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# REINFORCED CONCRETE DESIGN 1

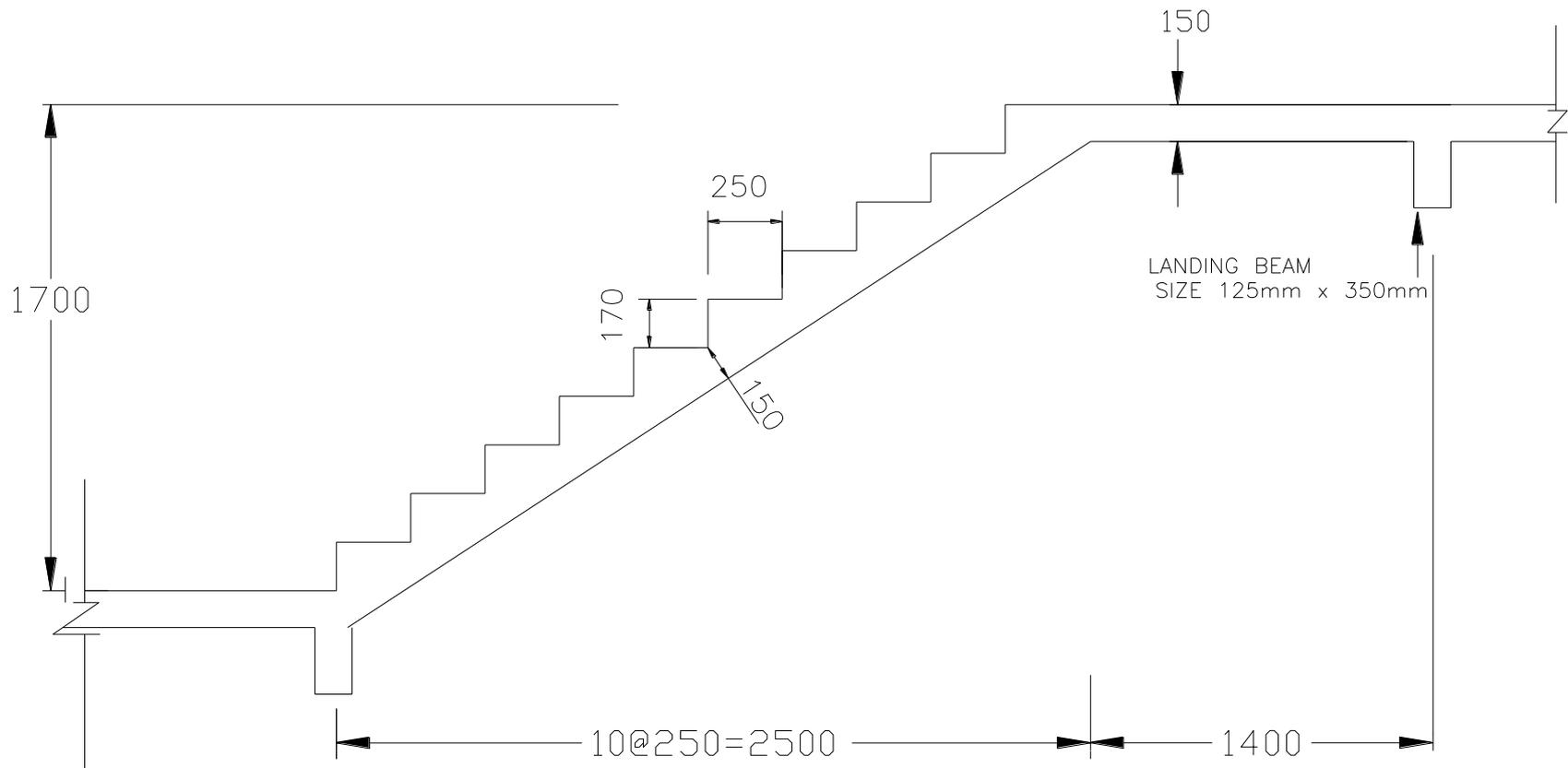
## Design of Staircase (Examples and Tutorials) by

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# Example 1: Straight staircase design

A reinforced concrete staircase for office use is shown in **Figure 1**. It is connected to a landing at upper part and supported by a beam at the end of the landing. At the end lower the stair supported by a beam and continuous with the floor slab. Design the staircase by using concrete grade 25 and strength of reinforcement of 500 N/mm<sup>2</sup>. The imposed load is 2.5 kN/m<sup>2</sup> and finishes is 0.5 N/m<sup>2</sup>. Nominal cover,  $c_{nom}$  is 25 mm. The width of staircase is 1500 mm, the thickness of landing is 150 mm and the waist thickness ( $h$ ) is 150 mm. Design the reinforcement for the stairs. Use diameter bar = 10 mm.

