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CHAPTER 4

ANALYSIS OF VARIANCE (ANOVA)

Expected Outcomes

- ✓ Able to use the one way ANOVA technique to determine if there is a significant difference among three or more means when sample sizes are equal or unequal.
- ✓ Able to use the two way ANOVA technique to determine if there is an effect of interaction between two factors in an experiment.

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CONTENT

4.1 Introduction to Analysis of Variance (ANOVA)

4.2 One-Way ANOVA

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4.1 INTRODUCTION TO ANALYSIS OF VARIANCE (ANOVA)

- **Purpose:** To compare the differences of more than two populations means.
- Also known as factorial experiments.
- The approach that allows us to use sample data to see if the values of more than two populations means are likely to be different from each other.
- This name is derived from the fact that in order to test for statistical significance between means, we are actually comparing variances. (so F-distribution will be used).
- There are two-types of ANOVA:
    - One-way ANOVA – involve one factor
    - Two-way ANOVA – involve two factors

Example of problems involving ANOVA

1. A manager want to evaluate the performance of three (or more) employees to see if any performance different from others.

2. A marketing executive want to see if there's a difference in sales productivity in the 5 company region.

3. A teacher wants to see if there's a difference in student's performance if he use 3 or more approach to teach.

Example 1

a) Will **three different levels of a chemical concentration** have different effect on **an electroplating process**?

H_0 : The mean effect of electroplating process is the same for all three concentration levels.

→ One-way ANOVA

b) Will **three different levels of a chemical concentration** have different effect on **four electroplating processes**?

H_0 : Different levels of a chemical concentration have no effect on the electroplating processes.

(There is no interaction effect between chemical concentration and electroplating processes)

→ Two-way ANOVA

The Procedural Steps for an ANOVA Test

1. State the Null (H_0) and Alternative (H_1) hypotheses.

2. Determine the test statistic to be used - F_{test}.

3. Establish the test criterion by determining the critical value (point) and rejection region, based on significance level, α – $F_{critical}$.

4. Make a decision to reject or fail to reject the null hypothesis based on the comparison between test statistic and critical values. **The test is right tailed only**. Graphical illustration can help to determine where the statistic value lies either within the rejection or acceptance region.

5. Draw a conclusion to reject or not to reject the claim.

General procedure of ANOVA

State H_0 and H_1

↓

Determine test statistic, f_{test}

↓

Find critical value at α

↓

Test the hypothesis (rejection region). Graphical illustration can help to determine where the statistic value lies either within the rejection or acceptance region.

↓

Make a conclusion:
There is enough/sufficient evidence to reject/accept the claim

4.2: ONE-WAY ANOVA

Assumptions for one-way ANOVA:

To use the one-way ANOVA test, the following assumptions of data should be considered:

1. The populations under study follow normal distribution.

2. The samples are drawn randomly, and each sample is independent of the other samples.

3. All the populations from which the samples values are obtained, have the same unknown population variances, that is for *k* number of populations,

$$\boxed{\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2}$$

Tabulation of Data for One-Way ANOVA

	Replicates						Total
Treatment/The level of factor (variable) considered under study/Number of population	x_{11}	x_{12}	$\cdots$	x_{1j}	$\cdots$	x_{1n}	$x_{1.}$
	x_{21}	x_{22}	$\cdots$	x_{2j}	$\cdots$	x_{2n}	$x_{2.}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	x_{i1}	x_{i2}	$\cdots$	x_{ij}	$\cdots$	x_{in}	$x_{i.}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	x_{k1}	x_{k2}	$\cdots$	x_{kj}	$\cdots$	x_{kn}	$x_{k.}$
Total	$x_{.1}$	$x_{.2}$	$\cdots$	$x_{.j}$	$\cdots$	$x_{.n}$	$x_{..}$

where

x_{ij} : the *j*th observation from the *i*th treatment

$x_{i.}$: the total of all observations from the *i*th treatment

$x_{.j}$: the total of all treatments from the *j*th observation

$x_{..}$: the total of all observations

x_{kn} : the *n*th observation from the *k*th treatment

Reminder: The roles of *i* and *j* can be interchanged.

Model for one-way ANOVA

$$x_{ij} = \mu + \alpha_i + \varepsilon_{ij} \qquad , i = 1, 2, \ldots k \qquad j = 1, 2, \ldots n$$

where

x_{ij} : the jth observation from the ith treatment

μ : the overall mean

α_i : the ith treatment effect

ε_{ij} : the random error

k : number of populations (treatments)

n_i : sample size from ith population

Two rules should be fulfilled which are:

a) Assume $\varepsilon_{ij} \sim NID(0, \sigma^2)$
where *NID* is abbreviated for Normally Identically Distributed.

b) $\sum_{i=1}^{k} \alpha_i = 0$, all treatment effects are the same.

The Null and Alternative hypotheses

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$

- All population means are equal
  OR
- No treatment effect (no variation in means among groups)

$$H_1 : \mu_i \neq \mu_j \text{ for at least one } i \neq j$$

- At least one population mean is different
  OR
- There are differences between the population means
  OR
- Not all population means are equal
  OR
- There is a treatment effect between treatment *i* and treatment *j*

One-way ANOVA Table

Source of Variation	Sum Squares (SS)	Degrees of Freedom	Mean Squares (MS)	f_{test}
Treatment	$SSTr = \sum_{i=1}^{k} \frac{x_{i.}^2}{n} - \frac{x_{..}^2}{N}$	$k-1$	$MSTr = \frac{SSTr}{k-1}$	$f_{test} = \frac{MSTr}{MSE}$
Error	$SSE = SST - SSTr$	$N-k$	$MSE = \frac{SSE}{N-k}$	
Total	$SST = \sum_{i=1}^{k}\sum_{j=1}^{n} x_{ij}^2 - \frac{x_{..}^2}{N}$	$N-1$		

Reject H_0 if $f_{test} > f_{\alpha,k-1,N-k}$

Example 2

An experiment was performed to determine whether the annealing temperature of ductile iron affects its tensile strength. Five specimens were annealed at each of four temperatures. The tensile strength (in kilo-pounds per square inch, ksi) was measured for each temperature. The results are presented in the following table.

Temperature (^oC)	Tensile strength (in ksi)				
750	19.72	20.88	19.63	18.68	17.89
800	16.01	20.04	18.10	20.28	20.53
850	16.66	17.38	14.49	18.21	15.58
900	16.93	14.49	16.15	15.53	13.25

Example 2: solution

a) Write down the model, assumptions and rules of the one-way ANOVA for the given data.

$$\textbf{Model:}\ x_{ij} = \mu + \alpha_i + \varepsilon_{ij}\ ,\ i = 1,2,3,4, \quad j = 1,2,3,4,5.$$

Assumption:

1. The populations under study follow normal distribution.
2. The samples are drawn randomly, and each sample is independent of the other samples.
3. $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

$$\textbf{Rules:}\ \text{Assume } \varepsilon_{ij} \sim NID(0, \sigma^2) \ \text{ and } \sum_{i=1}^{k} \alpha_i = 0$$

Example 2: solution

(b) Perform the ANOVA and test the hypothesis at 0.01 level of significance that the mean data provided by the tensile strength is the same for all four temperatures.

- From the model above, clearly can be seen that $k = 4$ and $n = 5$ thus the table above can be explained as

Temperature (ºC)	Sample Values					Total
	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	
750, $i=1$	19.72	20.88	19.63	18.68	17.89	$x_{1.} = 96.80$
800, $i=2$	16.01	20.04	18.10	20.28	20.53	$x_{2.} = 94.96$
850, $i=3$	16.66	17.38	14.49	18.21	15.58	$x_{3.} = 82.32$
900, $i=4$	16.93	14.49	16.15	15.53	13.25	$x_{4.} = 76.35$
Total						$x_{..} = 350.43$

Example 2: solution

$\sum_{i=1}^{4} \frac{x_{i.}^2}{5}$

$$SSTrt = \frac{96.80^2}{5} + \frac{94.96^2}{5} + \frac{82.32^2}{5} + \frac{76.35^2}{5} - \frac{350.43^2}{20} = 58.6501$$

$\sum_{j=1}^{5} \sum_{i=1}^{4} x_{ij}^2$

$$SST = 19.72^2 + 20.88^2 + \ldots + 13.25^2 - \frac{350.43^2}{20} = 95.4875$$

$$SSE = 95.4875 - 58.6501 = 36.8374$$

Source of Variation	Sum of Squares (SS)	Degrees of Freedom	Mean of Squares (MS)	f_{test}
Treatment	58.6501	3	$\frac{58.6501}{3} = 19.5500$	$f_{test} = \frac{19.5500}{2.3023} = 8.4915$
Error	36.83.74	16	$\frac{36.8374}{16} = 2.3023$	
Total	95.4875	19		

Example 2: solution

The hypothesis is:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$
$$H_1 : \mu_i \neq \mu_j, \text{ for at least one } i \neq j$$

From statistical table, test statistic $f_{0.01,3,16} = 5.2922$.

Clearly that

$$\left(f_{test} = 8.4915 \right) > \left(f_{0.01,3,16} = 5.2922 \right)$$

Thus, H_0 is rejected.

At $\alpha = 0.01$, at least one of population means of temperature is different

Solve one-way ANOVA using Microsoft EXCEL

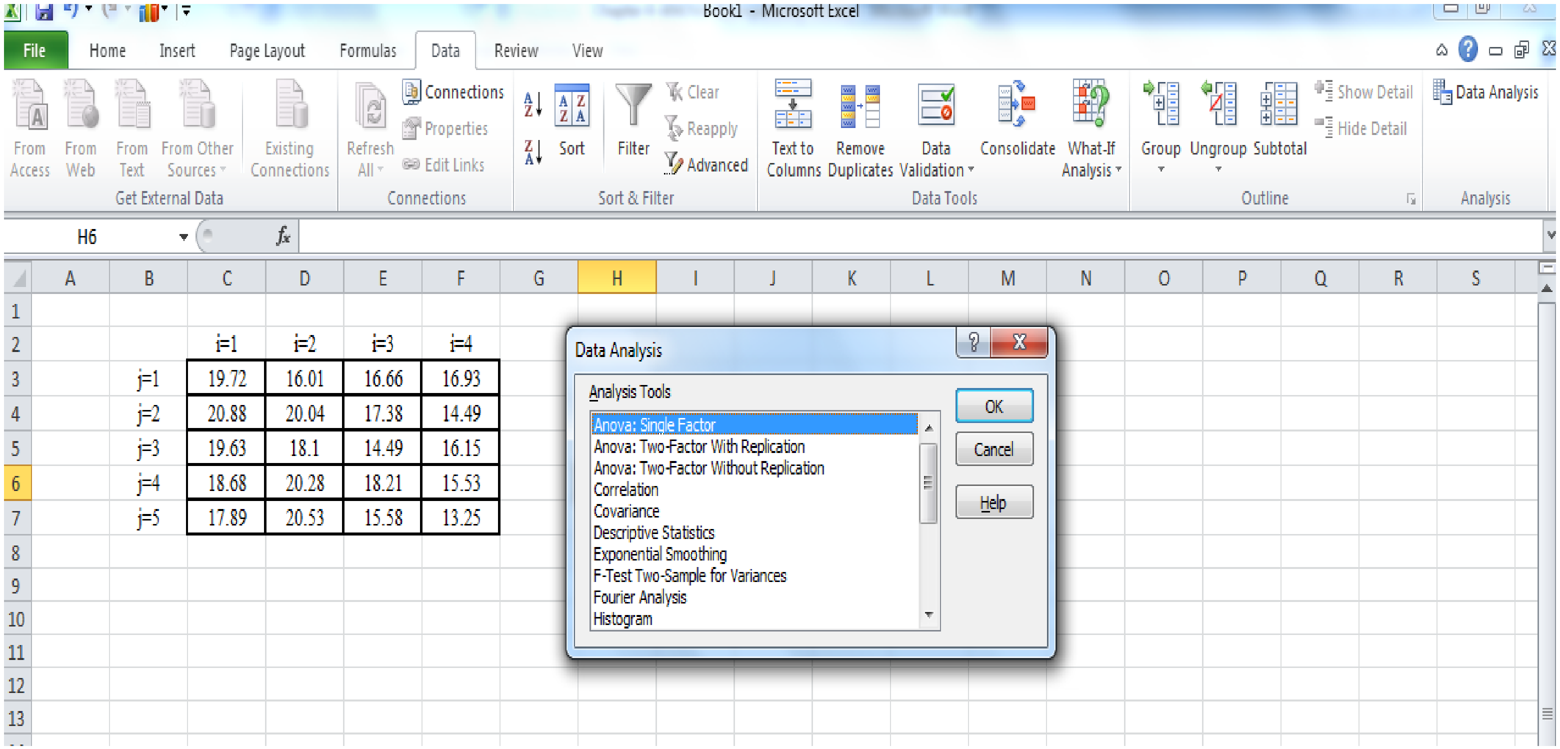
1. Excel - Key in data:

Temperature (°C)	Sample Values				
750	19.72	20.88	19.63	18.68	17.89
800	16.01	20.04	18.10	20.28	20.53
850	16.66	17.38	14.49	18.21	15.58
900	16.93	14.49	16.15	15.53	13.25

	i=1	i=2	i=3	i=4
j=1	19.72	16.01	16.66	16.93
j=2	20.88	20.04	17.38	14.49
j=3	19.63	18.1	14.49	16.15
j=4	18.68	20.28	18.21	15.53
j=5	17.98	20.53	15.53	13.25

2. Follow the steps below:

Click menu: Data → Data Analysis → ANOVA single factor → enter the data range → set a value of α → OK



3. Output:

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	58.65006	3	19.55002	8.491378	0.001327	5.292214
Within Groups	36.8374	16	2.302338			
Total	95.48746	19				

Reject H_0 if P-value $\leq \alpha$ or $F > F\ crit$

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_1 : \mu_i \neq \mu_j, \text{ for at least one } i \neq j$$

Since $(P - value = 0.0013) < (\alpha = 0.01)$, thus, H_0 is rejected.

At $\alpha = 0.01$, at least one of population means of temperature is different

4.2 TWO-WAY ANOVA

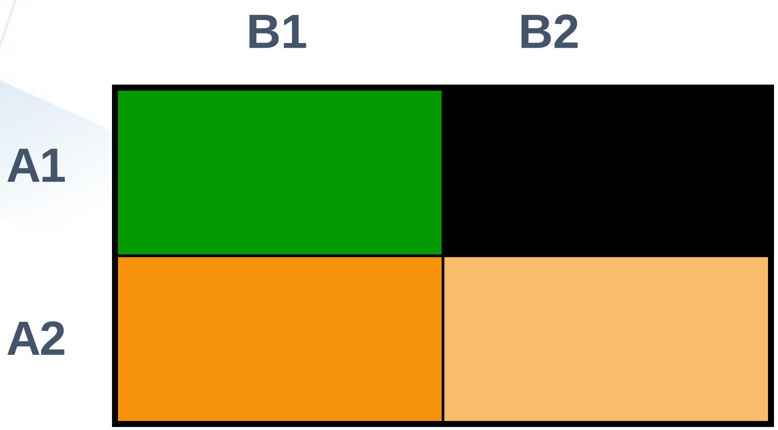
- Two classification factor is considered

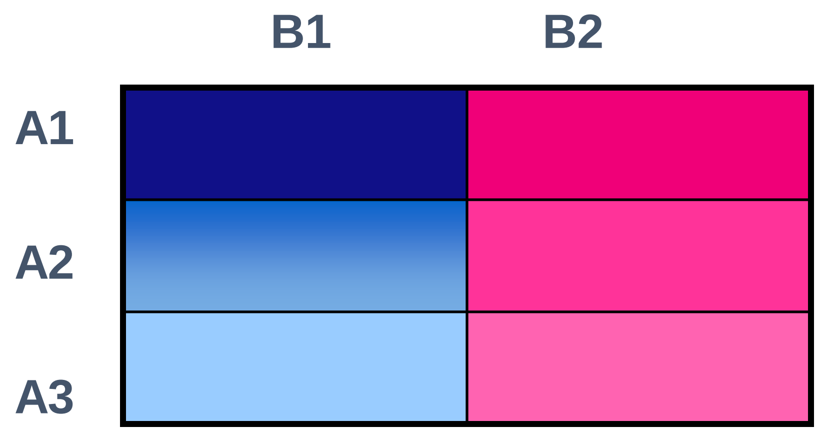
		Factor B			
		$j = 1$	2		b
Factor A	$i = 1$	$k = 1,\ldots r$			
	2				
	a				

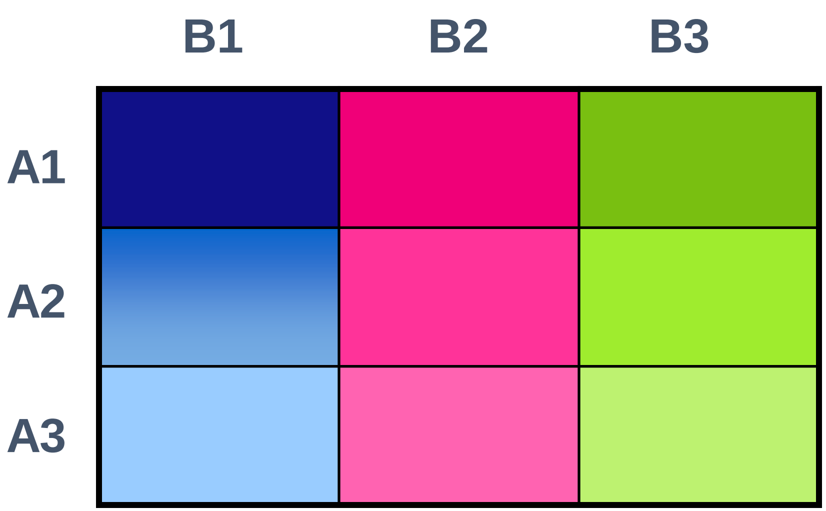
- **Example**

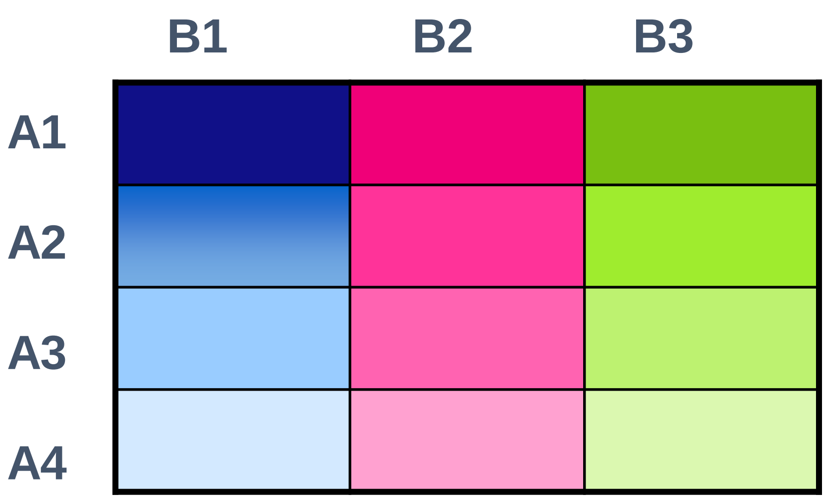
  A researcher whishes to test the effects of two different types of plant food and two different types of soil on the growth of certain plant.

Some types of two way ANOVA design









Tabulation of Data for Two-way ANOVA with Replication

Factor	B					Total
A	$x_{111}\ x_{112}\ \ldots\ x_{11k}\ \ldots\ x_{11r}$	$\cdots$	$x_{1j1}\ x_{1j2}\ \ldots\ x_{1jk}\ \ldots\ x_{1jr}$	$\cdots$	$x_{1b1}\ x_{1b2}\ \ldots\ x_{1bk}\ \ldots\ x_{1br}$	$x_{1..}$
	$x_{211}\ x_{212}\ \ldots\ x_{21k}\ \ldots\ x_{21r}$	$\cdots$	$x_{2j1}\ x_{2j2}\ \ldots\ x_{2jk}\ \ldots\ x_{2jr}$	$\cdots$	$x_{2b1}\ x_{2b2}\ \ldots\ x_{2bk}\ \ldots\ x_{2br}$	$x_{2..}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$x_{i11}\ x_{i12}\ \ldots\ x_{i1k}\ \ldots\ x_{i1r}$	$\cdots$	$x_{ij1}\ x_{ij2}\ \ldots\ x_{ijk}\ \ldots\ x_{ijr}$	$\cdots$	$x_{ib1}\ x_{ib2}\ \ldots\ x_{ibk}\ \ldots\ x_{ibr}$	$x_{i..}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$x_{a11}\ x_{a12}\ \ldots\ x_{a1k}\ \ldots\ x_{a1r}$	$\cdots$	$x_{aj1}\ x_{aj2}\ \ldots\ x_{ajk}\ \ldots\ x_{ajr}$	$\cdots$	$x_{ab1}\ x_{ab2}\ \ldots\ x_{abk}\ \ldots\ x_{abr}$	$x_{a..}$
Total	$x_{.1.}$	$\cdots$	$x_{.j.}$	$\cdots$	$x_{.b.}$	$x_{...}$

Where;

a : Number of levels of factor A

b : Number of levels of factor B

r : Number of replicates

x_{ijk} : the *k*th observation from the (*ij)*th cell

$x_{i..}$: the total of all observations from the *i*th level of factor A

$x_{.j.}$: the total of all observations from the *j*th level of factor B

$x_{...}$: the total of all observations

Assumptions for two-way ANOVA with replication

1. The populations under study follow normal distribution.
2. The samples are drawn randomly, and each sample is independent of the other samples.
3. All the populations from which the samples values are obtained, have the same unknown population variances, that is for *k* number of populations,

$$\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$$

4. Observations are taken on every possible treatment.
5. The number of replication is the same for each treatment.
6. The number of replication per treatment, *k* must be at least 2.

Model for Two-way ANOVA with replication

$$x_{ijk} = \mu + \alpha_i + \beta_j + \left(\alpha\beta\right)_{ij} + \varepsilon_{ijk}$$

where

x_{ijk}	: the k^{th} observation from the $(ij)^{\text{th}}$ cell
μ	: the overall mean
α_i	: the effect of the i^{th} level of factor A
β_j	: the effect of the j^{th} level of factor B
$(\alpha\beta)_{ij}$	: the interaction effect of the i^{th} level of factor A and the j^{th} level of factor B

Rules to construct the model

1. Assume $\varepsilon_{ijk} \sim NID(0, \sigma^2)$

2. $\sum_{i=1}^{a} \alpha_i = 0 \quad \sum_{j=1}^{b} \beta_j = 0 \quad \sum_{i=1}^{a} (\alpha\beta)_{ij} = 0 \quad \sum_{j=1}^{b} (\alpha\beta)_{ij} = 0$

Null & Alternative Hypothesis

Interaction effect

H_{0AB}: There is no interaction effect between factor A and factor B.

H_{1AB}: There is an interaction effect between factor A and factor B.

Row effect

H_{0A} : There is no effect of factor A.

H_{1A} : There is an effect of factor A.

Column effect

H_{0B} : There is no effect of factor B.

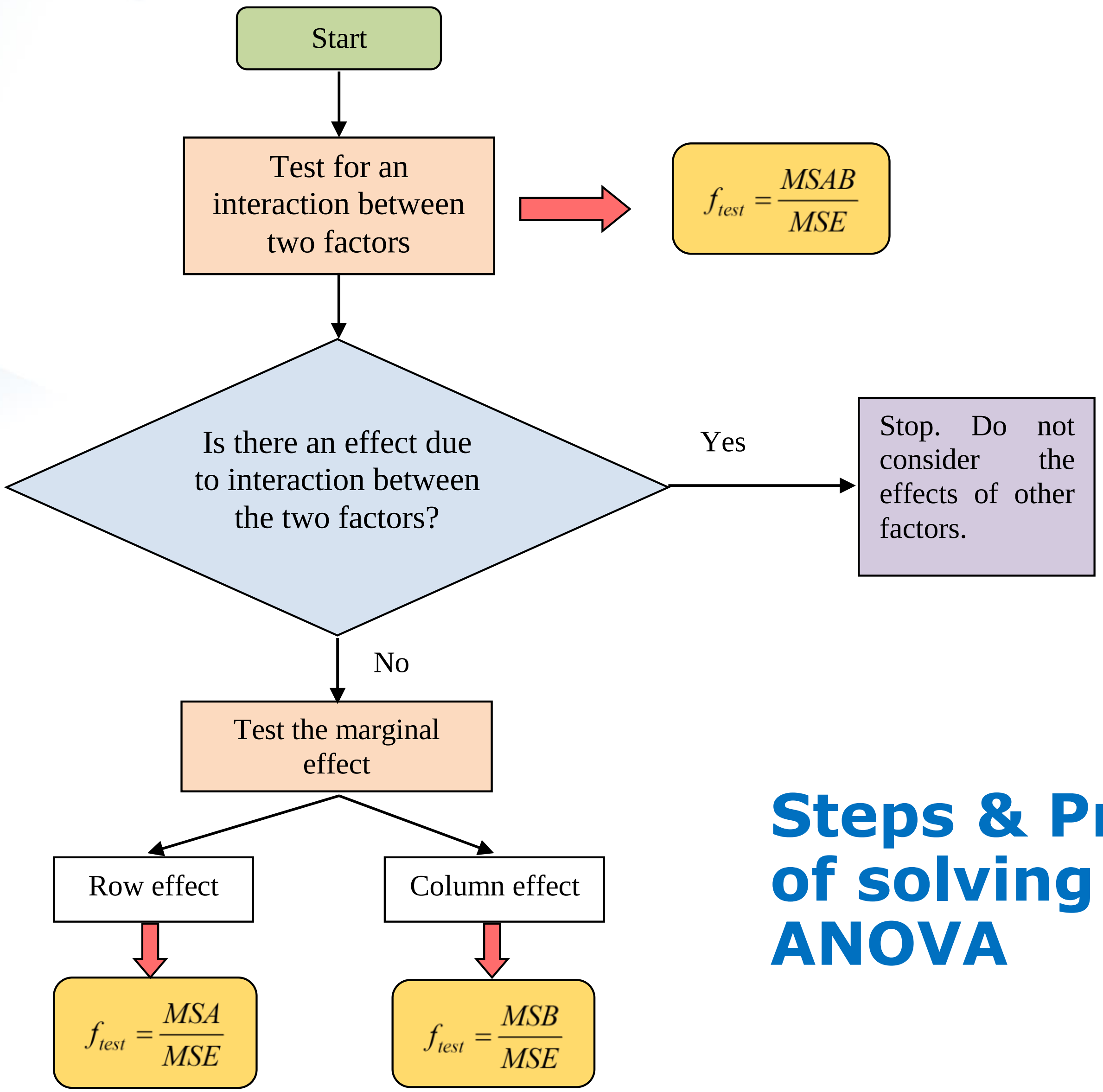
H_{1B} : There is an effect of factor B.

Marginal effect

Two-way ANOVA Table

Source	Sum Squares	Degree of Freedom	Mean Squares	f_{test}	Critical value
A (row effect)	$SSA = \frac{1}{br}\sum_{i=1}^{a} x_{i..}^2 - \frac{x_{...}^2}{abr}$	$a-1$	$MSA = \frac{SSA}{a-1}$	$f_{test} = \frac{MSA}{MSE}$	$f_{\alpha,a-1,ab(r-1)}$
B (column effect)	$SSB = \frac{1}{ar}\sum_{j=1}^{b} x_{.j.}^2 - \frac{x_{...}^2}{abr}$	$b-1$	$MSB = \frac{SSB}{b-1}$	$f_{test} = \frac{MSB}{MSE}$	$f_{\alpha,b-1,ab(r-1)}$
AB (interaction effect)	$SSAB = \frac{1}{r}\sum_{i=1}^{a}\sum_{j=1}^{b} x_{ij.}^2 - \frac{x_{...}^2}{abr} - SSA - SSB$	$(a-1)(b-1)$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$f_{test} = \frac{MSAB}{MSE}$	$f_{\alpha,(a-1)(b-1),ab(r-1)}$
Error	$SSE = SST - SSA - SSB - SSAB$	$ab(r-1)$	$MSE = \frac{SSE}{ab(r-1)}$		
Total	$SST = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r} x_{ijk}^2 - \frac{x_{...}^2}{abr}$	$abr-1$			

Steps & Procedures of solving Two-Way ANOVA



Example 3

A chemical engineer studies the effects of various reagents and catalysts on the yield of a chemical process. Yield is expressed as a percentage of a theoretical maximum. Two runs of the process are made for each combination of three reagents and four catalysts.

Catalyst	Reagent		
	1	**2**	**3**
A	86.8 82.4	93.4 85.2	77.9 89.6
B	71.9 72.1	74.5 87.1	87.5 82.7
C	65.5 72.4	66.7 77.1	72.7 77.8
D	63.9 70.4	73.7 81.6	79.8 75.7

(a) How many treatments involved in this experiment?

(b) Construct an ANOVA table.

(c) Test if there is an interaction effect between reagents and catalysts on the yield of a chemical process at 5% significance level

Example 4.3: solution

a) How many treatments involved in this experiment?

Factor A has 4 levels, factor B has 3 levels and each sample has 2 replicates.

Thus, a = 4, b = 3, and r = 2. Total treatment = 12.

b) ANOVA

Catalyst	Reagent			Total
	1	2	3	
A	$x_{11.} = 169.2$	$x_{12.} = 178.6$	$x_{13.} = 167.5$	$x_{1..} = 515.3$
B	$x_{21.} = 144.0$	$x_{22.} = 161.6$	$x_{23.} = 170.2$	$x_{2..} = 475.8$
C	$x_{31.} = 137.9$	$x_{32.} = 143.8$	$x_{33.} = 150.5$	$x_{3..} = 432.2$
D	$x_{41.} = 134.3$	$x_{42.} = 155.3$	$x_{43.} = 155.5$	$x_{4..} = 445.1$
Total	$x_{.1.} = 585.4$	$x_{.2.} = 639.3$	$x_{.3.} = 643.7$	$x_{...} = 1868.4$

Example 4.3: solution

Calculate sum of square values:

$\sum_{i=1}^{4}\sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk}^2$

$$SST = \boxed{86.8^2 + 82.4^2 + \ldots + 75.7^2} - \frac{1868.4^2}{(4)(3)(2)} = 1440.0400$$

$\sum_{i=1}^{4} x_{i..}^2$

$$SSA = \frac{1}{(3)(2)}\boxed{\left(515.3^2 + 475.8^2 + 432.2^2 + 445.1^2\right)} - \frac{1868.4^2}{(4)(3)(2)} = 683.4900$$

$\sum_{j=1}^{3} x_{.j.}^2$

$$SSB = \frac{1}{(4)(2)}\boxed{\left(585.4^2 + 639.3^2 + 643.7^2\right)} - \frac{1868.4^2}{(4)(3)(2)} = 263.4775$$

$\sum_{i=1}^{4}\sum_{j=1}^{3} x_{ij.}^2$

$$SSAB = \frac{1}{2}\boxed{\left(169.2^2 + 178.6^2 + \ldots + 155.5^2\right)} - \frac{1868.4^2}{(4)(3)(2)} - 683.4900 - 263.4775 = 138.7825$$

$$SSE = 1440.0400 - 683.4900 - 263.4775 - 138.7825 = 354.29$$

Example 4.3: solution

ANOVA table:

Source	Sum Squares	Degree of Freedom	Mean Squares	f_{test}
A (row effect)	683.4900	3	$MSA = \frac{683.4900}{3} = 227.8300$	$f_{testA} = \frac{227.8300}{29.5242} = 7.7167$
B (column effect)	263.4775	2	$MSB = \frac{263.4775}{2} = 131.7388$	$f_{testB} = \frac{131.7388}{29.5242} = 4.4621$
AB (interaction effect)	138.7825	6	$MSAB = \frac{138.7825}{6} = 23.1304$	$f_{testAB} = \frac{23.1304}{29.5242} = 0.7834$
Error	354.2900	12	$MSE = \frac{354.2900}{12} = 29.5242$	
Total	1440.0400	23		

Example 4.3: solution

(d) Test if there is an interaction effect between reagents and catalysts on the yield of a chemical process at 5% significance level

H_{0AB} : There is no interaction effect between reagent and catalyst

H_{1AB} : There is an interaction effect between reagent and catalyst

From statistical table, $f_{0.05,6,12} = 2.9961$.

Clearly that, $\left(f_{testAB} = 0.7834 \right) < \left(f_{0.05,6,12} = 2.9961 \right)$. Thus, do not reject H_0.

At $\alpha = 0.05$, there is no interaction effect between catalyst and reagent on the yield of a chemical process

Since there is no interaction effect, test of row effect and column effect should be conducted as follows.

Example 4.3: solution

Row effect

H_{0A}: There is no effect of catalysts

H_{1A}: There is an effect of catalyst

From statistical table, $f_{0.05,3,12} = 3.4903$.

Clearly that, $\left(f_{testA} = 7.7167\right) > \left(f_{0.05,3,12} = 3.4903\right)$. Thus, reject H_0

At $\alpha = 0.05$, there is an effect of catalyst on the yield of a chemical process .

Column effect

H_{0B}: There is no effect of reagents

H_{1B}: There is an effect of reagents

From statistical table, $f_{0.05,2,12} = 3.8853$.

Clearly that, $\left(f_{testB} = 4.4621\right) > \left(f_{0.05,2,12} = 3.8853\right)$. Thus, reject H_0

At $\alpha = 0.05$, there is an effect of reagents on the yield of a chemical process.

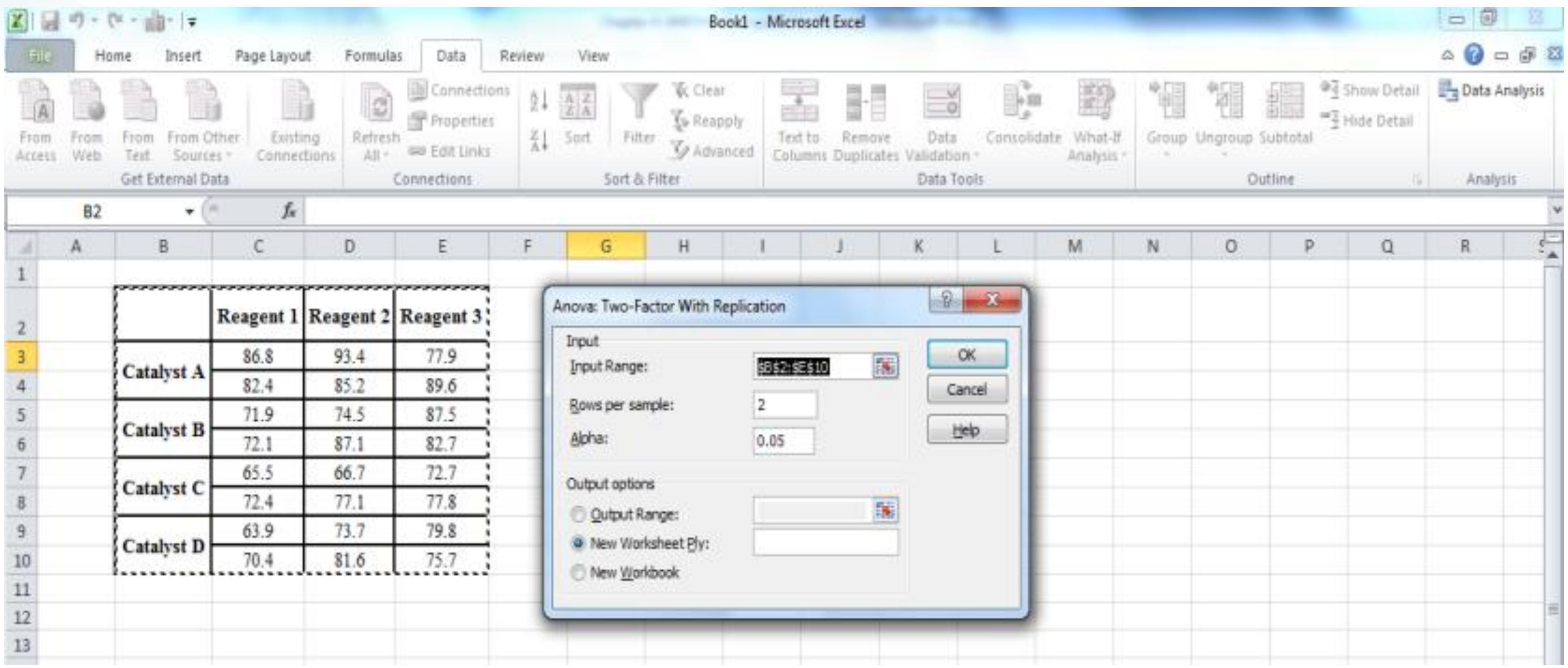
Solve two-way ANOVA by using Microsoft EXCEL

1. