Universiti Malaysia PAHANG
Engineering • Technology • Creativity

Computer Graphics

Projections

(Viewing Transformations)

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Chapter Description

- Aims
  - Basic of Computer Graphics.

- Expected Outcomes
  - Understand the basic concept of computer graphics. (CO1: Knowledge)
  - Ability to use the computer graphics technology. (CO1: Knowledge)

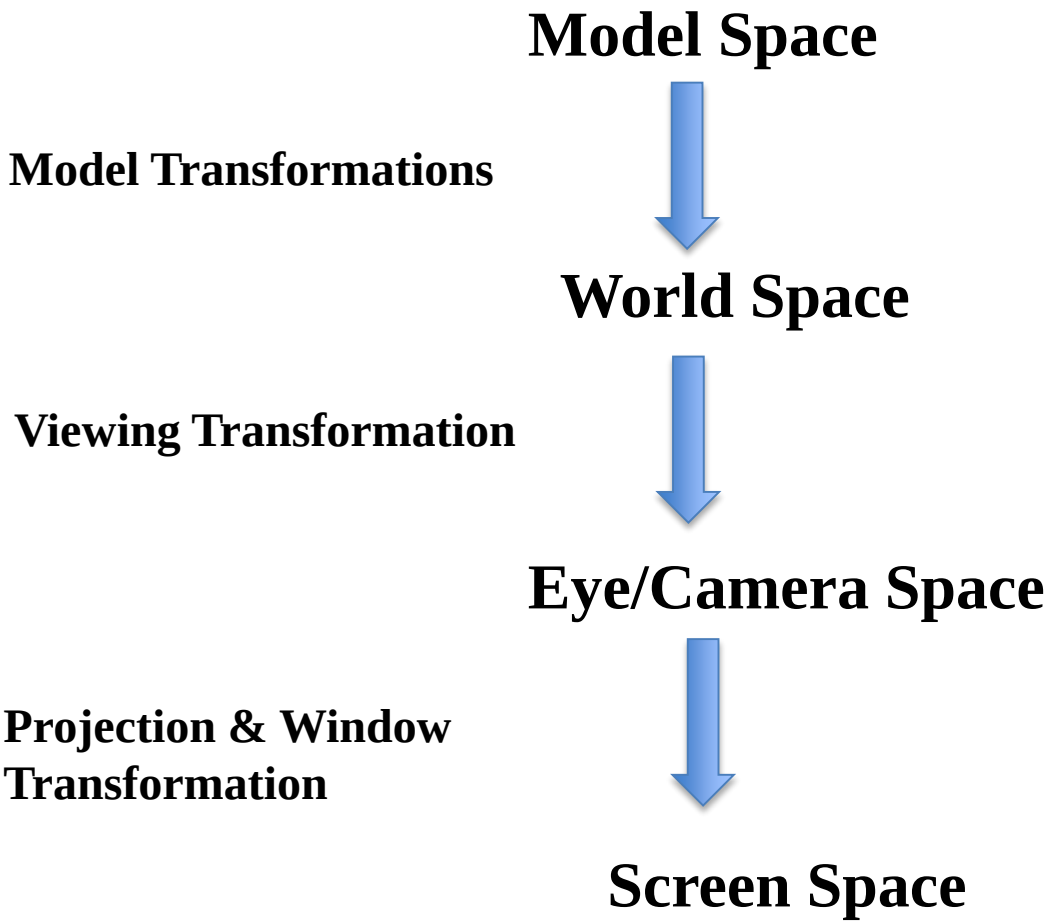
- References
  - Computer Graphics by Zhigang Xiang, Schaum's Outlines.
  - Donald Hearn & M. Pauline Baker, Computer Graphics with OpenGL, 4th Edition, Boston : Addison Wesley, 2011.

