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BCS3323 – Software Testing and Maintenance

Combinatorial Testing

Editors

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Chapter 3.3

- Aims is to discover
 - The Combinatorial testing need
 - The efficiency of Combinatorial testing
 - The type of Combinatorial testing
 - Different Combinatorial testing approaches
- Expected Outcomes
 - Students can use Combinatorial testing to design test List
 - Differentiate between all Combinatorial testing approaches
- References
 - ISTQB
 - MSTB/GTB
 - <http://www.softwaretestingclass.com/software-testing-tools-list/>
 - <http://www.softwaretestinggenius.com/articalDetails.php?qry=572#commentsList>



Is Combinatorial Testing Important?



How much to test?

- 34 hardware switches = $2^{34} = 1.7 \times 10^{10}$ possible input configurations = 1.7×10^{10} tests.
- **Is it possible to test every possible combinations** of inputs?

Combinatorial Explosion Problem in Hardware Product Testing

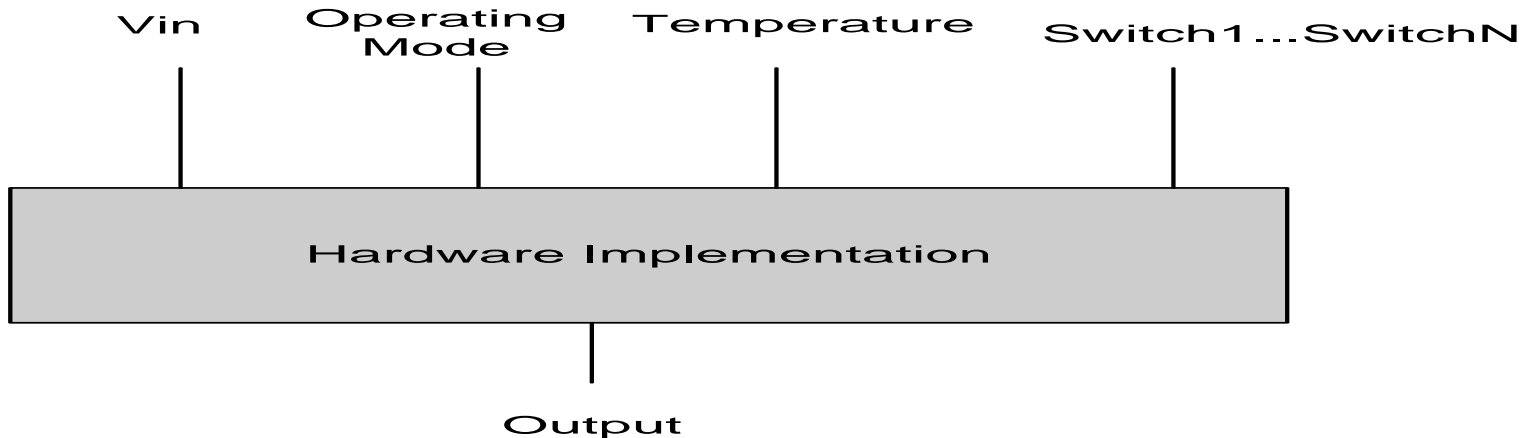
PRODUCT TESTING

$V_{in} = \{-5, 0, +5\}$ Volts

Operating Mode = {Reverse, Forward}

Temperature = {Low, Medium, High}

Switch1.. SwitchN = {On, Off}



Let us assume $N=5$

Exhaustive possibilities = $3 \times 2 \times 3 \times 2^5 = 576$

What if we need to test at least 100 products a day?

576x100 tests per day!

If we are late, we will lose our business to our competitors.....

Approach to address combinatorial explosion problem

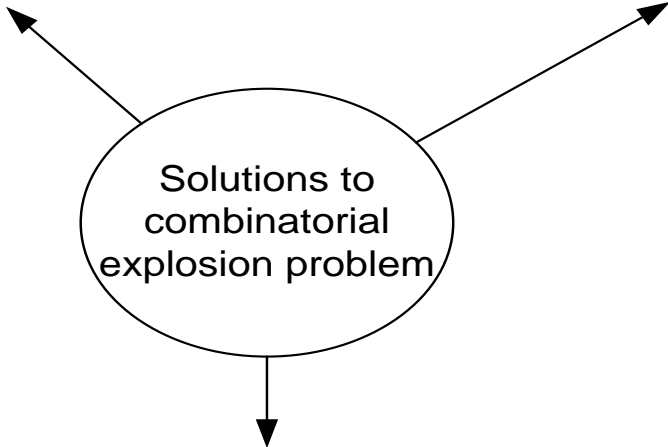
Random testing (UNFAIR DISTRIBUTION)

- Non systematic
- Tend to stress on certain configurations

Manual testing process (IMPOSSIBLE)

- Test engineers need to write write test cases
- Test engineers need to write test drivers for each test cases
- Test engineers need to execute the test driver

Solutions to
combinatorial
explosion problem



Parallel Testing (EXPENSIVE)

- Software applications are getting more complex and larger in terms of line of codes
- Test engineers need to test more and more codes
- Many more combinations of input parameters and system conditions need to be tested

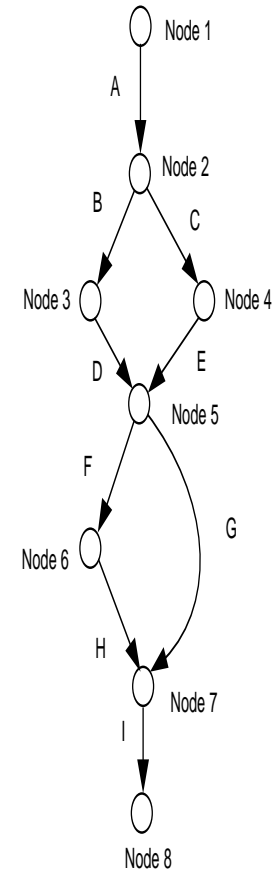
Given limited time and resources, what are the optimum sets of smaller test cases to be used for testing?

On Sampling Strategy

- As highlighted earlier, **testing of every possibilities is impractical**. There is a need for a sampling strategy.
- Is the idea of **sampling strategy** for software/hardware testing new?
- No!!!**
- Some of existing sampling strategies: Equivalent Partitioning, Boundary Value, Robustness, Cause and Effect etc.
- Existing sampling strategies does not sufficiently cater for **fault possibilities due to interaction**

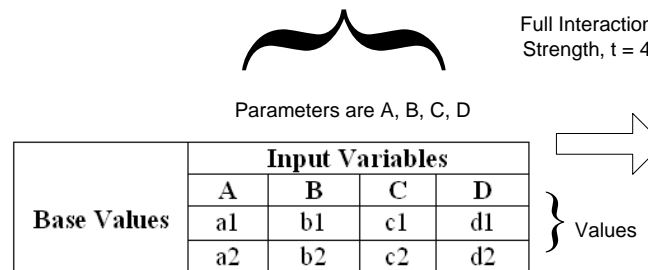
```
void ValveControl (int pressure, int temperature)
{
    if (pressure >=100)
    {
        OpenTheValve();
        printf ("Valve opened \n");
    }
    else
    {
        CloseTheValve();
        printf ("Valve closed \n");
    }

    if (temperature > 27)
    {
        EnableCoolingCoil();
        printf ("Cooling coil enabled \n");
    }
}
```



Motivating Example - Interaction Testing

- Interaction testing (a.k.a. t-way testing) complements existing sampling strategies.
- Consider a system with 4 parameters with each with 2 values. At full **interaction strength, $t = 4$** , correspond to all combinations



Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	

Exhaustive Combinations	A	B	C	D
	a1	b1	c1	d1
	a1	b1	c1	d2
	a1	b1	c2	d1
	a1	b1	c2	d2
	a1	b2	c1	d1
	a1	b2	c1	d2
	a1	b2	c2	d1
	a1	b2	c2	d2
	a2	b1	c1	d1
	a2	b1	c1	d2
	a2	b1	c2	d1
	a2	b1	c2	d2
	a2	b2	c1	d1
	a2	b2	c1	d2
	a2	b2	c2	d1
a2	b2	c2	d2	

- What if we **relax or compromise** the interaction strength?

