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REINFORCED CONCRETE DESIGN 1

Analysis of Section (Examples and Tutorials)

by

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Example 1: Singly reinforced rectangular section

A rectangular reinforced concrete beam has to support a design moment of 45 kNm. Determine the area of reinforcement required if the beam dimension is 150 x 315 mm (bxd), concrete strength $f_{ck} = 25 \text{ N/mm}^2$ and steel strength $f_{yk} = 500 \text{ N/mm}^2$

$$K = M/f_{ck}bd^2 = 45 \times 10^6 / (25 \times 150 \times 315^2) = 0.121$$

Redistribution = 0%

Redistribution ratio, $\delta = 1.0$

$$\begin{aligned} K_{bal} &= 0.454(\delta - k_1)/k_2 - 0.182 [(\delta - k_1)/k_2]^2 \\ &= 0.167 \end{aligned}$$

$K < K_{bal}$: Compression reinforcement not required

$$\begin{aligned} z &= d[0.5 + (0.25 - K/1.134)^{1/2}] \\ &= 0.88 d \end{aligned}$$

$$\begin{aligned} A_{s,req} &= M/0.87f_{yk}z \\ &= 45 \times 10^6 / (0.87 \times 500 \times 0.88 \times 315) \\ &= 373 \text{ mm}^2 \end{aligned}$$

Thus provide : 2H16
($A_s = 402 \text{ mm}^2$)

Example 2: Singly reinforced rectangular section

Determine the ultimate moment of resistance of a rectangular beam if the beam dimension is 150 x 315 mm (bxd), concrete strength $f_{ck} = 25 \text{ kN/mm}^2$ and steel strength $f_{yk} = 500 \text{ N/mm}^2$. 2H16 ($A_s = 402 \text{ mm}^2$) of tension reinforcement bar are provided.

Forces

$$\begin{aligned} F_{cc} &= 0.567f_{ck} b(0.8)x = 0.454f_{ck} bx \\ &= 1702.5 x \end{aligned}$$

$$\begin{aligned} F_{st} &= 0.87 f_{yk} A_s \\ &= 174870 \end{aligned}$$

Example 2: Singly reinforced rectangular section

Equilibrium of forces

$$F_{st} = F_{cc}$$

$$174870 = 1702.5 x$$

$$x = 102.7 \text{ mm}$$

Moment of resistance

$$M = F_{cc} \cdot z$$

$$= (1702.5 \times 102.7)(315 - 0.4(102.7))$$

$$= \mathbf{47.9 \text{ kNm}}$$

Example 3: Doubly reinforced rectangular section

A rectangular reinforced concrete beam has to support a design moment of 60 kNm. Determine the area of reinforcement required if the beam dimension is 150 x 315 mm (bxd), concrete strength $f_{ck} = 25 \text{ N/mm}^2$ and steel strength $f_{yk} = 500 \text{ N/mm}^2$. The effective depth of compression bar (d') is taken as 37 mm.

$$K = M/f_{ck}bd^2 = 70 \times 10^6 / (25 \times 150 \times 315^2) = 0.188$$

$K > K_{bal} = 0.167$: Compression reinforcement is required

$$\begin{aligned} z &= d [0.5 + (0.25 - K_{bal}/1.134)^{1/2}] \\ &= 0.82 d \end{aligned}$$

Example 3: Doubly reinforced rectangular section

$$x = (d - z) / 0.4 = 141.8 \text{ mm}$$

$$d'/x = 37 / 141.8 = 0.261 < 0.35$$

$$\begin{aligned} A_{s',\text{req}} &= (K - K_{\text{bal}})f_{\text{ck}}bd^2/0.87f_{\text{yk}}(d - d') \\ &= (0.188 - 0.167)(25)(150)(315^2)/ \\ &\quad (0.87 \times 500 \times (315 - 37)) \\ &= 65 \text{ mm}^2 \end{aligned}$$

Thus provide : 2H12 ($A_s = 226 \text{ mm}^2$)

Example 3: Doubly reinforced rectangular section

$$\begin{aligned}A_{s,req} &= K_{bal} f_{ck} b d^2 / 0.87 f_{yk} z_{bal} + A_{s',req} \\ &= 0.167(25)(150)(315^2) / (0.87 \times 500 \times 0.82 \times 315) + \\ &\quad 65 \\ &= 618 \text{ mm}^2\end{aligned}$$

Thus provide : 2H20 ($A_s = 628 \text{ mm}^2$)

Example 4: Doubly reinforced rectangular section

Determine the ultimate moment of resistance of a rectangular beam if the beam dimension is 150 x 315 mm (bxd), compression effective depth (d') = 37 mm, concrete strength $f_{ck} = 25 \text{ kN/mm}^2$ and steel strength $f_{yk} = 500 \text{ N/mm}^2$. 2H12 and 2H20 are provided for compression and tension reinforcement bar, respectively.