Motivation

- We want to see our “virtual 3-D world” on a 2-D monitor (screen)

- map the object from 3D space to 2D screen

Graphics Pipeline



Projection

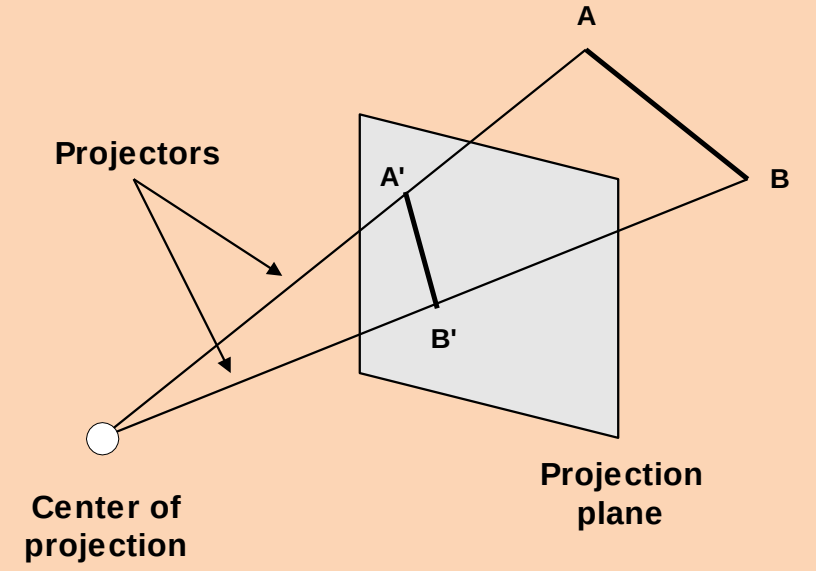
- **PROJECTIONS** transform objects or points in a coordinate system from dimension *m* into a coordinate system of dimension n where m < n.

- Focus: Projection from 3D to 2D.

Projections (key terms)

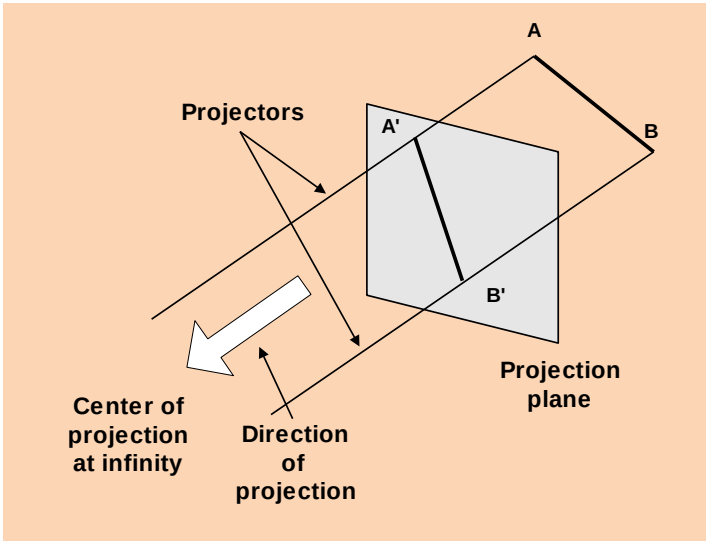
Let, AB is a straight line.

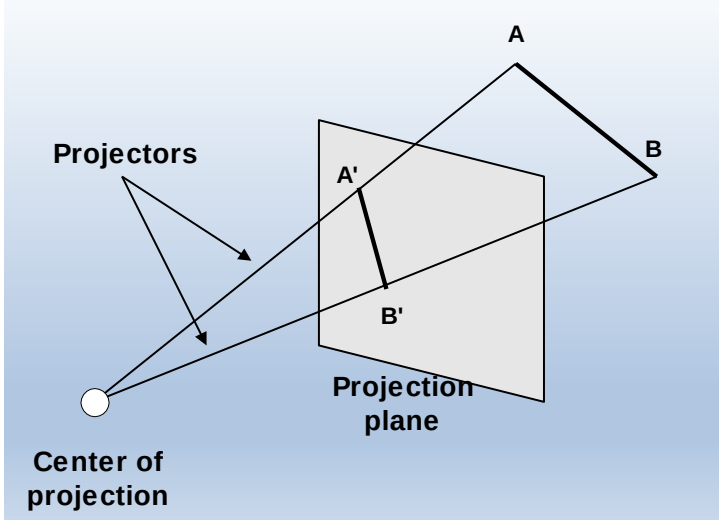
- ***Projectors:*** straight projection rays
- ***Center of projection:*** Its emanating from a ***Projectors***,
- ***Projection plane: Projectors*** passing through each point of the object, an ***ojection plane***.



