Universiti Malaysia PAHANG
Engineering • Technology • Creativity

# Computer Graphics

## 3D Transformations

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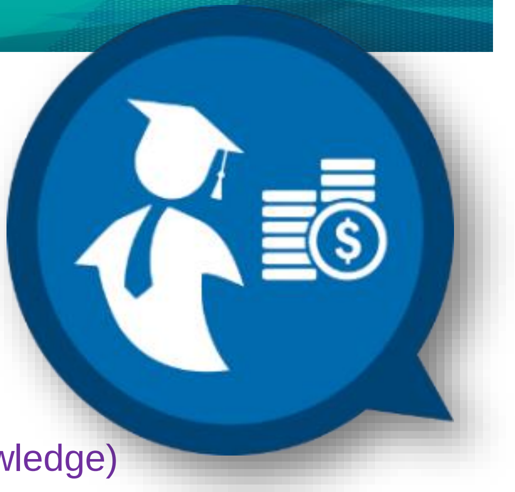
# Chapter Description



- Aims
  - Basic of Computer Graphics.

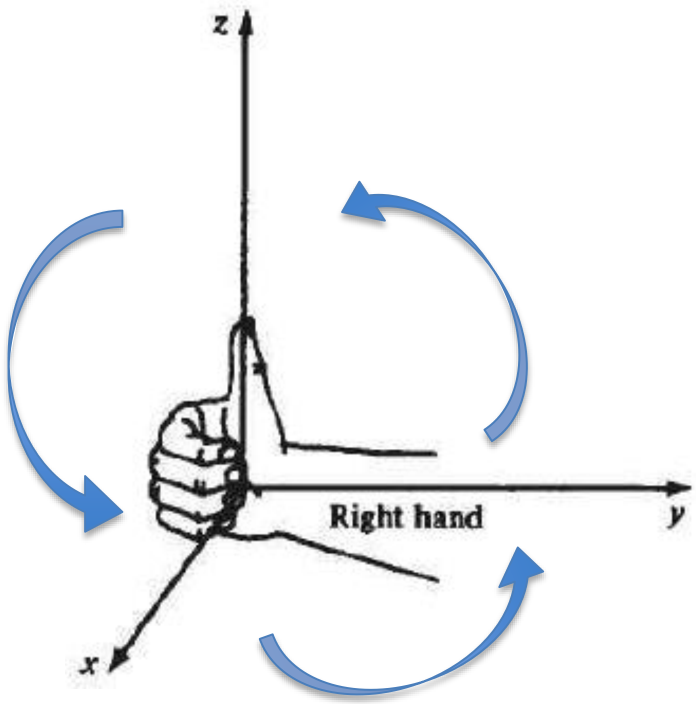
- Expected Outcomes
  - Understand the basic concept of computer graphics. (CO1: Knowledge)
  - Ability to use the computer graphics technology. (CO1: Knowledge)

- References
  - Computer Graphics by Zhigang Xiang, Schaum's Outlines.
  - Donald Hearn & M. Pauline Baker, Computer Graphics with OpenGL, 4th Edition, Boston : Addison Wesley, 2011.

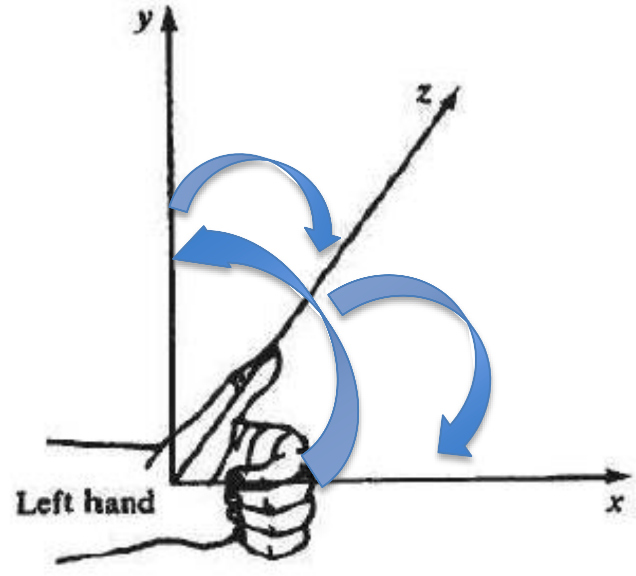
# 3D Transformations

- In this Chapter, we will learn the basics of
  - 3D geometry
  - And its 3D Transformations
  - Composite Transformations for 3D geometry.

# Orientation




Right-handed Orientation



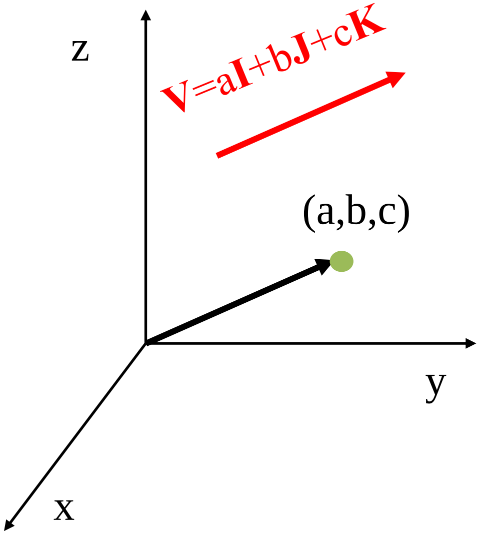

Left-handed Orientation

# Vectors in 3D

$V = [a, b, c]$

**Length** using Euclidean Norm

$|V| = \sqrt{a^2 + b^2 + c^2}$




**Dot Product:** $V \bullet U$

$[x_v, y_v, z_v] \bullet [x_u, y_u, z_u]$

$= x_v * x_u + y_v * y_u + z_v * z_u$

And, $V \bullet U = |V|\ |U| \cos ß$

**Cross Product :** $V \times U$

$[y_v * z_u - z_v\, y_u\ , -x_v * z_u + z_v * x_u\ , x_v * y_u - y_v * x_u\ ]$

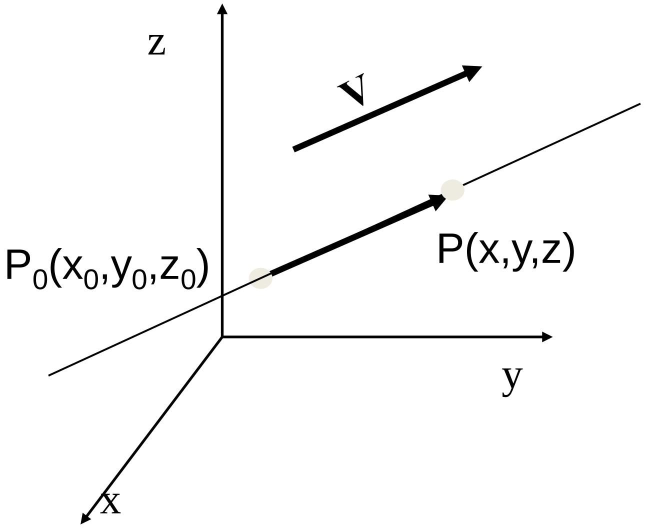
And, $V \times U = \eta\ |V|\ |U| \sin ß$