Compression : **2H12** $A_s' = 226 \text{ mm}^2$

Tension : **2H20** $A_s = 628 \text{ mm}^2$

Example 4: Doubly reinforced rectangular section

Assuming initially that the steel stresses f_{st} and f_{sc} are the design yield value $0.87 f_{yk}$

Forces

$$\begin{aligned} F_{cc} &= 0.567 f_{ck} b (0.8)x = 0.454 f_{ck} b x \\ &= 1702.5 x \end{aligned}$$

$$\begin{aligned} F_{sc} &= 0.87 f_{yk} A'_s \\ &= 98310 \end{aligned}$$

$$\begin{aligned} F_{st} &= 0.87 f_{yk} A_s \\ &= 273180 \end{aligned}$$

Example 4: Doubly reinforced rectangular section

Equilibrium of forces

$$F_{st} = F_{cc} + F_{sc}$$

$$273180 = 1702.5 x + 98310$$

$$x = 102.7 \text{ mm} < 0.617d = 194 \text{ mm}$$

Thus, tension steel has been yield as assumed

$$d'/x = 37/102.7$$

$$= 0.36 < 0.38$$

Thus, compression steel has yield as assumed

Example 4: Doubly reinforced rectangular section

Lever arm

$$z = d - 0.4x = 273.9 \text{ mm}$$

$$z_1 = d - d' = 278 \text{ mm}$$

Moment of resistance

$$\begin{aligned} M &= F_{cc} \cdot z + F_{sc} \cdot z_1 \\ &= (1702.5 \times 102.7 \times 273.9) + (98310 \times 278) \\ &= \mathbf{75.2 \text{ kNm}} \end{aligned}$$

Example 5: Singly reinforced flanged section

Determine the area of reinforcement required in a T beam with the following dimension for an applied moment of 250 kNm. Depth of slab 100 mm, width of flange 800 mm, width of web 200 mm, effective depth 400 mm. Use $f_{ck} = 25 \text{ N/mm}^2$ and $f_{yk} = 500 \text{ N/mm}^2$.

$$\begin{aligned}M_f &= 0.567f_{ck}bh_f(d - 0.5h_f) \\ &= 0.567 \times 25 \times 800 \times 100 (400 - 0.5 \times 100) \\ &= 396 \text{ kNm}\end{aligned}$$

$M < M_f$ Thus, Neutral axis lies in flange

Example 5: Singly reinforced flanged section

$$\begin{aligned}K &= M/f_{ck}bd^2 \\ &= 250 \times 10^6 / (25 \times 800 \times 400^2) \\ &= 0.078\end{aligned}$$

$$K_{bal} = 0.167$$

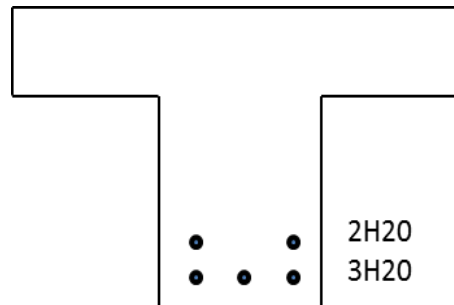
$K < K_{bal}$: Thus, Compression reinforcement is not required

$$\begin{aligned}z &= d[0.5 + (0.25 - K/1.134)^{1/2}] \\ &= 0.93 d\end{aligned}$$

Example 6: Singly reinforced flanged section

$$\begin{aligned}A_s &= M/0.87f_{yk}z \\ &= 250 \times 10^6 / (0.87 \times 500 \times 0.93 \times 400) \\ &= 1545 \text{ mm}^2\end{aligned}$$

Thus provide : 5H20 ($A_s = 1571 \text{ mm}^2$)



Example 6: Singly reinforced flanged section

Determine the area of reinforcement required in a T beam with the following dimension for an applied moment of 250 kNm. Depth of slab 100 mm, width of flange 500 m, width of web 200 mm, effective depth 400 mm. Use $f_{ck} = 25$ N/mm² and $f_{yk} = 500$ N/mm².