Types of viewing transforms

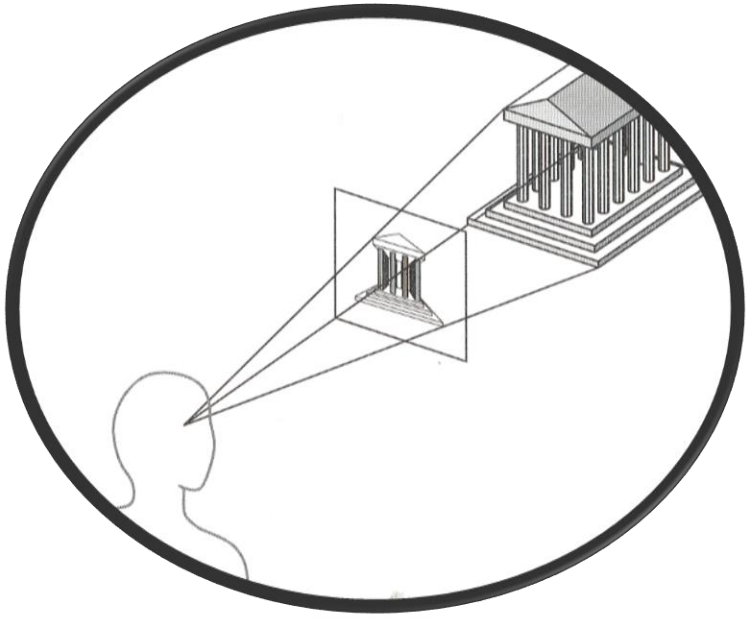
- Two types of viewing transforms
  - *Orthographic (parallel projection)*
  - *Perspective (convergent projection)*
- Key factor: ***center of projection***.
  - if distance to center of projection is finite : perspective projection
  - if distance to center of projection is infinite : parallel or Orthographic