Moreover,

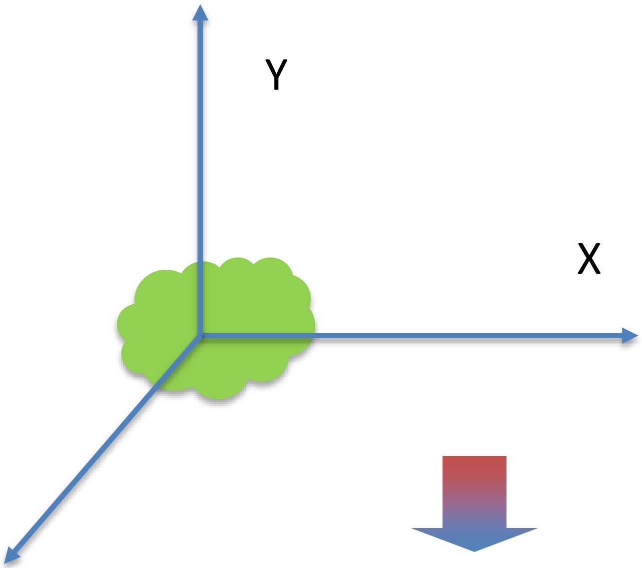
$V \times U = -(U \times V)$

# Vector Equation

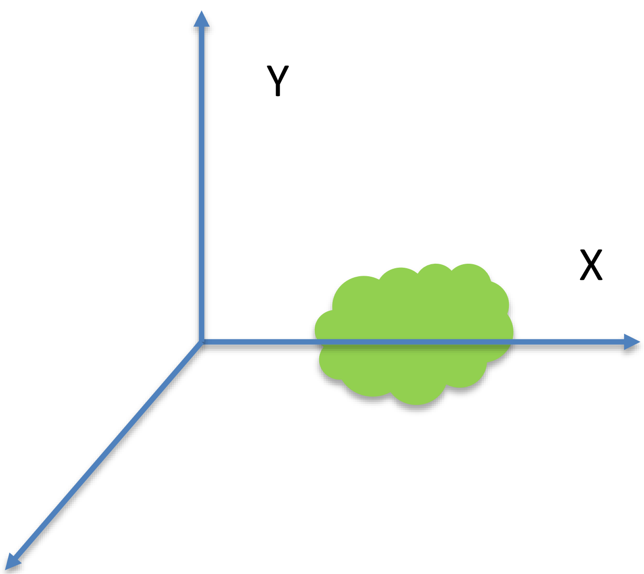
- **Line** is defined by direction and point
  - direction vector, $\vec{V}$
  - A point, $P_0(x_0,y_0,z_0)$



# Translation – 3D




$$x' = x + d_x$$
$$y' = y + d_y$$
$$z' = z + d_z$$

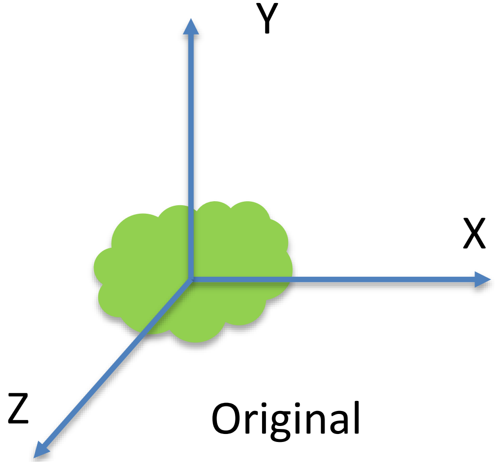


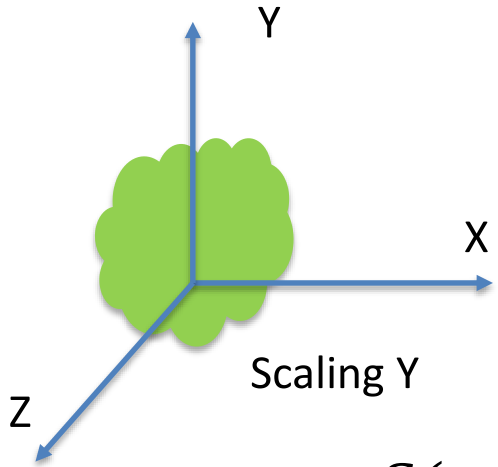

$$\begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + d_x \\ y + d_y \\ z + d_z \\ 1 \end{bmatrix}$$

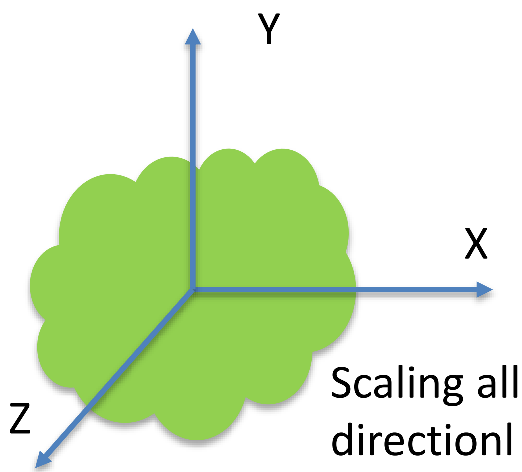
$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$T(d_x, d_y, d_z) \quad * \quad P \quad = \quad P'$$