What if we consider all possible 3 way interaction: ABC, ABD, ACD, and BCD

Combining each 3-way combinations

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for ABC, t=3	a1	b1	c1	d1
	a1	b1	c2	d2
	a1	b2	c1	d1
	a1	b2	c2	d2
	a2	b1	c1	d1
	a2	b1	c2	d2
	a2	b2	c1	d1
	a2	b2	c2	d1

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for ACD, t=3	a1	b1	e1	d1
	a1	b2	e1	d2
	a1	b1	e2	d1
	a1	b1	e2	d2
	a2	b1	e1	d1
	a2	b1	e1	d2
	a2	b2	e2	d1
	a2	b2	e2	d2

All 3-way combinations

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial Values with t=3	a1	b1	c1	d1
	a1	b1	c2	d2
	a1	b2	c1	d1
	a1	b2	c2	d2
	a2	b1	c1	d1
	a2	b1	c2	d2
	a2	b2	c1	d1
	a2	b2	c2	d1
	a2	b2	c2	d2
	a1	b1	c1	d2
	a1	b2	c1	d2
	a2	b1	c1	d2
	a2	b2	c1	d2

Removing repetitions

=

Total test data = 13

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for ABD, t=3	a1	b1	e1	d1
	a1	b1	e1	d2
	a1	b2	e2	d1
	a1	b2	e2	d2
	a2	b1	e1	d1
	a2	b1	e1	d2
	a2	b2	e2	d1
	a2	b2	e2	d2
Combinatorial values for BCD, t=3	a1	b1	e1	d1
	a1	b1	e1	d2
	a1	b2	e2	d1
	a1	b2	e2	d2
	a2	b1	e1	d1
	a2	b1	e1	d2
	a2	b2	e2	d1
	a2	b2	e2	d2

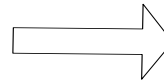
The reduction is now from 16 to 13.....at t=3

Demonstration of Correctness

All 3 Way Pairs

Interaction	3 Way pairs	Occurrences
ABC	a1, b1,c1	2
	a1,b1,c2	1
	a1,b2,c1	2
	a1,b2,c2	2
	a2, b1,c1	2
	a2,b1,c2	1
	a2,b2,c1	1
	a2,b2,c2	2
ABD	a1, b1,d1	1
	a1,b1,d2	2
	a1,b2,d1	2
	a1,b2,d2	2
	a2, b1,d1	1
	a2,b1,d2	2
	a2,b2,d1	2
	a2,b2,d2	1
BCD	b1,c1,d1	2
	b1,c1,d2	2
	b1,c2,d1	1
	b1,c2,d2	2
	b2, c1,d1	2
	b2,c1,d2	1
	b2,c2,d1	2
	b2,c2,d2	2
ACD	a1,c1,d1	2
	a1,c1,d2	2
	a1,c2,d1	1
	a1,c2,d2	2
	a2, c1,d1	2
	a2,c1,d2	1
	a2,c2,d1	1
	a2,c2,d2	2

All 3-Way Interactions



Test suite based on coverage strength t=3

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
All T-way Combinatorial Values	a1	b1	c1	d1
	a1	b1	c2	d2
	a1	b2	c1	d1
	a1	b2	c2	d2
	a1	b2	c1	d1
	a1	b2	c2	d2
	a2	b1	c1	d1
	a2	b1	c2	d2
	a2	b2	c1	d1
	a2	b2	c2	d2
	a1	b1	c1	d2
	a1	b2	c2	d1
	a1	b2	c1	d2
	a2	b1	c1	d2
	a2	b2	c2	d2

Notice that all the 3 way pairs have been covered by at least one test

All 4 Way (i.e. t= 4, full strength) => 16
 All 3 Way (i.e. t=3) => 13
 Hence, the reduction => $\frac{3}{16} * 100 \%$
 = 18.75 %

What if we relaxed the interaction to t=2?

Combining each 2-way combinations

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for AB, t=2	a1	b1	c1	d1
	a1	b2	c2	d2
	a2	b1	c1	d1
	a2	b2	c2	d2

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for AC, t=2	a1	b1	c1	d1
	a1	b2	c2	d2
	a2	b1	c1	d1
	a2	b2	c2	d2

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for AD, t=2	a1	b1	c1	d1
	a1	b2	c2	d2
	a2	b1	c1	d1
	a2	b2	c2	d2

+

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for BC, t=2	a1	b1	c1	d1
	a1	b1	c2	d2
	a2	b2	c1	d1
	a2	b2	c2	d2

Removing repetitions

=

All 2-way combinations

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial Values with t=2	a1	b1	c1	d1
	a1	b2	c2	d2
	a2	b1	c1	d1
	a2	b2	c2	d2
	a2	b1	c1	d1
	a1	b1	c2	d2
	a2	b2	c1	d1
	a1	b2	c1	d2
	a2	b1	c2	d1

Total test data = 9

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for BD, t=2	a1	b1	c1	d1
	a1	b1	c2	d2
	a2	b2	c1	d1
	a2	b2	c2	d2

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	
Combinatorial values for CD, t=2	a1	b1	c1	d1
	a1	b2	c1	d2
	a2	b1	c2	d1
	a2	b2	c2	d2

More reduction now from 16 to 9.....at t=2

Why not Pairwise t=2?