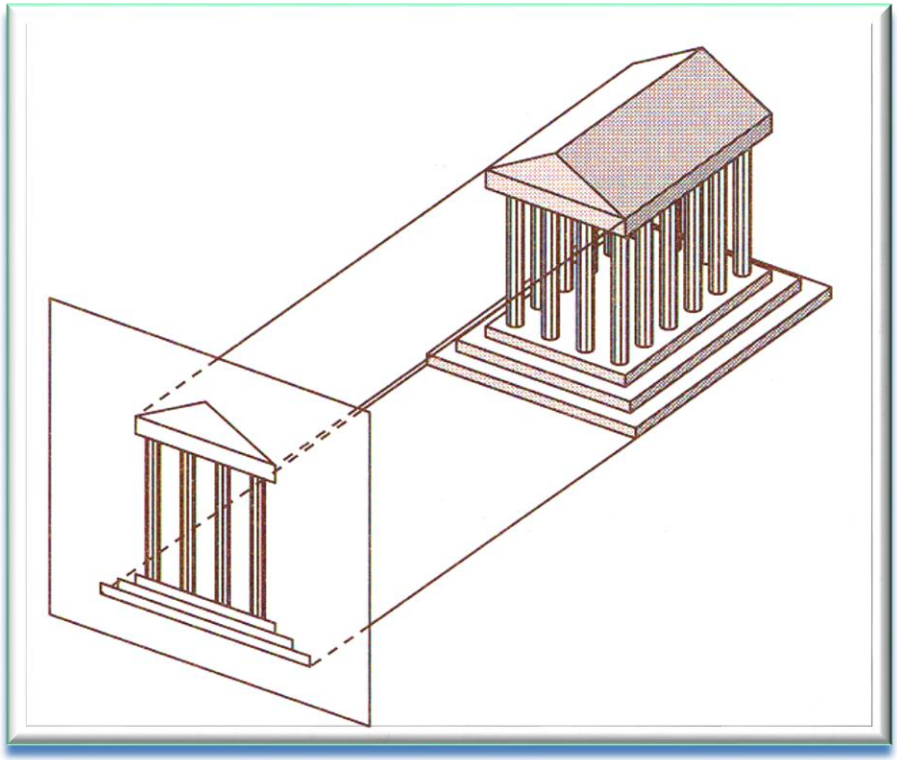
Perspective v Orthographic

- Perspective Projection:
  - visual effect: similar to visual system of human
  - existence of the “foreshortening”
    - size of object inversely proportional with the distance of the COP (center of projection).



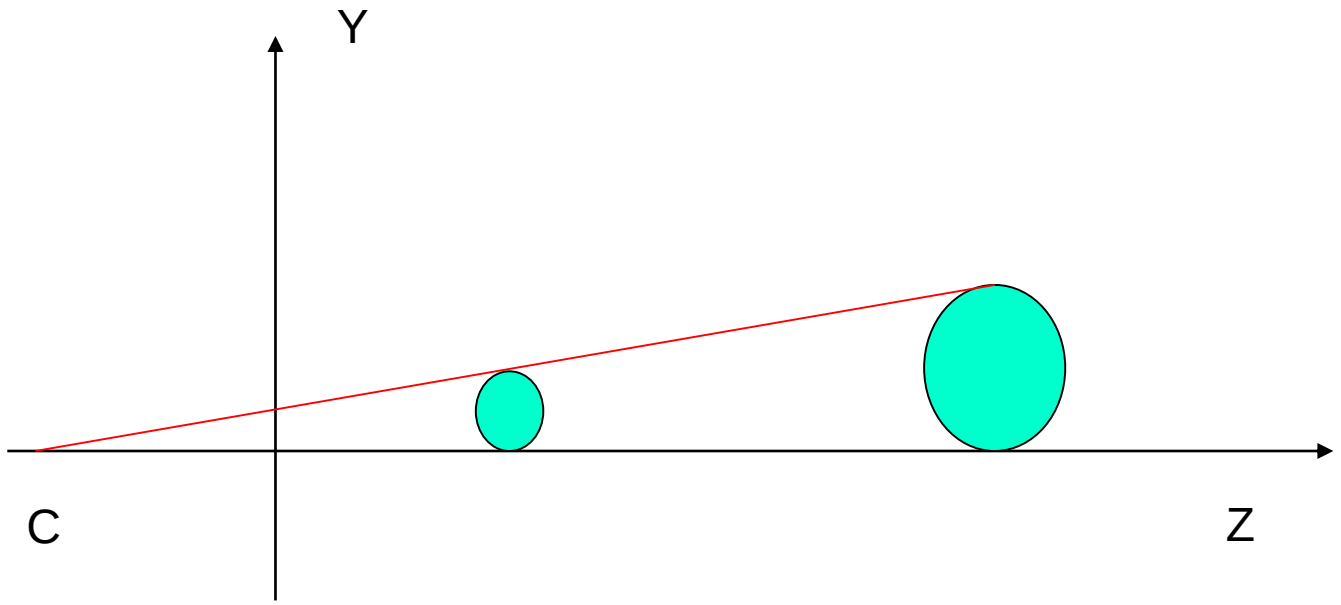
Perspective v Orthographic

- Orthographic Projection :
  - It is a less realistic view because it not consider “foreshortening”
  - parallel lines continue as parallel.