# Example 1: Straight staircase design



**Figure 1**

# Example 1: Straight staircase design

## Load Analysis

$$\begin{aligned}\text{Average thickness of flight, } y &= h(G^2 + R^2)^{1/2}/G \\ &= 150 (250^2 + 170^2)^{1/2}/250 \\ &= \mathbf{181.4 \text{ mm}}\end{aligned}$$

$$\begin{aligned}\text{Average thickness, } t &= y + (R/2) = 181.4 + (170/2) \\ &= 266.4 \text{ mm}\end{aligned}$$

## Actions

Landing permanent action,

$$\begin{aligned}\text{Self-weight staircase} &= 0.15 \times 25 \\ &= 3.75 \text{ kN/m}^2\end{aligned}$$

$$\text{Finishes} = 0.5 \text{ kN/m}^2$$

$$\mathbf{\text{Total } g_k} = 4.25 \text{ kN/m}^2$$

$$\text{Variable action, } q_k = 2.5 \text{ kN/m}^2$$

# Example 1: Straight staircase design

$$\begin{aligned}\text{Design Action, } w &= 1.35g_k + 1.5q_k \\ &= 9.49 \text{ kN/m}^2\end{aligned}$$

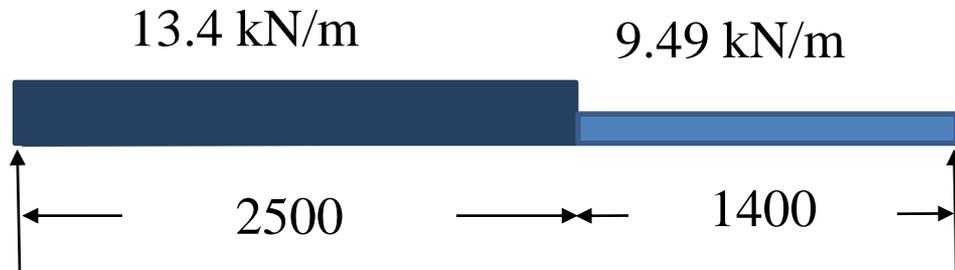
$$\begin{aligned}\text{Flight permanent action,} \\ \text{Self-weight staircase} &= 0.266 \times 25 \\ &= 6.65 \text{ kN/m}^2\end{aligned}$$

$$\begin{aligned}\text{Finishes} &= 0.5 \text{ kN/m}^2 \\ \text{Total } g_k &= 7.15 \text{ kN/m}^2\end{aligned}$$

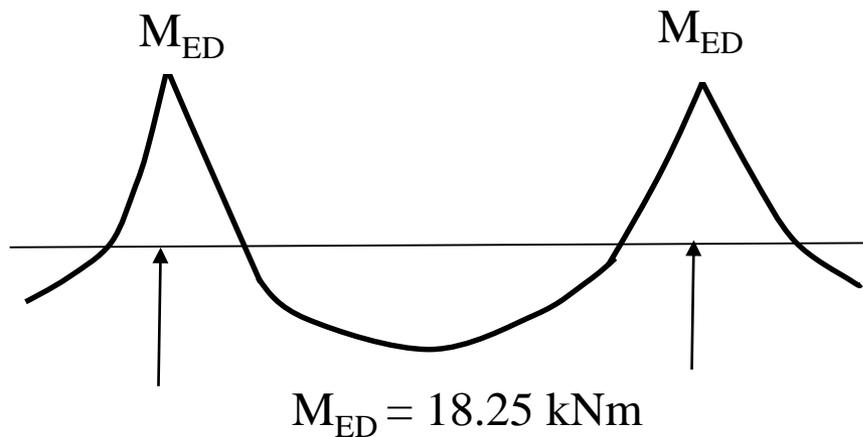
$$\begin{aligned}\text{Variable action, } q_k &= 2.5 \text{ kN/m}^2\end{aligned}$$

$$\begin{aligned}\text{Design Action, } w &= 1.35g_k + 1.5q_k \\ &= 13.4 \text{ kN/m}^2\end{aligned}$$

# Example 1: Straight staircase design



$$\text{Total load, } F = 13.4(2.5) + 9.49(1.4) = 46.79 \text{ kN}$$



$$\begin{aligned} M_{ED} &= wL/10 \\ &= 46.79 (3.9)/10 \\ &= 18.25 \text{ kNm} \end{aligned}$$

# Example 1: Straight staircase design

Effective depth,

$$d = h - c_{\text{nom}} - 0.5\phi_{\text{bar}} = 150 - 25 - 10/2 = 120 \text{ mm}$$

$$M_{\text{ED}} = 18.25 \text{ kNm}$$

$$\begin{aligned} K &= M/bd^2f_{\text{ck}} \\ &= 18.25 \times 10^6 / (1000 \times 120^2 \times 25) \\ &= 0.052 < k_{\text{bal}} = 0.167 \quad \text{no. compression reinforcement required} \end{aligned}$$

$$\begin{aligned} z &= d[0.5 + (0.25 - K/1.134)^{1/2}] \\ &= 0.98d > 0.95d \quad \text{use } 0.95d \end{aligned}$$

$$\begin{aligned} A_s &= M/0.87f_{\text{yk}}z \\ &= 18.25 \times 10^6 / (0.87 \times 500 \times 0.95 \times 120) \\ &= 368 \text{ mm}^2 \end{aligned}$$