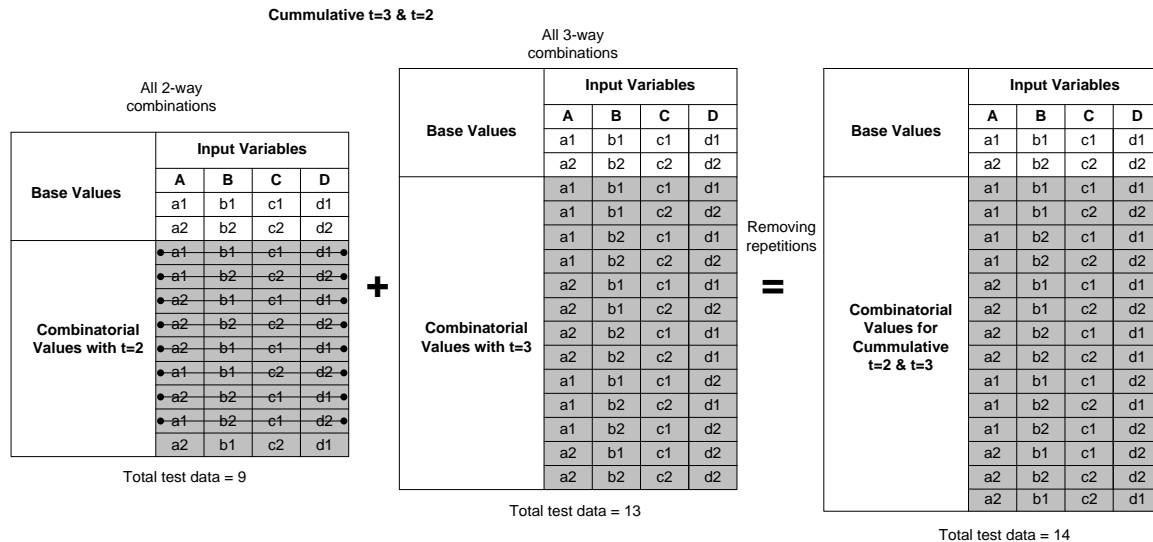
- Pairwise => gives the most reduction
 - 4 parameter, 3 values
 - Exhaustive = $3^4 = 3 \times 3 \times 3 \times 3 = 81$ test cases
 - Pairwise = 9 test cases
 - 13 parameter, 3 values
 - Exhaustive = $3^{13} = 3 \times 3 \times 3 \times \dots = 1,594,323$ test cases
 - Pairwise = 15 test cases
 - 20 parameter, 10 values
 - Exhaustive = $10^{20} = 10 \times 10 \times 10 \times \dots =$ a lot of test cases
 - Pairwise = 180 test cases

Pairwise saves time and costs.....

Why not Pairwise a.k.a. $t=2$?

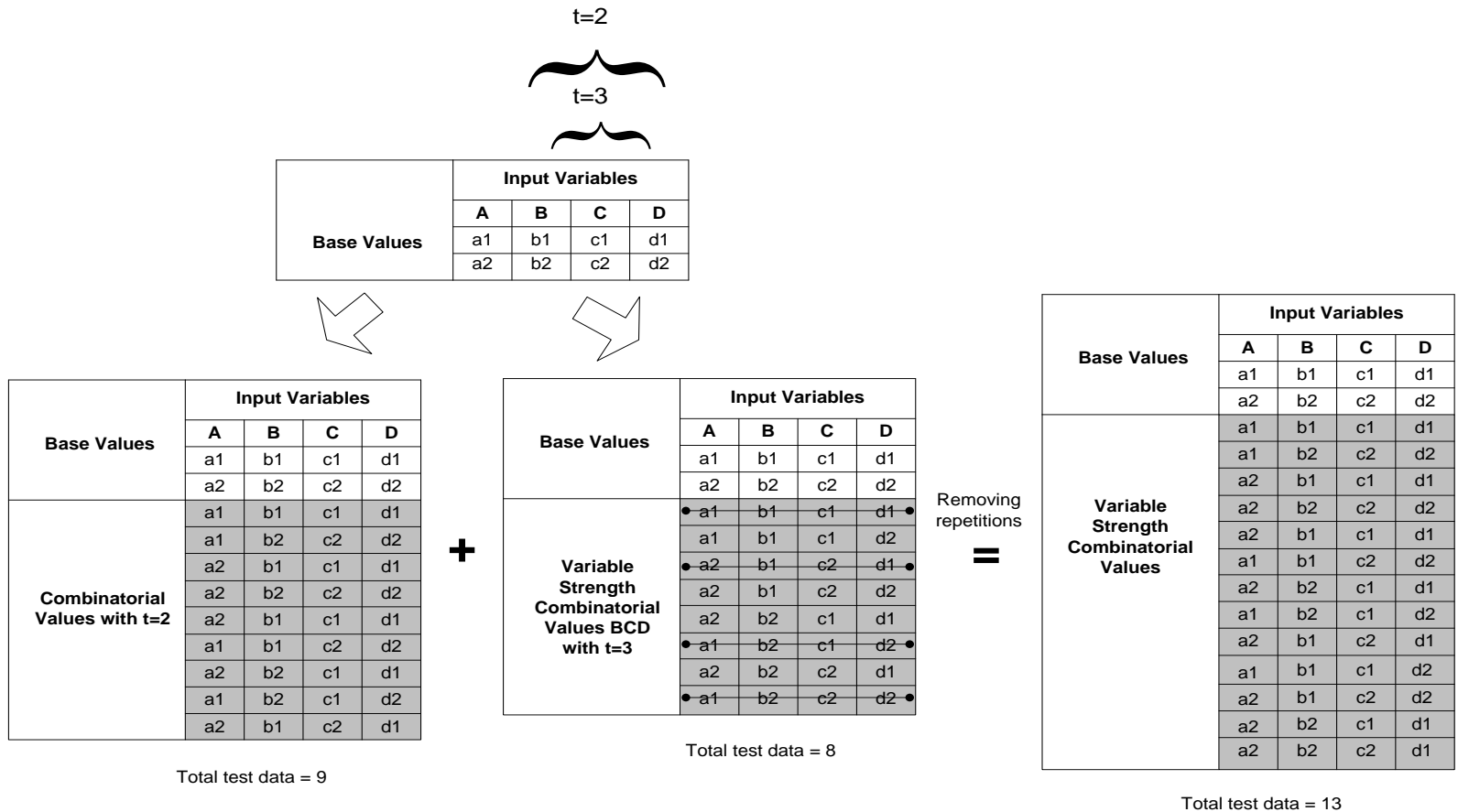
- In some system where there are **hardly any interaction of variables**, pairwise testing can generate a very efficient test case that are able to locate between **75%-85%** of faults.
- While such conclusion may be true for some system, it **cannot be generalized** to all software/hardware system faults (i.e. especially when there are **significant interactions** between variables)....

What if we do cumulative t=2 & t=3?