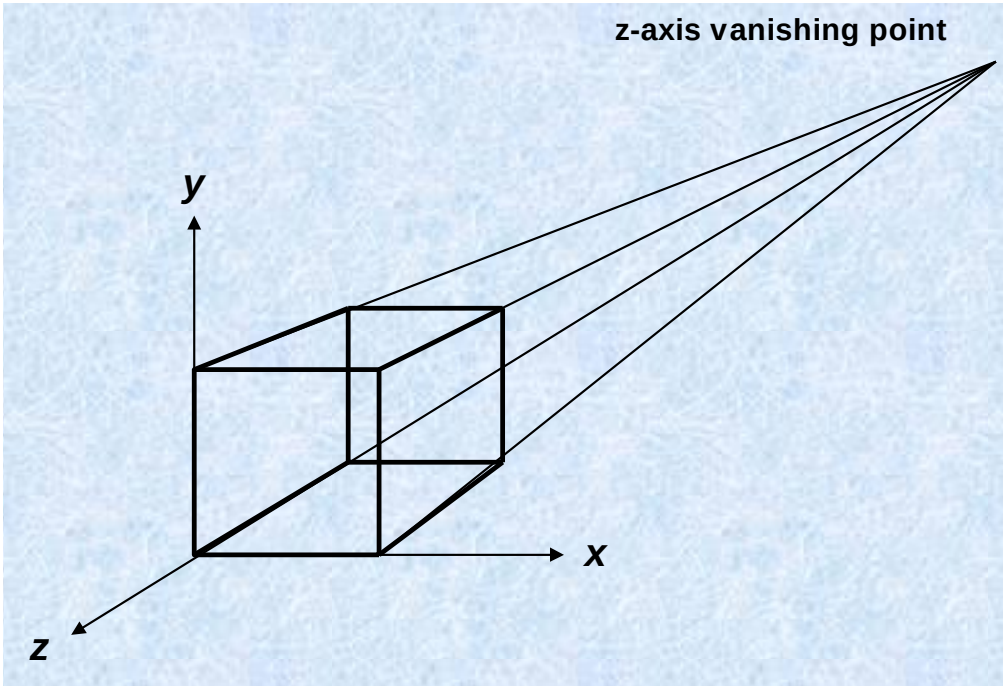
---- Anomalies
Perspective Projection

1) ***foreshortening*** An object appears smaller if it further from center of projection (COP)



