MECHANICS OF MATERIALS

Torsion

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Chapter Description

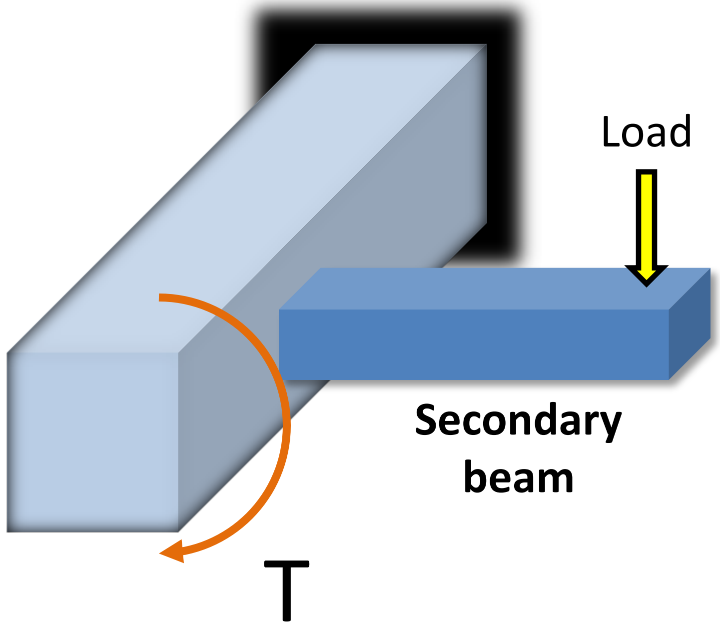
- Expected Outcomes
  - Define the concept on torsion for circular shaft – hollow and solid shaft
  - Describe the concept on deformation of a circular shaft – hollow and solid shaft
  - Calculate the stress and shear stress for the circular shaft
  - Calculate the polar moment of inertia
  - Apply the principle of torsion formula – determine the torsional deformations
  - Calculate the angle of twist for circular shaft

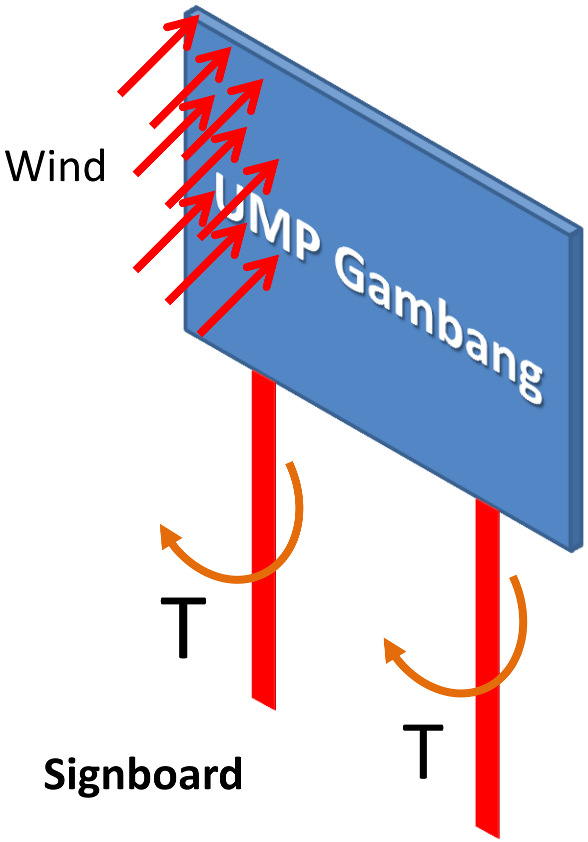
8.1 Introduction

- Stresses also can occur within a structural element due to **torsional** or **twisting effect**
- Torsion refers to the loading of a member that tends to cause it to **rotate** or **twist**
- Such a load is called a **torque, rotational moment, twisting moment** or **couple**
- **Torsional deformation** created when a torque is applied to a member, shearing stress is developed

- Torsion – Applications more to mechanical engineering : machineries, shaft transmitting power, turbine, motor etc
- Applications due to civil engineering : certain structures are considered to torsion (some cases torsion is unnoticed)
- Examples:
  - Secondary beam
  - Column for signboard
  - Beam supported gutter

Big Pictures





Torque

- **Torque** is a moment that tends to twist a member about its **longitudinal axis**
- This simplest device for accomplishing this function is called a **shaft**

8.2 Shaft Deformation

- As mention earlier, torque is a moment that tends to **twist** a member about its longitudinal axis
- If the angle of rotation is **small**, the **length of the shaft** and **its radius** will **remain unchanged**

- From observation, the angle of twist of the shaft is proportional to the applied torque and to the shaft length

$$\phi \propto T$$

$$\phi \propto L$$

- When subjected to torsion, every cross section of a circular shaft remains plane and undistorted

- Cross-sections for hollow and solid circular shafts remain plain and undistorted because a circular shaft is axisymmetric

- Cross-sections of noncircular (non-axisymmetric) shafts are distorted when subjected to torsion

8.3 Failure modes

Failure of **ductile** specimen:

A ductile specimen breaks along a plane of maximum shear, i.e., a plane perpendicular to the shaft axis

Failure of **brittle** specimen:

A brittle specimen breaks along planes perpendicular to the direction in which tension is a maximum, i.e., along surfaces at 45^o to the shaft axis

8.4 Torsion Formula

- When material is linear-elastic, Hooke's law applies
- A **linear variation in shear strain** leads to a corresponding **linear variation in shear stress** along any radial line on the cross section
- To determine the shear stress:

$$\tau_{\max} = \frac{Tc}{J} \quad \text{or} \quad \tau = \frac{T\rho}{J}$$

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$$\tau_{\max} = \frac{Tc}{J} \quad \text{or} \quad \tau = \frac{Tp}{J}$$

$\tau_{\max}$ = maximum shear stress in the shaft

τ = shear stress

T = resultant internal torque

J = polar moment of inertia of cross-sectional area

c = outer radius of the shaft

p = intermediate distance

- If the shaft has a **solid circular** cross section:

$$J = \frac{\pi}{2} c^4$$

- If a shaft has a **tubular** cross section:

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$

Torsional Deformations

- If the shear stresses in a shaft are below the proportional limit of the shaft material (elastic action), then Hooke's Law relates shear stress and shear strain in the torsion member is:

$$\tau = G\gamma$$

- Using Hooke's Law:

8.5 Angle of Twist

- Integrating over the entire length *L* of the shaft, we have

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G}$$

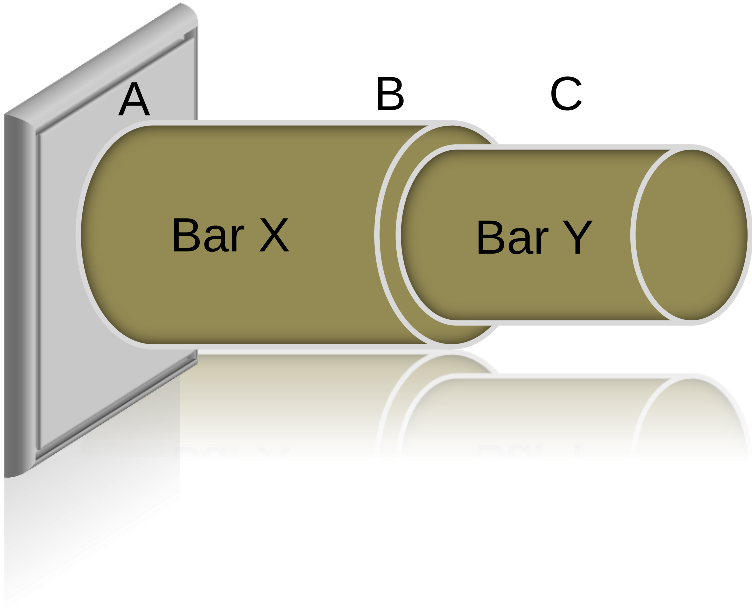
Φ = angle of twist
T(x) = internal torque
J(x) = shaft's polar moment of inertia
G = shear modulus of elasticity for the material

- Assume material is homogeneous, G is constant, thus

$$\phi = \frac{T_i L_i}{J_i G_i}$$

- Sign convention is determined by **right hand rule**

Angle Of Twist For Two Bars



- External torsion applied is equal to internal torsion on bar

$$T = T_{bar1} + T_{bar2}$$

- Angle of twist for bar X is equal to bar Y

$$\phi_{bar1} = \phi_{bar2}$$

- Total angle of twist can be determine using angle of twist formula $\phi = \frac{TL}{JG}$

References

- Hibbeler, R.C., Mechanics Of Materials, 9th Edition in SI units, Prentice Hall, 2013.
- Ferdinand P. Beer, E. Russell Johnston, Jr., John T. DeWolf, David F. Mazurek, Mechanics of materials 5th Edition in SI Units, McGraw Hill, 2009.

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