**Provide H10-200**  
**( $A_s = 393 \text{ mm}^2$ )**

# Example 1: Straight staircase design

Minimum and maximum reinforcement area,

$$\begin{aligned}A_{s,\min} &= 0.26(f_{ctm}/f_{yk})bd \\ &= 0.0151 bd > 0.013bd \\ &= 179.45 \text{ mm}^2\end{aligned}$$

$$\begin{aligned}A_{s,\max} &= 0.04 A_c \\ &= 0.04(1000)(150) \\ &= 6000 \text{ mm}^2\end{aligned}$$

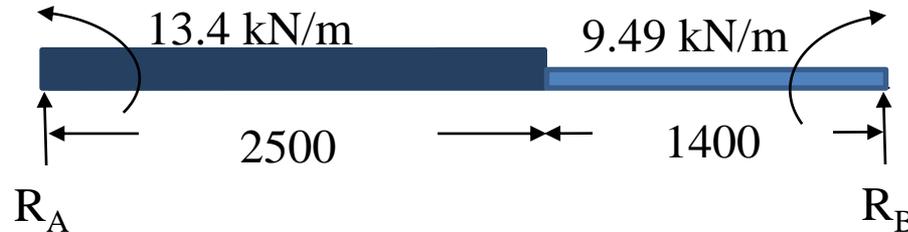
Secondary Reinforcement = 20%  $A_s$

$$A_s = 20\% (A_s) = 0.2 (503) = 100.6 \text{ mm}^2$$

**Provide H10-400 ( $A_s = 196.5 \text{ mm}^2$ )**

# Example 1: Straight staircase design

## SHEAR



$$R_A = 29.56 \text{ kN} \quad R_B = 17.22 \text{ kN}$$
$$\text{Shear force, } V_{ED} = 29.54 \text{ kN}$$

$$V_{Rdc} = [0.12 k (100\rho_1 f_{ck})^{1/3}] bd$$

$$k = 1 + (200/d)^{1/2} \leq 2.0$$
$$= 2.29 > \mathbf{2.0}$$

$$\rho_1 = (A_{sl} / b_w d) = 503 / (1000 \times 120) = 0.0033 \leq 0.02$$
$$= 0.0033 > 0.002$$

$$V_{Rdc} = 34.03 \text{ kN}$$

$$V_{min} = [0.035 k^{3/2} f_{ck}^{1/2}] bd = 59.4 \text{ kN}$$

**Thus,  $V_{Rdc} = 54.9 \text{ kN} > V_{ED} = 29.56 \text{ kN}$ , OK!**

# Example 1: Straight staircase design

## Deflection

$$\rho = A_{s,req}/bd = 368/(1000 \times 120) = 0.0031$$

$$\rho_o = \sqrt{f_{ck}}^{(10-3)} = \sqrt{25}^{(10-3)} = 0.005 \times 10^{-3}$$

$$\rho_o > \rho, \text{ structural system, } K = 1.5$$

$$L/d = k (11 + 1.5\sqrt{f_{ck}} (\rho_o / \rho) + 3.2\sqrt{f_{ck}} ((\rho_o / \rho) - 1)^{3/2})$$

$$L/d = 1.5 (11 + 12.1 + 7.68)$$

$$(L/d)_{basic} = 46.17$$

$$\text{Modification factor, } A_{s,prov}/A_{s,req} = 393/368 = 1.07 < 1.5$$

$$(L/d)_{allow} = 46.17 \times 1.07 = 49.4$$

$$(L/d)_{actual} = 3900/120 = 32.5 < (L/d)_{allow}, \text{ OK!}$$

# Example 1: Straight staircase design

## Check Crack

Slab thickness,  $h = 150 \text{ mm} < 200 \text{ mm}$

Main bar:

$$S_{v,\max, \text{ slab}} = 3h \leq 400 \text{ mm}$$
$$= 3(150) \leq 400 \text{ mm}, 450 \text{ mm} \geq 400 \text{ mm}, \text{ Use } 400$$

Actual bar spacing =  $200 \text{ mm} < 400 \text{ mm}$  **OK**

Secondary bar:

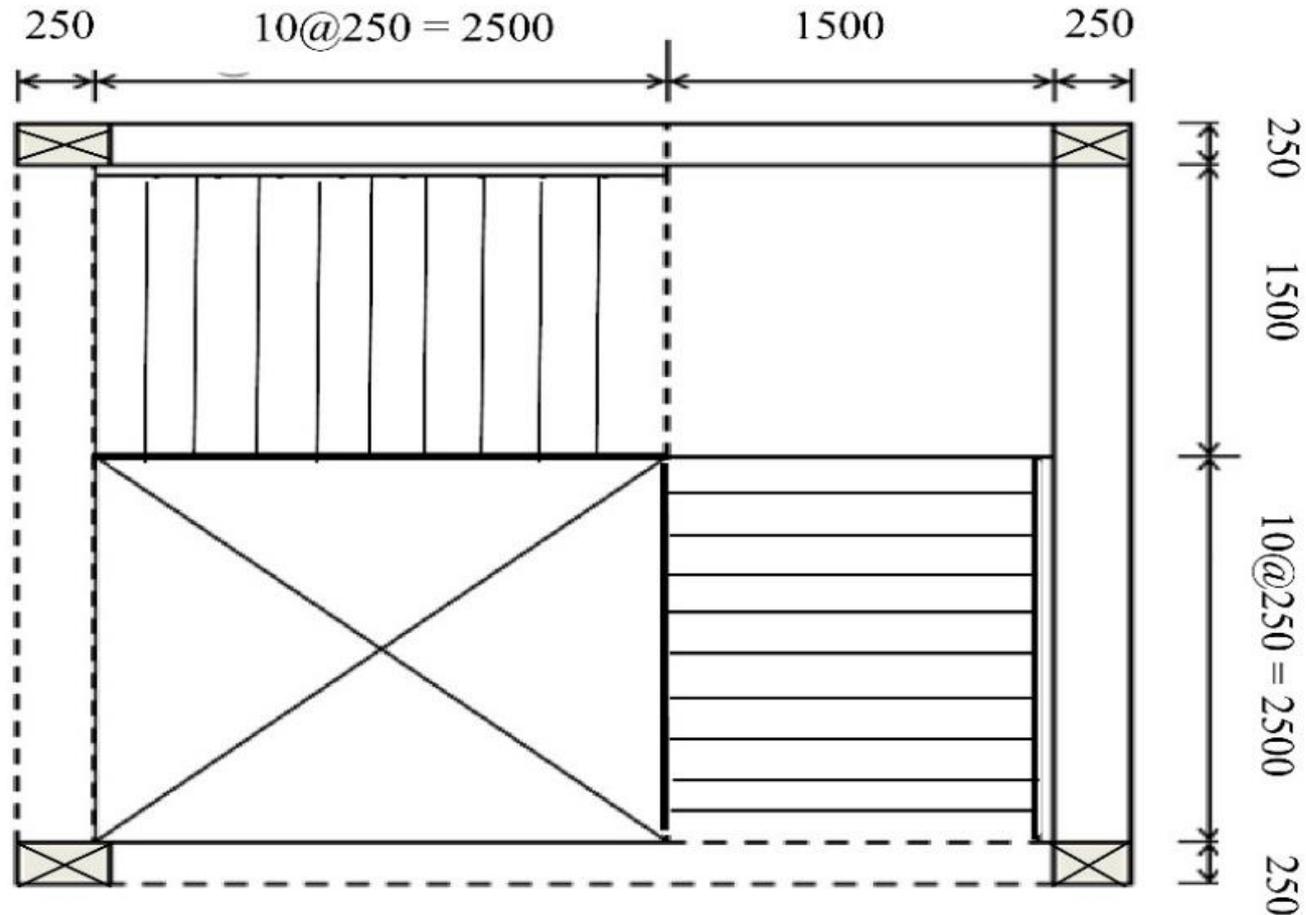
$$S_{\max, \text{ slab}} = 3.5h \leq 450 \text{ mm}$$
$$= 450 \text{ mm}, \text{ use } 450 \text{ mm}$$

