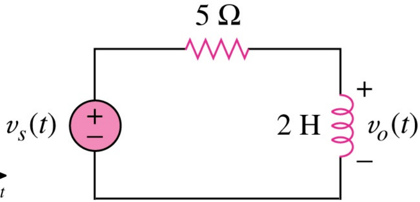
$$\begin{aligned} i(t) =& i_0(t) + i_1(t) + i_2(t) + \dots \\ =& I_0 + \sum_{n=1}^{\infty} |\mathbf{I}_n| \cos(n\omega_0 t + \psi_n) \end{aligned}$$



Example 17.6, pg. 775 (Alexander & Sadiku, 2009)

Let the function in Example 17.1 be the voltage source in the circuit of Fig. 17.20. Find the response of the circuit.



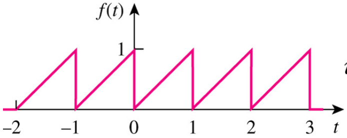


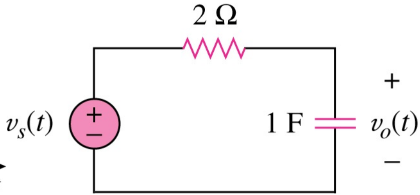
Answer:

$$v_o(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{\sqrt{25 + 4n^2\pi^2}} \cos\left(n\pi t - \tan^{-1}\frac{2n\pi}{5}\right) \text{ V}$$

Practice Problem 17.6, pg. 776 (Alexander & Sadiku, 2009)

If the sawtooth waveform in Fig. 17.9 is the voltage source in the circuit of Fig. 17.22, find the response $v_o(t)$.



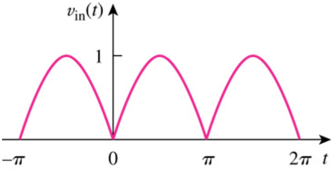


Answer:

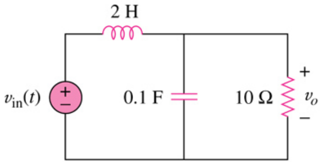
$$v_o(t) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n\pi t - \tan^{-1} 4n\pi)}{n\sqrt{1+16n^2\pi^2}}\ \text{V}$$

Problem 17.41, pg. 804 (Alexander & Sadiku, 2009)

The full-wave rectified sinusoidal voltage in Fig. 17.77(a) is applied to the lowpass filter in Fig. 17.77(b). Obtain the output voltage $v_o(t)$ of the filter.



(a)



Problem 17.41, pg. 804 (cont.)

Answer:

$$v_o(t) = \frac{10}{\pi} + \sum_{n=1}^{\infty} A_n \cos(2nt + \theta_n)$$

$$A_n = \frac{100}{\pi(4n^2 - 1)\sqrt{16n^2 - 40n + 29}}$$

$$\theta_n = 90^\circ - \tan^{-1}(2n - 2.5)$$

Average power and rms values

- For the Fourier series expansion

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \theta_n)$$

its rms value is given by

$$F_{\text{rms}} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)}$$

- The power dissipated by the 1 Ω resistance is (Parseval's theorem)

$$P_{1\text{-}\Omega} = F_{\text{rms}}^2 = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2 = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

Example 17.9, pg. 781 (Alexander & Sadiku, 2009)

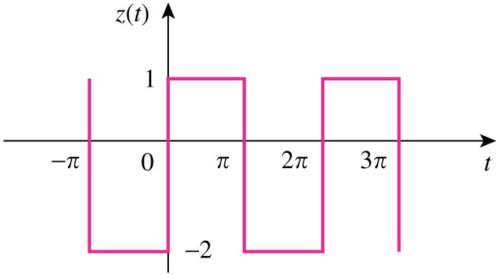
Find an estimate for the rms value of the voltage in Example 17.7 (pg. 776):

$$v(t) = 1 + \sum_{n=1}^{\infty} \frac{2(-1)^n}{1+n^2}(\cos nt - n \sin nt)$$

Answer: $V_{\rm rms} = 1.649$ V (up to 4th harmonics)

Problem 17.49, pg. 805 (Alexander & Sadiku, 2009)

(a) For the periodic waveform in Prob. 17.5 (pg. 799), find the rms value.



(b) Use the first five harmonic terms of the Fourier series in Prob. 17.5 to determine the effective value of the signal.

Problem 17.49, pg. 805 (cont.)

(c) Calculate the percentage error in the estimated rms value of $z(t)$ if

$$\%\text{error} = \left(\frac{\text{estimated value}}{\text{exact value}} - 1\right) \times 100\%$$

Answer: (a) 1.5811, (b) 1.5328, (c) -3.16% (corrected)

Spectrum analyzers

- The Fourier series provides the spectrum of a signal
- As we have seen, the spectrum consists of the amplitudes and phases of the harmonics versus frequency
- By providing the spectrum of a signal $f(t)$, the Fourier series helps us identify the pertinent features of the signal
- It demonstrates which frequencies are playing an important role in the shape of the output and which ones are not
- For example, audible sounds have significant components in the frequency range of 20 Hz to 15 kHz, while visible light signals range from 105 GHz to 106 GHz

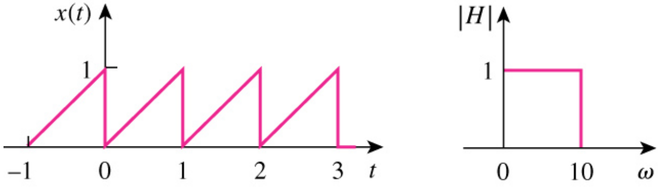
- A spectrum analyzer is an instrument that displays the amplitude of the components of a signal versus frequency
- In other words, it shows the various frequency components (spectral lines) that indicate the amount of energy at each frequency
- It is unlike an oscilloscope, which displays the entire signal (all components) versus time
- An oscilloscope shows the signal in the time domain, while the spectrum analyzer shows the signal in the frequency domain
- An analyzer can conduct noise and spurious signal analysis, phase checks, electromagnetic interference and filter examinations, vibration measurements, radar measurements, and more

Filters

- Here, we investigate how to design filters to select the fundamental component (or any desired harmonic) of the input signal and reject other harmonics
- This filtering process cannot be accomplished without the Fourier series expansion of the input signal

Example 17.14, pg. 795 (Alexander & Sadiku, 2009)

If the sawtooth waveform in Fig. 17.45(a) is applied to an ideal lowpass filter with the transfer function shown in Fig. 17.45(b), determine the output.



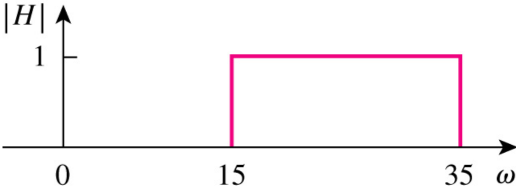
(a) (b)

Answer:

$$y(t) = \frac{1}{2} - \frac{1}{\pi}\sin 2\pi t$$

Practice Problem 17.14, pg. 796 (Alexander & Sadiku, 2009)

Rework Example 17.14 if the lowpass filter is replaced by the ideal bandpass filter shown in Fig. 17.46.



Answer:

$$y(t) = -\frac{1}{3\pi}\sin 6\pi t - \frac{1}{4\pi}\sin 8\pi t - \frac{1}{5\pi}\sin 10\pi t$$

List of References

1. C.K. Alexander and M.N.O. Sadiku (2009), Fundamentals of Electric Circuits 4th ed., New York: McGraw-Hill.