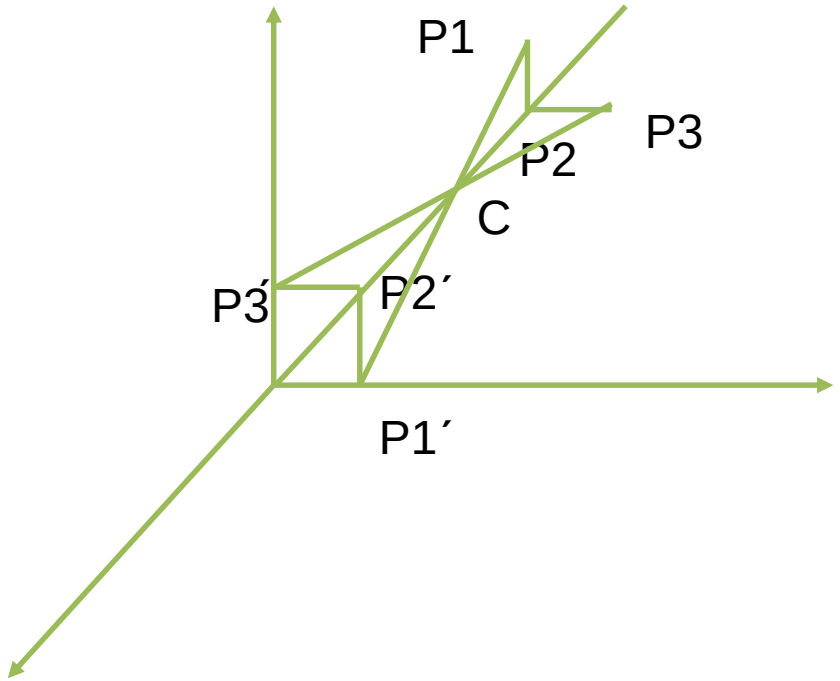
---- Anomalies
Perspective Projection

2) Vanishing Points: Any set of parallel lines that are not perpendicular to view plane normal (or not parallel to view plane), can be appeared to meet at vanishing point.



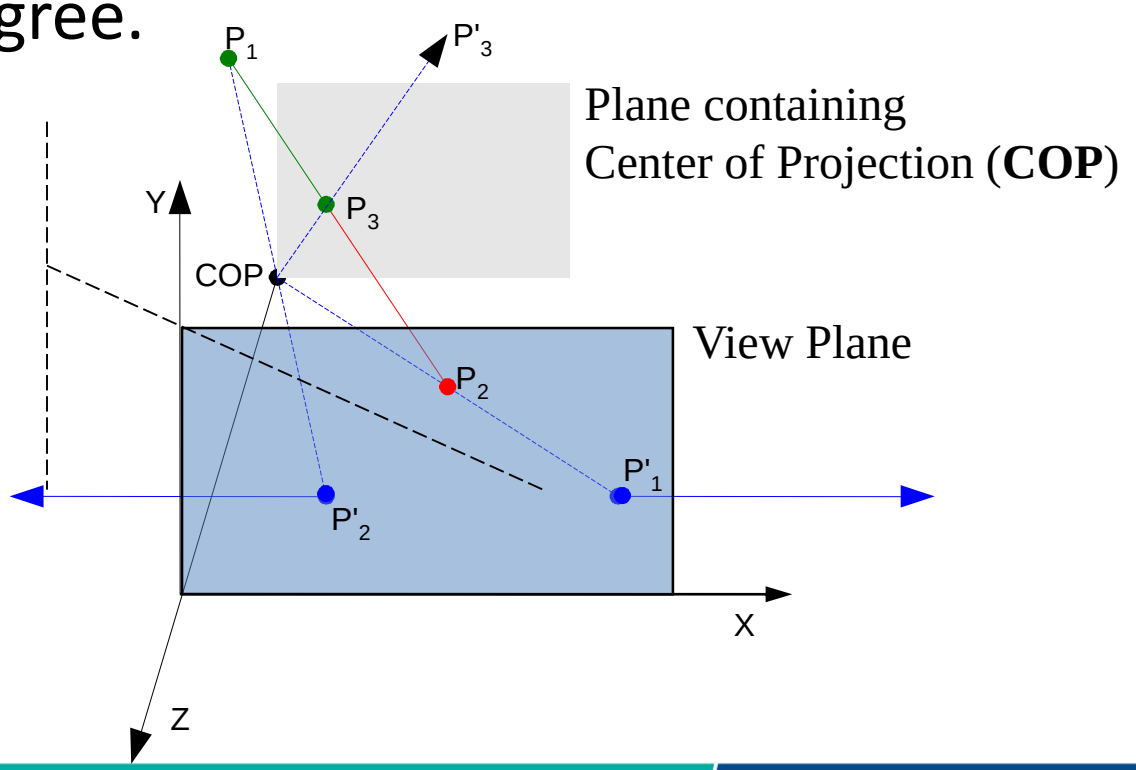
---- Anomalies
Perspective Projection

3) View Confusion: If any object exist behind the COP (center of projection), then it can be projected onto the view-plane seems like upside down and backward.