Actual bar spacing =  $400 < 450 \text{ mm}$ , **OK**

## Example 2: Open-well staircase design

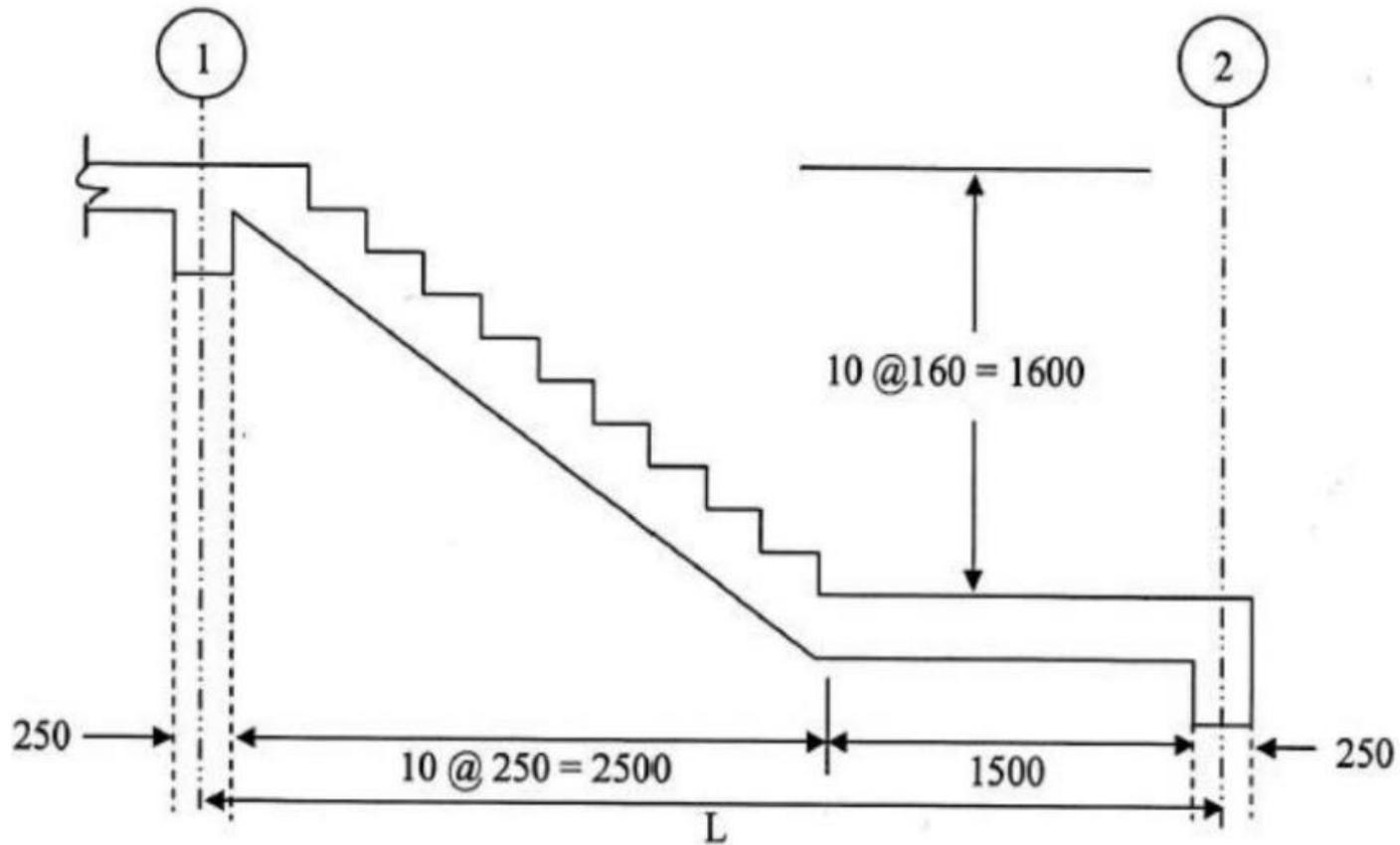
A staircase of 1.5 m width for an office building with slab supported on a beam at the top and on the landing of the flight at right angles at the bottom is shown in **Figure 2**. The riser and goings of the stairs are 160 mm and 250 mm, respectively. The variable load is 3.0 kN/m<sup>2</sup> and the permanent action from finishes, baluster and railing about 1.0 kN/m<sup>2</sup>. Materials used in this construction consist of concrete with characteristic strength,  $f_{ck} = 30 \text{ N/mm}^2$  and steel strength,  $f_{yk} = 500 \text{ N/mm}^2$ . The thickness of the landing is 150 mm and waist thickness (h) is 150 mm. Design the stairs if the concrete cover = 25 mm and the main bar diameter,  $\phi_{\text{main}} = 10 \text{ mm}$ .

# Example 2: Open-well staircase design



**Figure 2**

# Example 2: Open-well staircase design



**Figure 2 (continue)**

# Example 2: Open-well staircase design

## Load Analysis

$$\begin{aligned}\text{Average thickness, } y &= h(G^2 + R^2)^{1/2}/G \\ &= 150 (250^2 + 160^2)^{1/2}/250 \\ &= \mathbf{178 \text{ mm}}\end{aligned}$$

$$\text{Average thickness, } t = [y + (y + R)]/2 = 258 \text{ mm}$$

## Actions

### **Landing**

Slab self-weight	$= 0.15 \times 25 = 3.75 \text{ kN/m}^2$
Permanent load excluding self-weight	$= 1.00 \text{ kN/m}^2$
Characteristic permanent action	$= 4.75 \text{ kN/m}^2$
Characteristic variable action	$= 3.00 \text{ kN/m}^2$

# Example 2: Open-well staircase design

$$\text{Design action } n_d = 1.35 (4.75) + 1.5 (3.0) = 10.91 \text{ kN/m}^2$$

## **Flight**

$$\text{Slab self-weight} = 0.258 \times 25 = 6.45 \text{ kN/m}^2$$

$$\text{Permanent load excluding self-weight} = 1.00 \text{ kN/m}^2$$

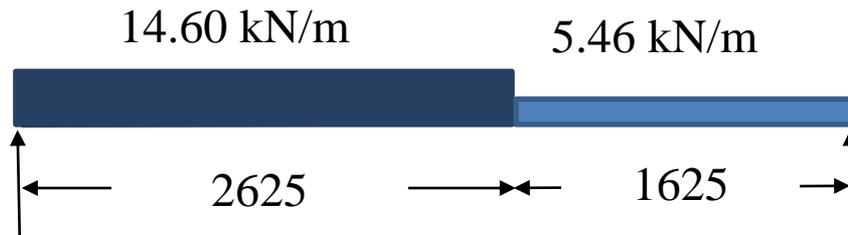
$$\text{Characteristic permanent action} = 7.45 \text{ kN/m}^2$$