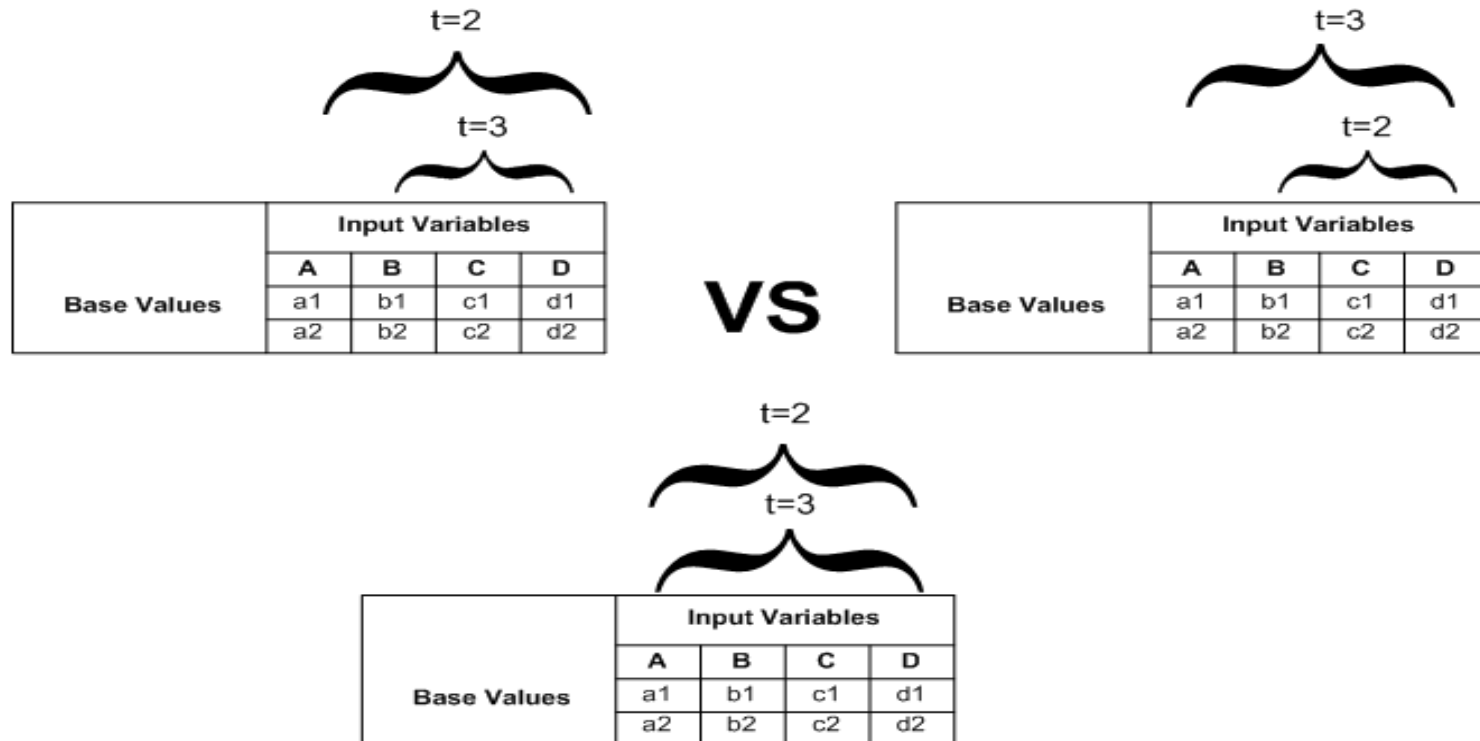
Reduction is now from 16 to 14.....for cumulative t=2 & t=3.....

How about if we consider variable strength? In real life, interaction is not always uniform



Reduction is now from 16 to 13.....

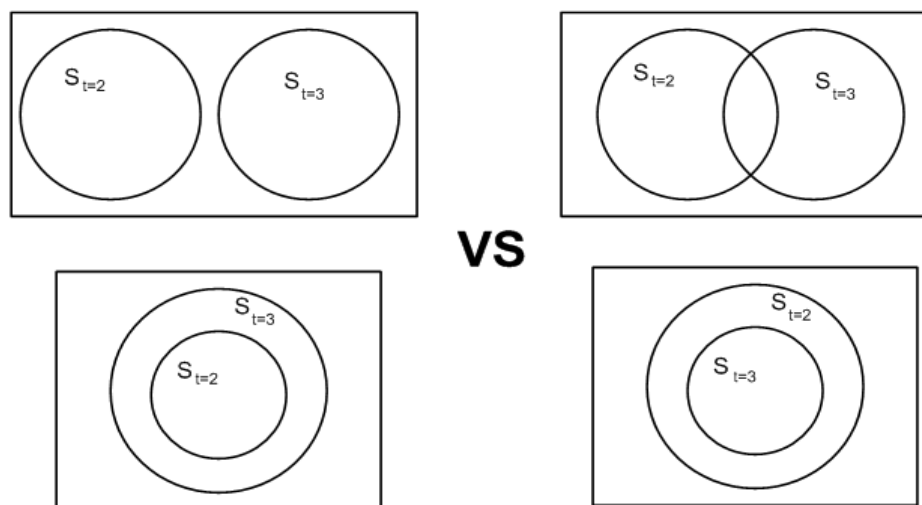
Something to ponder?



- Which is the good choice for applying variable strength?

Something to ponder?

Venn Diagram



- Let $S_{t=2} = \{\text{the set of test cases for } t=2\}$ and $S_{t=3} = \{\text{the set of test cases for } t=3\}$
- Is $S_{t=2}$ is the subset of $S_{t=3}$?

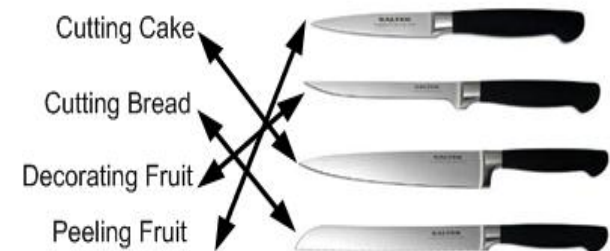
Observation on our application of t-way testing strategies - contd.

- There are four possibilities for applying t-way strategies for test reduction
 - Uniform Strength Interaction
 - Cumulative Strength Interaction
 - Variable Strength Interaction
- Which is the most effective strategy for testing?
 - It depends on circumstances. Refer to the chef's knife analogy.



Recommendation??

- Which is the most effective strategy for testing?
 - Uniform strength – We have no idea on the system under test...We have no knowledge on the effects of inputs on output(s).
 - Cumulative strength/Variable strength – Based on our experience, we have some knowledge on the contribution of particular parameters. Hence, we want to relax interaction on overall strength but focus on specific interaction. We have no knowledge on the effects of inputs on output(s).
 - The choice of interaction strength (t)???
 - Literature suggests $2 < t < 6$.. See www.pairwise.org
 - We can debate this?



Existing Tools and Strategies

- Some Existing strategies used
Optimization Algorithms:

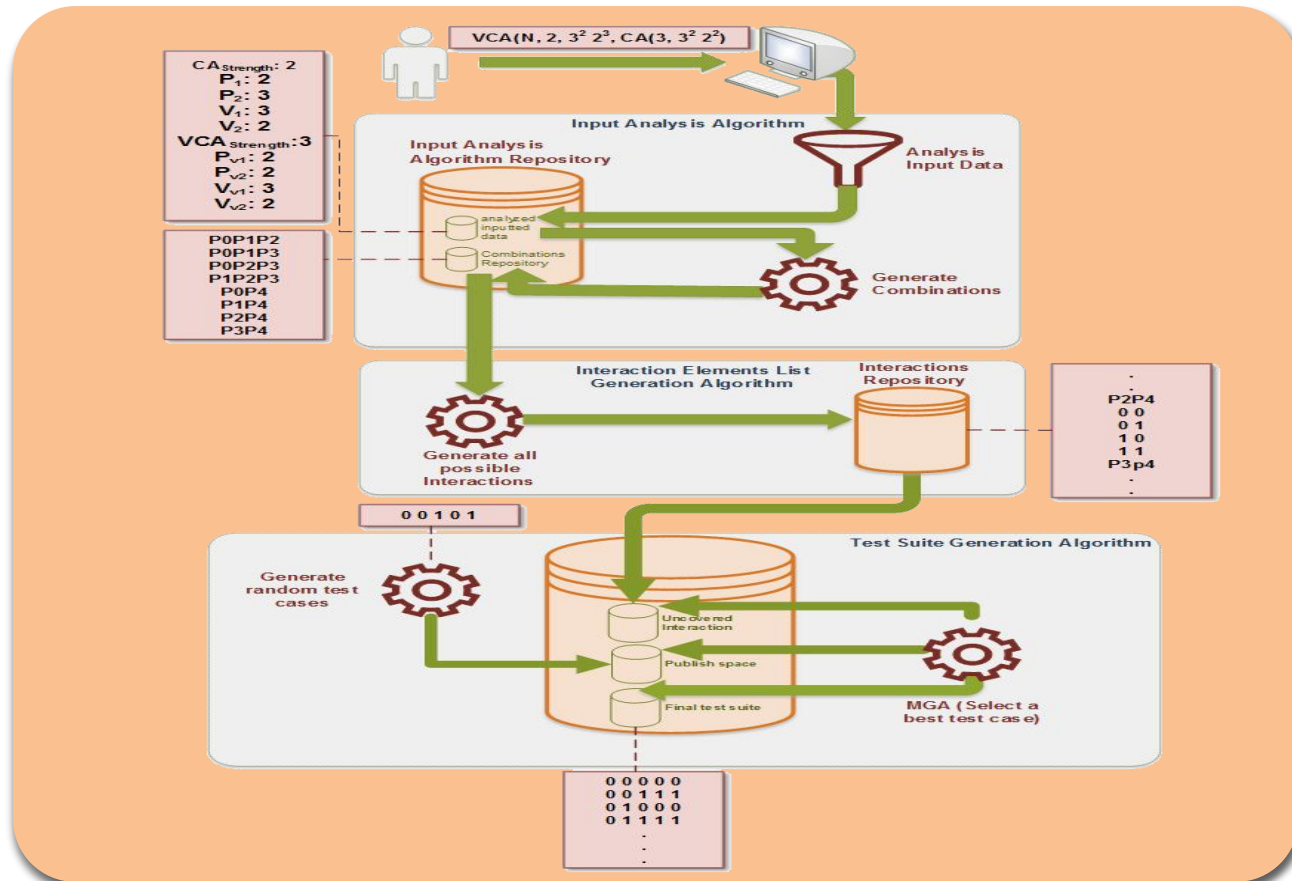
- ACO
- PSO
- HS
- GA
- SA
- HHH
- Greedy Algorithm
- FP
- FF

Only published in the research papers

- Source: <http://www.pairwise.org/tools.asp>

Available Tools		
1. CATS (Constrained Array Test System) *)	[Sherwood] Bell Labs.	
2. OATS (Orthogonal Array Test System) *)	[Phadke] ATT	
3. AETG	Telecordia	Web-based, commercial
4. IPO (PairTest) *)	[Tai/Lei]	
5. TConfig	[Williams]	Java-applet
6. TCG (Test Case Generator) *)	NASA	
7. AllPairs	Satisfice	Perl script, free, GPL
8. Pro-Test	SigmaZone	GUI, commercial
9. CTS (Combinatorial Test Services)	IBM	Free for non-commercial use
10. Jenny	[Jenkins]	Command-line, free, public-domain
11. ReduceArray2	STSC, U.S. Air Force	Spreadsheet-based, free
12. TestCover	Testcover.com	Web-based, commercial
13. DDA *)	[Colburn/Cohen/Turban]	
14. Test Vector Generator		GUI, free
15. OA1	k sharp technology	
16. CTE-XL	Berner & Mattner	GUI, free

Test list generation based on Optimization Algorithms



MGS : Intelligent Automatic Test Cases Generator Tool

File Help

System Configuration **Generate Test Cases**

Input the System configuration Read From File:

Input the configuration system information:

Uniform values Mixed values

Number of Paramters e.g. 3 , or 100, ...etc...

Number of parameter's values e.g: 1,2,3,4,5 or 10, ..etc..

Number of parameter's values e.g: 2,3,3,5 (no. of Parameters, their values)

Mixed values: 2 paramters has 3 values, and the another 3 parameters has 5 values

Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...	Par...
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9

MGS : Intelligent Automatic Test Cases Generator Tool

File Help

System Configuration Generate Test Cases

Main Configuration

Reaptation of running
1

Select the combinatorial Type:

Uniform Strength Variable Strength

Main Strength: 2

Variable Strength: 3

Generate Final Test Cases

Variable Interaction Strength

Identify the system's configuration parameters

- Parameter 9
- Parameter 10
- Parameter 11
- Parameter 12
- Parameter 13
- Parameter 14
- Parameter 15
- Parameter 16
- Parameter 17
- Parameter 18
- Parameter 19
- Parameter 20

Select ->

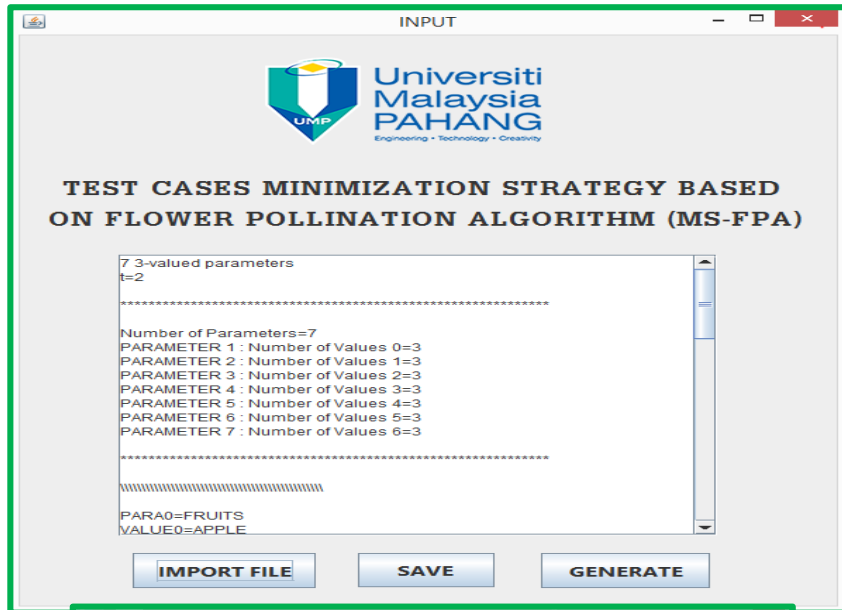
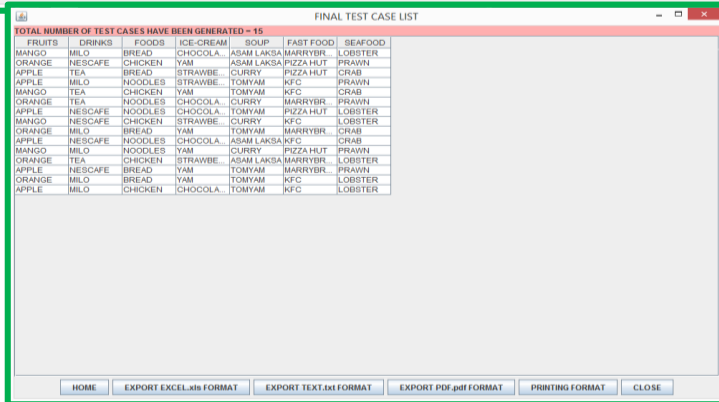
<- Remove