$$\begin{aligned}M_f &= 0.567f_{ck}bh_f(d - 0.5 h_f) \\ &= 0.567 \times 25 \times 500 \times 100 (400 - 0.5 \times 100) \\ &= 248 \text{ kNm}\end{aligned}$$

$M < M_f$ Thus, neutral axis lies in web

Example 6: Singly reinforced flanged section

$$\begin{aligned}\beta_f &= 0.167(b_w/b) + 0.567(h_f/d)(1-b_w/b)(1-h_f/2d) \\ &= 0.167 \times 0.4 + 0.567 \times 0.25 \times (1 - 0.4) (1 - 0.125) \\ &= 0.141\end{aligned}$$

$$\begin{aligned}M_{bal} &= \beta_f f_{ck} b d^2 \\ &= 0.141 \times 25 \times 500 \times 400^2 \\ &= 282 \text{ kNm}\end{aligned}$$

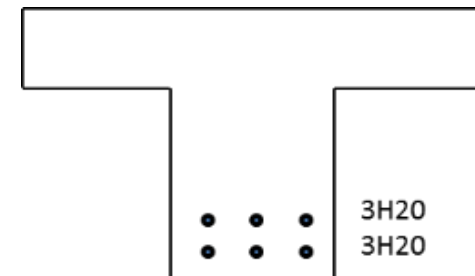
$M < M_{bal}$ Thus, compression reinforcement is not required!

Example 6: Singly reinforced flanged section

Area of tension reinforcement

$$\begin{aligned} A_s &= [M + 0.1f_{ck}b_w d(0.36d - h_f)] / 0.87f_{yk}(d - 0.5h_f) \\ &= \frac{250 \times 10^6 + 0.1 \times 25 \times 200 \times 400 \times (0.36 \times 400 - 100)}{0.87 \times 500 (400 - 0.5 \times 100)} \\ &= 1699 \text{ mm}^2 \end{aligned}$$

Thus provide : 6H20 ($A_s = 1885 \text{ mm}^2$)



Example 7: Doubly reinforced flanged section

Determine the ultimate moment resistance of the following T beam: $h_f = 100$ mm, $b = 700$ mm, $b_w = 225$ mm, $d = 500$ mm and $d' = 50$ mm. The tension reinforcement provided is 6H25 and the compression reinforcement is 3H12. Used $f_{ck} = 25$ N/mm² and $f_{yk} = 500$ N/mm²

Compr. reinforcement = 3H12

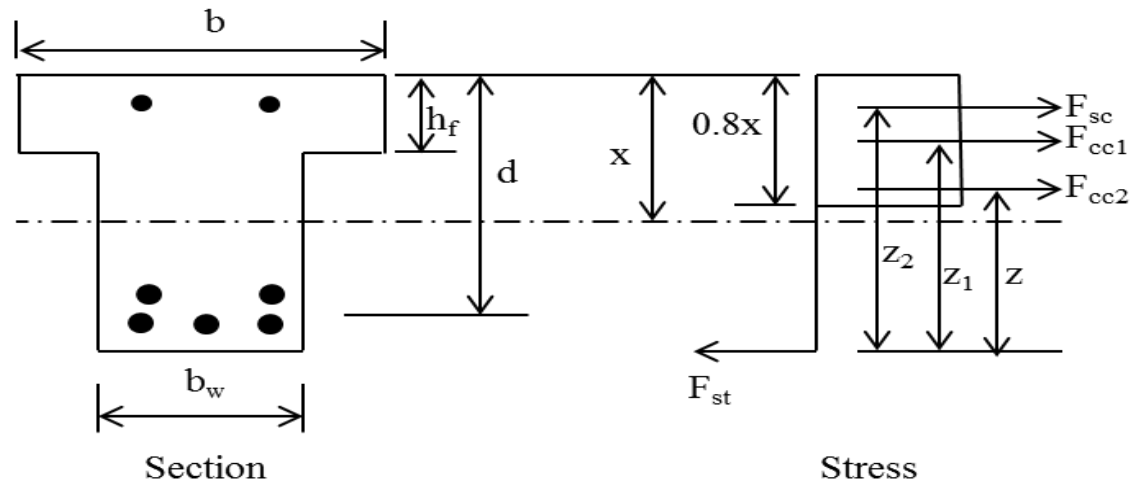
$$A_s = 339 \text{ mm}^2$$

Tension reinforcement = 6H25

$$A_s' = 2946 \text{ mm}^2$$

Example 7: Doubly reinforced flanged section

Assuming initially that steel stress f_{st} is the design yield value $0.87f_{yk}$ and neutral axis in the web