$$\text{Characteristic variable action} = 3.00 \text{ kN/m}^2$$

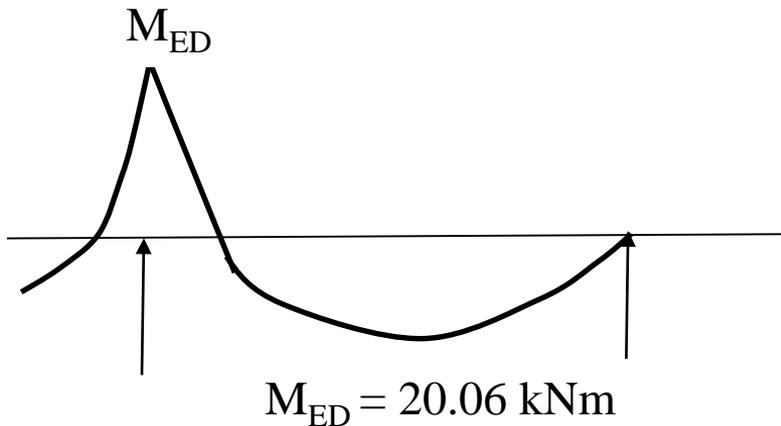
$$\text{Design action } n_d = 1.35 (7.45) + 1.5 (3.0) = 14.60 \text{ kN/m}^2$$

# Example 2: Open-well staircase design

## Analysis

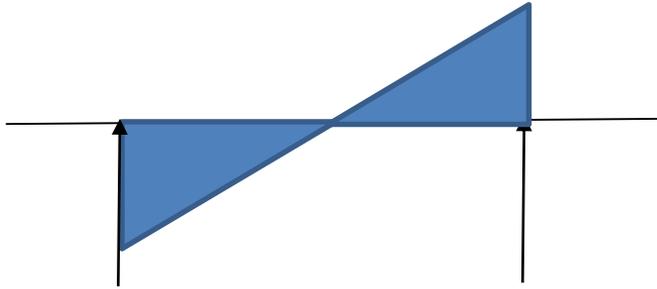


$$\text{Total action} = (5.46 \times 1.625) + (14.6 \times 2.625) = 47.2 \text{ kN/m}$$



$$\begin{aligned} M_{ED} &= FL/10 \\ &= 47.2 (4.25)/10 \\ &= 20.06 \text{ kNm} \end{aligned}$$

# Example 2: Open-well staircase design



Shear force,

$$V_A = [(14.6 \times 2.625 \times 2.94) + (5.46 \times 1.625 \times 0.81) - 20.06]/4.25 = 23.48 \text{ kN/m}$$

$$V_B = [(14.6 \times 2.625) + (5.46 \times 1.625) - 23.48] = 23.72 \text{ kN/m}$$

# Example 2: Open-well staircase design

## MAIN REINFORCEMENT

Effective depth,

$$d = h - c_{\text{nom}} - 0.5\phi_{\text{bar}} = 150 - 25 - 10/2 = 120 \text{ mm}$$

$$\begin{aligned} K &= M/bd^2f_{\text{ck}} \\ &= 20.1 \times 10^6 / (1000 \times 120^2 \times 25) \\ &= 0.056 < k_{\text{bal}} = 0.167, \text{ no. compression reinforcement required} \end{aligned}$$

$$\begin{aligned} z &= d[0.5 + (0.25 - K/1.134)^{1/2}] \\ &= 0.95d \leq 0.95d \quad \text{use } 0.95d \end{aligned}$$

$$\begin{aligned} A_s &= M/0.87f_{\text{yk}}z \\ &= 20.1 \times 10^6 / (0.87 \times 500 \times 0.95 \times 120) \\ &= 405.3 \text{ mm}^2 \end{aligned}$$

# Example 2: Open-well staircase design

Minimum and maximum reinforcement area,

$$\begin{aligned}A_{s,\min} &= 0.26(f_{ctm}/f_{yk})bd \\ &= 0.0145 bd > 0.013bd \\ &= 181 \text{ mm}^2\end{aligned}$$

$$\begin{aligned}A_{s,\max} &= 0.04 A_c \\ &= 0.04(1000)(150) \\ &= 6000 \text{ mm}^2\end{aligned}$$

$$\text{Secondary bar} = 0.2 \times 405.3 = 81.1 \text{ mm}^2/\text{m}$$

Main Reinforcement → Provide: H10-175 (449 mm<sup>2</sup>)

Secondary Reinforcement → Use: H10 – 400 (196 mm<sup>2</sup>)