# Scaling – 3D




$$x' = s_x * x$$

$$y' = s_y * y$$

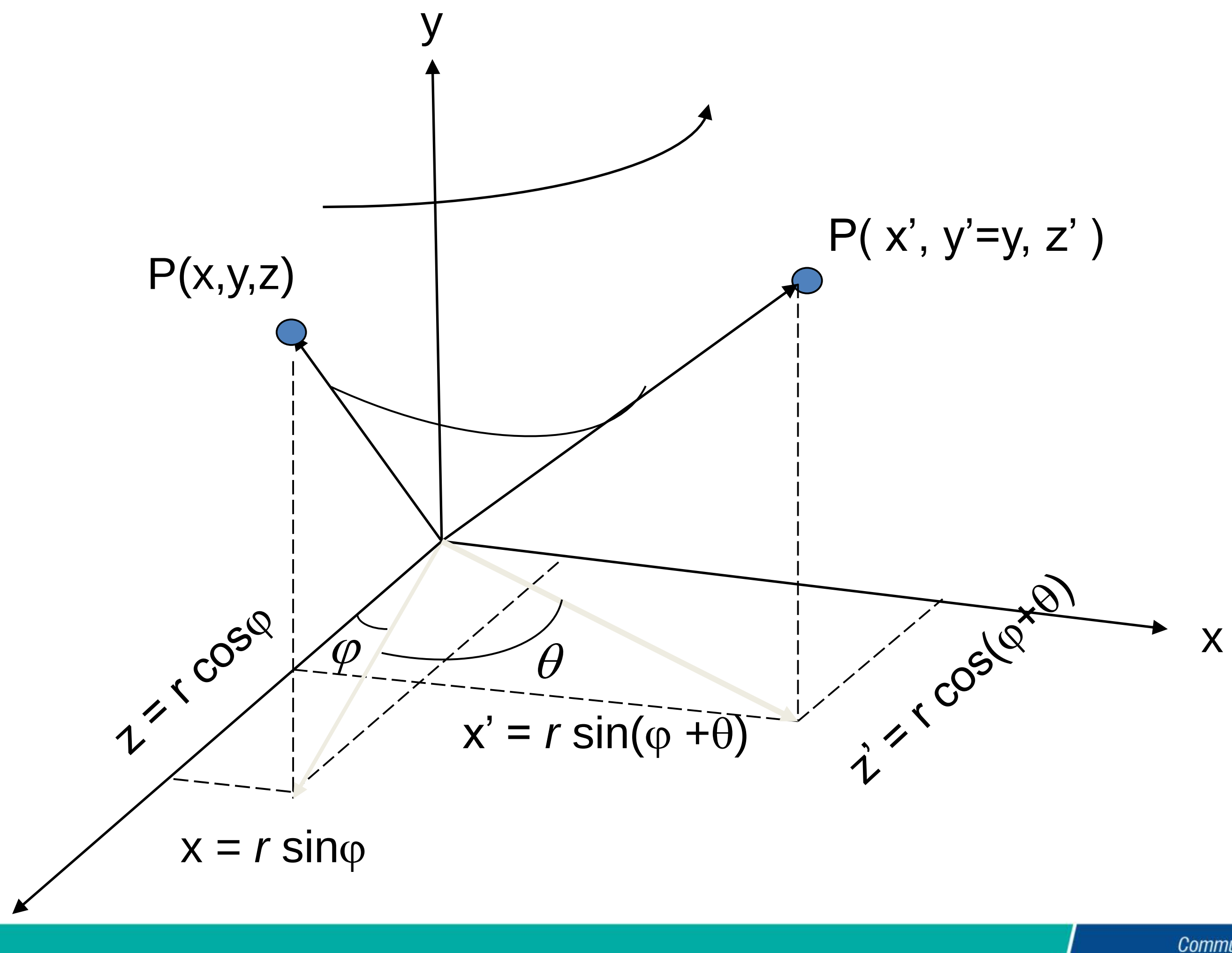
$$z' = s_z * z$$

$$S(s_x, s_y, s_z) \quad * \quad P \quad = \quad P'$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x * s_x \\ y * s_y \\ z * s_z \\ 1 \end{bmatrix}$$

# Rotation in 3D



# Rotation in 3D

$$x = r\sin\varphi$$

$$z = r\cos\varphi$$

$$x' = r\sin(\varphi+\theta) = r\sin\varphi\cos\theta + r\cos\varphi\sin\theta$$
$$= x\cos\theta + z\sin\theta$$

$$y' = y$$

$$z' = r\cos(\varphi+\theta) = r\cos\varphi\cos\theta - r\sin\varphi\sin\theta$$
$$= z\cos\theta - x\sin\theta$$
$$= -x\sin\theta + z\cos\theta$$

# Rotation in 3D

About y-axis

$$R_{\theta,j} \quad * \quad P \quad = \quad P'$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x*\cos\theta + z*\sin\theta \\ y \\ -x*\sin\theta + z*\cos\theta \\ 1 \end{bmatrix}$$

# Rotation – 3D

Rotation about z-axis:

$$R_{\theta,k} \quad * \quad P \quad = \quad P'$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x*\cos\theta - y*\sin\theta \\ x*\sin\theta + y*\cos\theta \\ z \\ 1 \end{bmatrix}$$

# Rotation about Y-axis & X-axis

About x-axis

$$R_{\theta,i} \quad * \quad P \quad = \quad P'$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y*\cos\theta - z*\sin\theta \\ y*\sin\theta + z*\cos\theta \\ 1 \end{bmatrix}$$