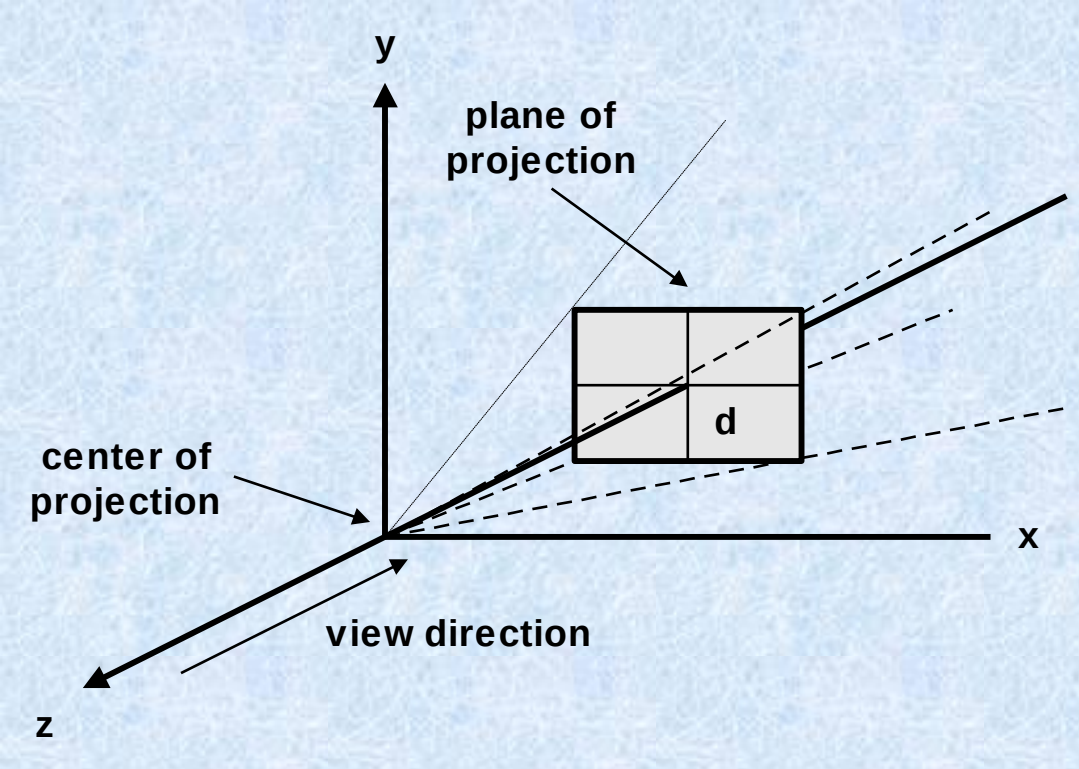
---- Anomalies
Perspective Projection

4) Topological distortion: Consider all points on a plan. If these points are parallel to view plane and passes through the COP, then these points are projected to a broken line of infinite degree.