# Example 2: Open-well staircase design

## SHEAR

Shear force,  $V_{ED} = 23.72 \text{ kN}$

$$V_{Rdc} = [0.12 k (100\rho_1 f_{ck})^{1/3}] bd$$

$$\begin{aligned} k &= 1 + (200/120)^{1/2} \leq 2.0 \\ &= 2.29 \leq \mathbf{2.0} \end{aligned}$$

$$\begin{aligned} \rho_1 &= (A_{sl} / b_w d) = 449 / (1000 \times 120) \leq 0.02 \\ &= 0.0037 \leq 0.002 \end{aligned}$$

$$V_{Rdc} = 64.2 \text{ kN}$$

$$V_{min} = [0.035 k^{3/2} f_{ck}^{1/2}] bd = 65.1 \text{ kN}$$

So,  $V_{Rdc} = 65.1 \text{ kN} > V_{ED} = 23.72 \text{ kN}$ , OK! . No shear reinforcement required

# Example 2: Open-well staircase design

## Deflection

$$\rho = A_{s,req}/bd = 405.3/(1000 \times 120) = 0.0034$$

$$\rho_o = \sqrt{f_{ck}}^{(10^{-3})} = \sqrt{25}^{(10^{-3})} = 0.005$$

$$\rho < \rho_o, \text{ structural system, } K = 1.3$$

$$L/d = k (11 + 1.5\sqrt{f_{ck}} (\rho_o / \rho) + 3.2\sqrt{f_{ck}} ((\rho_o / \rho) - 1)^{3/2})$$

$$L/d = 1.3 (11 + 11.03 + 5.17)$$

$$(L/d)_{basic} = 35.35$$

$$\text{Modification factor, } A_{s,prov}/A_{s,req} = 449/405.3 = 1.11 < 1.5$$

$$(L/d)_{allow} = 35.35 \times 1.11 = 39.24$$

$$(L/d)_{actual} = 4250/120 = \mathbf{35.42} < (L/d)_{allow}, \mathbf{OK!}$$

# Example 2: Open-well staircase design

## Check Crack

Slab thickness,  $h = 150 \text{ mm} < 200 \text{ mm}$

Main bar:

$$\begin{aligned} S_{\text{max, slab}} &= 3h \leq 400 \text{ mm} \\ &= 3(150) = 450 \text{ mm, use 400 mm} \end{aligned}$$

Max. bar spacing =  $175 \text{ mm} < 400 \text{ mm}$  OK.

Secondary bar:

$$\begin{aligned} S_{\text{max, slab}} &= 3.5h < 450 \text{ mm} \\ &= 3.5(150) = 525 \text{ mm, use 450 mm} \end{aligned}$$

Max. bar spacing =  $400 < 450 \text{ mm}$ , OK

# Tutorial 1: Staircase design

Design a straight reinforced concrete stairs supported by reinforced concrete beams at both ends. Landing slabs at both ends of the stairs are cast together connecting the stairs. Using the following information, design the staircase.

Concrete grade: C30/C37

Steel grade: 500

Perm. load from finishes, baluster and railing:  $1.5 \text{ kN/m}^2$

Variable load:  $3.5 \text{ kN/m}^2$

$C_{\text{nom}} = 25 \text{ mm}$

Bar size = 10 mm

$R = 180 \text{ mm}$

$G = 250 \text{ mm}$

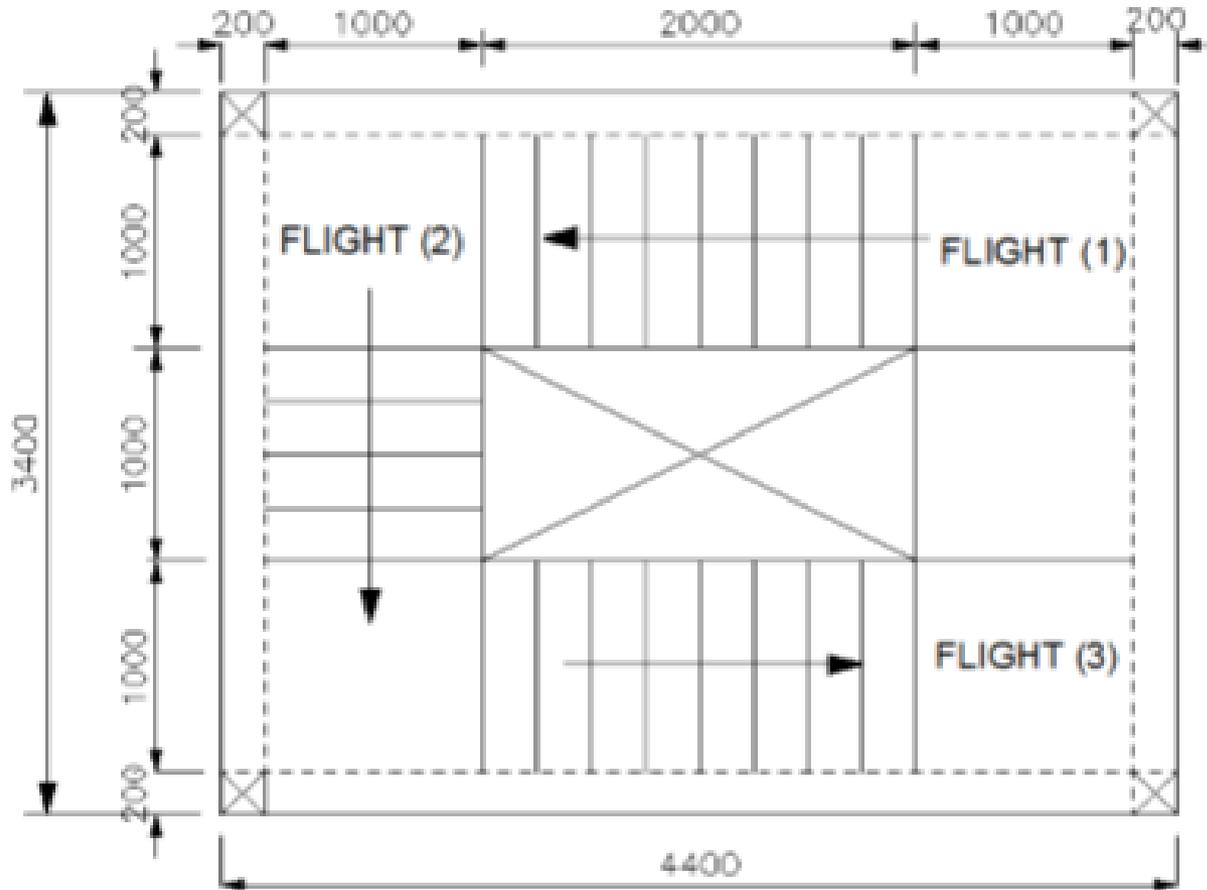
$h = 120 \text{ mm}$

## Tutorial 2: Staircase design

Figure 3 shows the plan view of open well stair support by beam at the end of its landing. The risers are 160 mm, goings are 250 mm as shown in Figure 4, and story height is 3.5 m. Goings are provided with 3 cm thick marble finish on cement mortar that weights 1.0 kN/m<sup>2</sup>. The landings are surface finished with terrazzo tiles on sand filling that weighs 1.2 kN/m<sup>2</sup>. The stair is to be designed for a variable action of 3.0 kN/m<sup>2</sup>. Design the staircase by providing the reinforcement. Check the shear, deflection, and crack and illustrate the curtailments of the staircase.

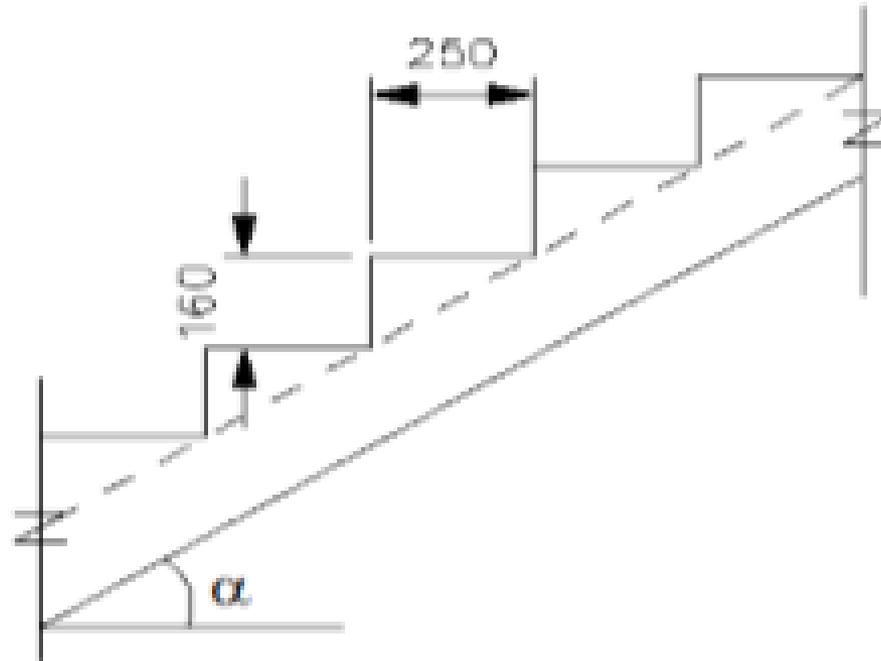
Use of concrete strength,  $f_{ck} = 30 \text{ N/mm}^2$ , steel strength,  $f_{yk} = 500 \text{ N/mm}^2$  and diameter of bar,  $\phi^{\text{main}} = 12 \text{ mm}$ . Nominal cover,  $c_{\text{nom}}$  for this stairs, is 25 mm.

# Tutorial 2: Staircase design



**Figure 3**

# Tutorial 2: Staircase design



**Figure 4**

# End of Examples and Tutorials