Export Best Result **Draw Line Chart**

```

117:[2, 9, 8, 7, 1, 8, 1, 7, 8, 1, 4, 7, 9, 4, 7, 4, 6, 0, 9, 2], Covers: 65 interactions, Covered: 77.94%.
118:[0, 6, 5, 4, 3, 3, 9, 9, 5, 5, 7, 5, 2, 7, 2, 8, 3, 5, 5, 9], Covers: 64 interactions, Covered: 78.27%.
119:[7, 2, 2, 5, 4, 1, 3, 0, 8, 0, 9, 6, 5, 2, 5, 7, 6, 4, 6, 0], Covers: 63 interactions, Covered: 78.61%.
120:[3, 1, 1, 6, 5, 9, 2, 4, 9, 6, 4, 6, 1, 7, 1, 0, 7, 0, 7, 1], Covers: 62 interactions, Covered: 78.93%.
121:[8, 9, 2, 8, 3, 0, 8, 2, 6, 2, 2, 4, 3, 8, 6, 7, 5, 7, 5, 5], Covers: 63 interactions, Covered: 79.26%.
122:[0, 3, 1, 1, 4, 1, 9, 5, 4, 6, 3, 9, 6, 4, 0, 9, 0, 2, 1, 1], Covers: 61 interactions, Covered: 79.58%.
123:[8, 0, 1, 9, 8, 5, 3, 7, 8, 1, 6, 8, 7, 2, 9, 9, 8, 7, 9, 0], Covers: 60 interactions, Covered: 79.9%.
124:[4, 5, 8, 9, 6, 9, 7, 8, 0, 8, 1, 7, 9, 1, 2, 9, 1, 4, 7, 8], Covers: 64 interactions, Covered: 80.24%.
125:[7, 3, 3, 3, 6, 5, 7, 9, 8, 3, 0, 6, 0, 6, 7, 1, 5, 1, 8, 4], Covers: 59 interactions, Covered: 80.55%.
126:[0, 8, 6, 2, 1, 3, 6, 3, 2, 9, 8, 5, 7, 8, 4, 2, 4, 3, 6, 6], Covers: 59 interactions, Covered: 80.86%.
127:[4, 8, 3, 0, 2, 0, 4, 6, 5, 7, 0, 7, 1, 0, 5, 6, 7, 4, 7, 6], Covers: 58 interactions, Covered: 81.16%.
128:[6, 5, 6, 2, 8, 7, 2, 5, 8, 7, 5, 9, 2, 6, 3, 8, 8, 9, 4, 4], Covers: 57 interactions, Covered: 81.46%.
129:[7, 8, 6, 7, 7, 1, 0, 2, 4, 8, 7, 5, 3, 9, 9, 8, 7, 4, 4, 7], Covers: 56 interactions, Covered: 81.76%.
130:[4, 4, 0, 4, 1, 5, 0, 7, 7, 6, 1, 3, 7, 5, 4, 7, 3, 0, 3, 4], Covers: 62 interactions, Covered: 82.08%.
131:[8, 3, 3, 9, 2, 3, 1, 1, 0, 0, 8, 9, 4, 7, 8, 3, 2, 4, 2, 0], Covers: 58 interactions, Covered: 82.39%.
132:[9, 6, 3, 6, 0, 1, 7, 0, 6, 9, 7, 3, 7, 1, 5, 9, 0, 3, 5, 1], Covers: 55 interactions, Covered: 82.68%.
133:[5, 9, 4, 9, 2, 4, 1, 5, 0, 7, 2, 1, 9, 0, 0, 5, 3, 0, 4, 7], Covers: 55 interactions, Covered: 82.97%.
134:[5, 1, 7, 4, 6, 2, 3, 2, 1, 5, 8, 9, 4, 3, 0, 9, 9, 0, 7], Covers: 53 interactions, Covered: 83.25%.

```

FRUITS	DRINKS	FOODS	ICE-CREAM	SOUP	FAST FOOD	SEAFOOD
MANGO	MILK	BREAD	CHOCOLATE	ASAM LAKSA	PIZZA HUT	LOBSTER
ORANGE	NESCAFE	CHICKEN	YAM	ASAM LAKSA	PIZZA HUT	PRAWN
APPLE	TEA	BREAD	STRAWBERRY	CURRY	PIZZA HUT	CRAB
APPLE	MILK	NOODLES	STRAWBERRY	TOMYAM	KFC	PRAWN
MANGO	TEA	CHICKEN	YAM	TOMYAM	KFC	CRAB
ORANGE	TEA	NOODLES	CHOCOLATE	CURRY	MARRRYBER	PRAWN
APPLE	NESCAFE	NOODLES	CHOCOLATE	TOMYAM	PIZZA HUT	LOBSTER
MANGO	NESCAFE	CHICKEN	STRAWBERRY	CURRY	KFC	LOBSTER
ORANGE	MILK	BREAD	YAM	TOMYAM	MARRRYBER	CRAB
APPLE	NESCAFE	NOODLES	CHOCOLATE	ASAM LAKSA	KFC	CRAB
MANGO	MILK	NOODLES	YAM	CURRY	PIZZA HUT	PRAWN
ORANGE	TEA	CHICKEN	STRAWBERRY	ASAM LAKSA	MARRRYBER	LOBSTER
APPLE	NESCAFE	BREAD	YAM	TOMYAM	MARRRYBER	PRAWN
ORANGE	MILK	BREAD	YAM	TOMYAM	KFC	LOBSTER
APPLE	MILK	CHICKEN	CHOCOLATE	TOMYAM	KFC	LOBSTER

Input: P: Parameter number n, and

V: set of values for each feature $V = [v_0 \dots v_j]$;

Output: Final test suite List *FTS*;

Let *FTS* be a set of candidate tests;

Generate all possible interactions elements IEL based on P and V

Generate initial population of pollens randomly

while IEL is not empty **do**

while t < MaxGeneration **or** stop criterion **do**

for i = 1 : n (all n pollens in the population)

if (rand < pa)