Example 7: Doubly reinforced flanged section

$$F_{cc1} = 0.454f_{ck}b_w x = 2554 x$$

$$F_{cc2} = 0.567f_{ck}(b-b_w)h_f = 673313$$

$$F_{sc} = 0.87f_{yk}A'_s = 147465$$

$$F_{st} = 0.87f_{yk}A_s = 1281510$$

Equilibrium of forces

$$F_{st} = F_{cc1} + F_{cc2} + F_{sc}$$

$$1281510 = 2554 x + 673313 + 147465$$

$$x = 180 \text{ mm} > 100 \text{ mm}$$

= Neutral axis in the web as assumed

Example 7: Doubly reinforced flanged section

Lever Arm

$$z = d - 0.4x = 500 - 0.4 \times 180 = 428 \text{ mm}$$

$$z_1 = d - d' = 500 - 50 = 450 \text{ mm}$$

$$z_2 = d - 0.5h_f = 500 - 0.5 \times 100 = 450 \text{ mm}$$

Moment of resistance

$$\begin{aligned} M &= F_{cc1} \cdot z + F_{sc} \cdot z_1 + F_{cc2} \cdot z_2 \\ &= (2554 \times 180 \times 428) + (673313 \times 450) + \\ &\quad (1281510 \times 450) \\ &= \mathbf{506 \quad kNm} \end{aligned}$$

Tutorial: Rectangular section

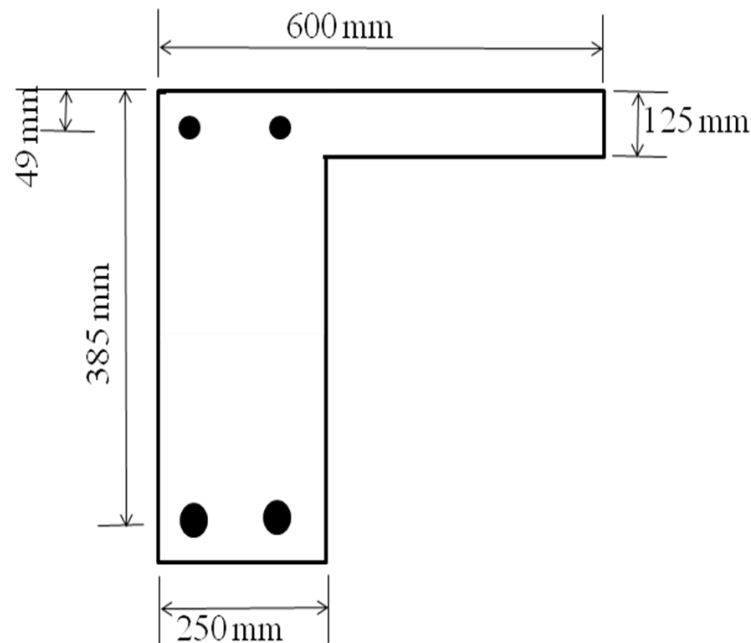
1. A reinforced concrete beam has breadth of 300 mm and effective depth of 500 mm. Tension and compression reinforcement provided are 6H25 and 3H12 respectively. Determine the ultimate moment capacity of the section, if $f_{ck} = 25 \text{ N/mm}^2$ and $f_{yk} = 500 \text{ N/mm}^2$. The effective depth to compression reinforcement is 55 mm.

Tutorial: Flanged Section

1. Determine the area of steel required in a T beam with the following dimensions for an applied moment of **250 kNm**; depth of slab = 110 mm, width of flange = 650 mm, width of web = 200 mm, effective depth = 400 mm. Consider concrete class 25/30 and high yield steel.
2. Determine the area of steel required in a T beam with the following dimensions for an applied moment of **400 kNm**; depth of slab = 110 mm, width of flange = 650 mm, width of web = 200 mm, effective depth = 400 mm, effective depth of compression reinforcement = 50 mm. Consider concrete class 25/30 and high yield steel.

Tutorial: Flanged Section

An L-section beam as shown below is required to resist an ultimate design moment of 380 kNm and design shear of 285 kN. Determine the area of steel required for longitudinal reinforcement. Use characteristic strength of concrete and steel reinforcement of 25 N/mm² and 500 N/mm², respectively.



End of Tutorial