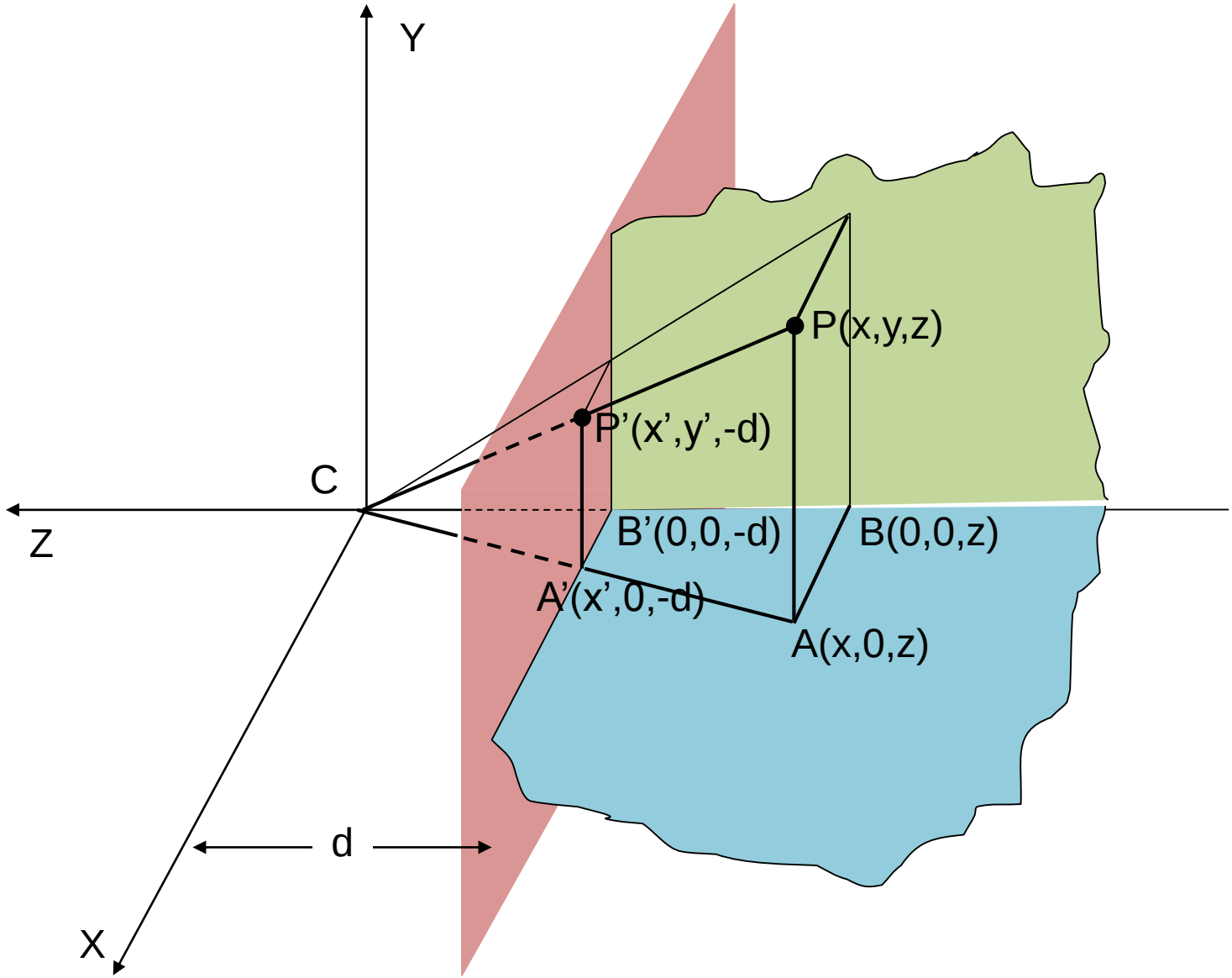
Projection Mathematics

Projective Transformations



Settings for perspective projection

Perspective Projection



Perspective Projection

- From triangle ABC and A'B'C

$$\frac{AB}{BC} = \frac{A'B'}{B'C}$$

$$\frac{x}{z} = \frac{x'}{-d} \Rightarrow x' = \frac{x}{-(z/d)}$$

$$\text{similarly, } y' = \frac{y}{-(z/d)} \text{ and,}$$

$$z' = -d$$

$$(x', y', z', 1) \Rightarrow \left(\frac{x}{-(z/d)}, \frac{y}{-(z/d)}, -d, 1 \right)$$

Projective Transformation

$$\begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \dfrac{x}{-(z/d)} \\ \dfrac{y}{-(z/d)} \\ -d \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ -\dfrac{z}{d} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \\ \dfrac{z}{d} \end{pmatrix} \xrightarrow{\text{perspective division}} \begin{pmatrix} \dfrac{x}{-(z/d)} \\ \dfrac{y}{-(z/d)} \\ -d \\ 1 \end{pmatrix} \longleftarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\dfrac{1}{d} & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ -\dfrac{z}{d} \end{pmatrix}$$