Global pollination via $x_t^{i+1} = x_t^i + L(\text{gbest} - x_t^i)$

Else

Do local pollination via $x_t^{i+1} = x_t^i + \phi(x_t^j - x_t^k)$

End if

Evaluate new solutions

If new solutions are better, update them in the population

End for

Find the current best solution gbest

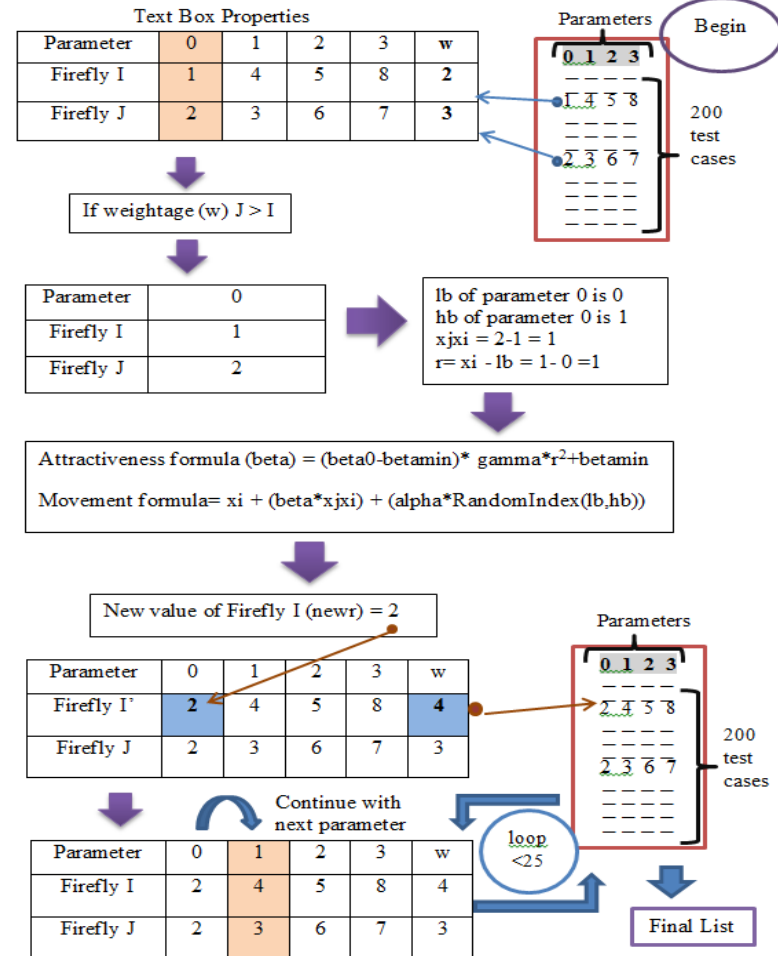
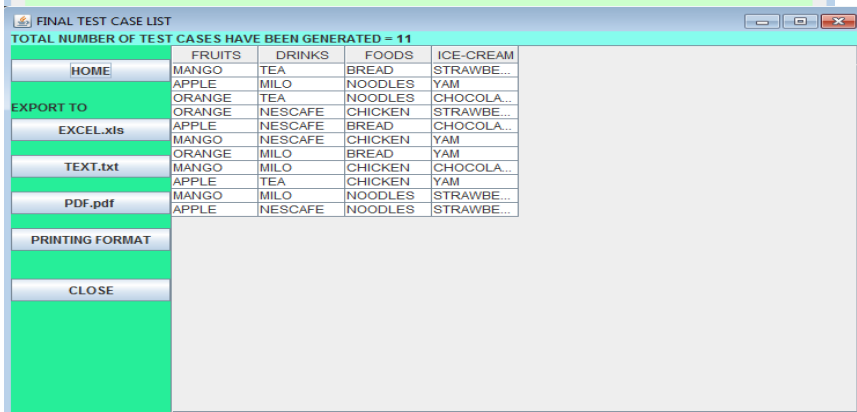
End while

Add the best test case, gbest, into *FTS*.

Remove covered interactions elements from IEL.

End while

End-Procedure



SOFTWARE PRODUCT LINE TESTING TOOLS BASED ON HARMONY SEARCH WITH CONSTRAINT SUPPORT

SOFTWARE PRODUCT LINE TESTING TOOLS BASED ON HARMONY SEARCH WITH CONSTRAINT SUPPORT


Features

Number of Feature:11;

Name of Feature:
Cellphone;
Wireless;
Infrared;
Bluetooth;
Accu_Cell;
Li_ion;
Ni_Ca;
Ni_Mh;
Display;
Color;
Monochrome;

Constraint

Excluded Constraint:
990099999999;
999991119999;
999991199999;
999991919999;
999999119999;
999999999111;



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T=3 T=4

Generate Test Case

Final Test List, 10 Test Cases Generated :

Cellphone	Wireless	Infrared	Bluetooth	Accu_Cell	Li_ion	Ni_Ca	Ni_Mh	Display	Color	Mon
Off	Off	On	On	Off	On	On	On	On	On	On
Off	On	Off	Off	On	Off	Off	Off	Off	Off	Off
On	Off	On	On	Off	Off	On	Off	Off	Off	Off
On	On	Off	Off	On	On	Off	On	On	On	On
On	Off	Off	On	On	On	On	Off	Off	Off	On
On	On	On	Off	Off	Off	Off	Off	On	On	Off
On	Off	Off	On	Off	On	Off	On	On	On	Off
Off	On	On	On	On	Off	On	On	On	Off	On
On	Off	On	Off	On	Off	On	On	On	Off	On

HOME EXPORT TO EXCEL.xls CLOSE

Constraints and Seeding



What about Seedings versus Constraints?



The screenshot shows a web browser window titled "Jalapeno Pizza" with a form for "Online Ordering". The form is divided into three sections: "Crust", "Flavour", and "Toppings".

- Crust:** Two radio buttons are present: "Classic Hand Tossed" and "Crunchy Thin".
- Flavour:** A dropdown menu labeled "Choose One" with a downward arrow, showing a list of options: "Vegetarian" and "Pepperoni Delight".
- Toppings:** A dropdown menu labeled "Choose One" with a downward arrow, showing a list of options: "Pineapples" and "Beef".

- Constraints – impossible combinations
 - {Crust Thin, Vegetarian, Beef},
 - {Classic Hand Hand Tossed, Vegetarian, Beef}
- Seedings – wanted combinations
 - {Crust Thin, Vegetarian, Pineapple}

What about Seedings versus Constraints?

Crust = { Classic Hand Tossed, Crunchy Thin }	→	A = {0,1}
Flavor = { Vegetarian, Pepperoni Delight }	→	B = {0,1}
Toppings = { Pineapples, Beef }	→	C = {0,1}

- Different perspective ...
- Now we use numbers

Equivalent base test cases

Base Values	Input Variables		
	A	B	C
0	0	0	0
1	1	1	1

Exhaustive Test Suite

Base Values	Input Variables		
	A	B	C
0	0	0	0
1	1	1	1
Exhaustive Combinations	0	0	0
	0	0	1
	0	1	0
	0	1	1
	1	0	0
	1	0	1
	1	1	0
	1	1	1

Base Values	Input Variables		
	A	B	C
0	0	0	0
1	1	1	1
2-way Combinations for AB	0	0	0
	0	1	0
	1	0	1
	1	1	1

Base Values	Input Variables		
	A	B	C
0	0	0	0
1	1	1	1
2-way Combinations for BC	0	0	0
	0	0	1
	0	1	0
	0	1	1

Base Values	Input Variables		
	A	B	C
0	0	0	0
1	1	1	1
2-way Combinations for AC	0	0	0
	0	0	1
	1	0	0
	1	1	1

Pairwise, t=2

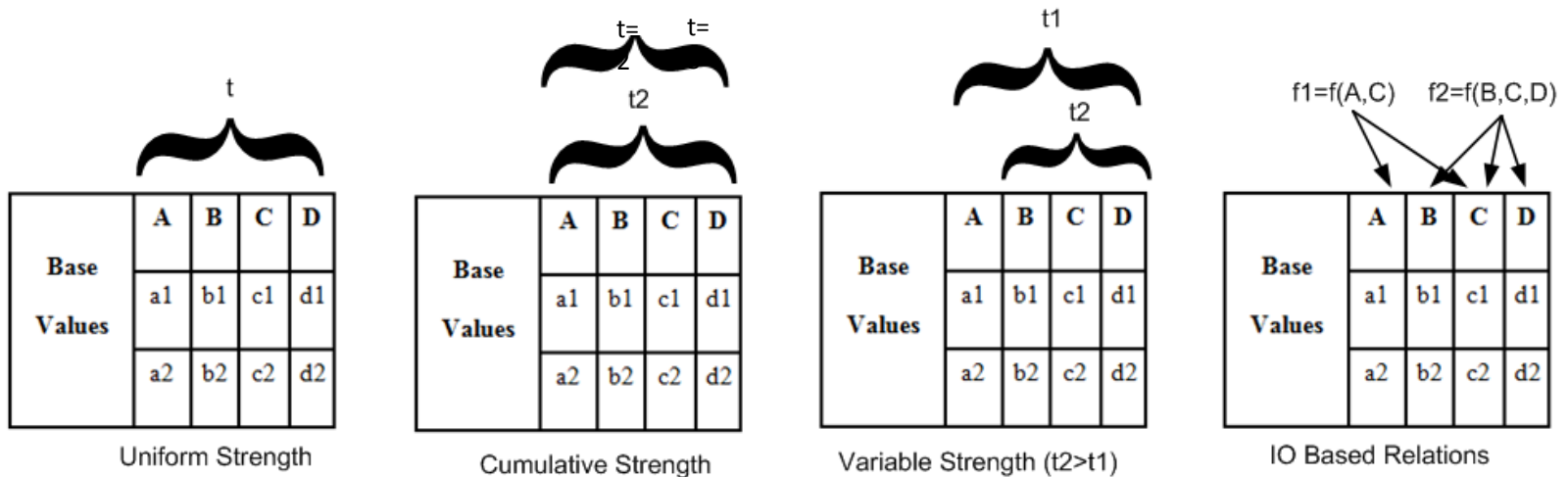
Base Values	Input Variables		
	A	B	C
0	0	0	0
1	1	1	1
All 2-way Combinatorial Values	0	0	0
	0	0	1
	0	1	0
	1	0	0
	1	0	1
	1	1	0
	1	1	1

Mathematical Notations



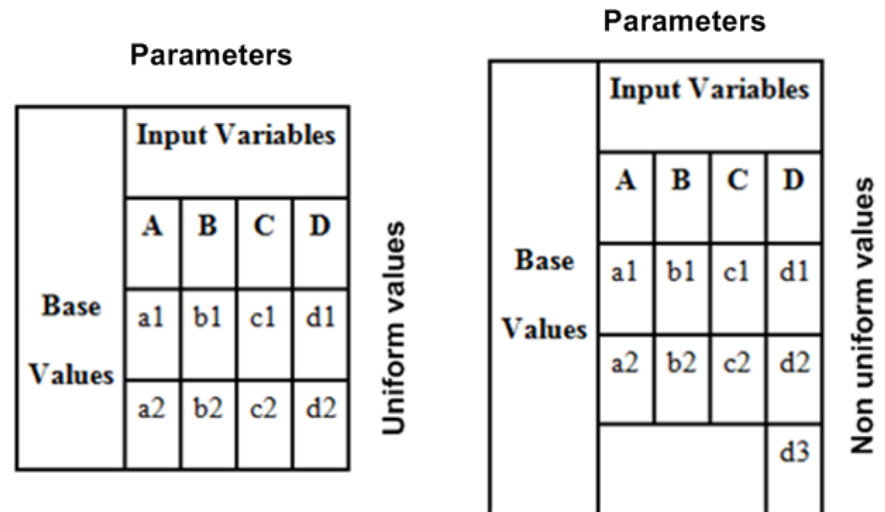
Covering Array vs Mixed Covering Array Notations

- The problem of t-way test generation can be abstracted as **covering array** or **mixed covering array problem**
- Recall, there are four possibilities for applying t-way testing for test reduction
 - Uniform Interaction
 - Cumulative Interaction
 - Variable Strength Interaction



Covering Array vs Mixed Covering Array Notations

- We use:
 - p , v , and t are used to refer to number of **parameters** (or factor), **values** (or levels) and interaction strength for the covering array respectively.
 - N is used to refer to test size
 - Covering Array (CA)** => values are uniform
 - Mixed Covering array (MCA)** => values are non uniform
- Example:
 - CA (N, t, v^p)
 - CAN (N, t, v^p) (i.e. CAN refers to the most minimum test size)
 - MCA ($N, t, v_1^{p1} v_2^{p2} v_3^{p3} \dots v_j^{pj}$)



Using the CA & MCA notations for Uniform Strength Interaction

- From earlier slides:

- CA (N,t,v^p)
- MCA (N, t, v₁^{p1} v₂^{p2} v₃^{p3}v_j^{pj})

- Example Use:

- CA (9, 2, 3⁴)
 - test size = 9, t=2, 4 parameters each with 3 values
- MCA (1265, 4, 10²4¹3²2⁷)
 - Test size =1265, t=4, 10 2 valued parameter, 4 1 valued parameter, 3 2 valued parameter, 7 2 valued parameter

- Our motivating example: CA (13, 3, 2⁴) but the most optimum is CAN (8, 3, 2⁴)



Parameters are A, B, C

Base Values	Input Variables			
	A	B	C	D
	a1	b1	c1	d1
a2	b2	c2	d2	

} Values

Variable Strength Interaction

- Using Covering array notation
 - Variable strength covering array (VCA)
 - $VCA(N, t, v_1^{p1} v_2^{p2} v_3^{p3} \dots v_j^{pj}, \{S_1 \dots S_k\})$
 - S consists of a multi-set of disjoint (mixed) covering array with strength larger t.
 - For example, $VCA(12, 2, 3^2 2^2, \{CA(3, 3^1 2^2)\})$, indicates the test size of 12 for pairwise interaction (with 2 3 valued parameter and 2 2 valued parameter) and 3-way interaction (with 1 3 valued parameter and 2 2 valued parameter).

Notations for Cumulative Strength Interaction

- Cumulative strength can be viewed as a special case of VCA
- Hence, we can use the same notation for cumulative strength interaction
 - For example, for a 4 parameters system with 2 values, with cumulative strength $t=2$, and $t=3$.
 - The notation: $VCA(N, 2, 2^4, \{CA(3, 2^4)\})$

Summary - Mathematical Notations for T-Way Strategies

- Uniform Strength Interaction
 - $CA(N, t, v^p) \Rightarrow$ uniform parameter values
 - $MCA(N, t, v_1^{p1} v_2^{p2} v_3^{p3} \dots v_j^{pj}) \Rightarrow$ non uniform parameter values
- Variable Strength Interaction (VCA) & Cumulative Strength Interaction
 - $VCA(N, t, v_1^{p1} v_2^{p2} v_3^{p3} \dots v_j^{pj}, \{S_1 \dots S_k\})$
 - S consists of a multi-set of disjoint (mixed) covering array with strength larger t.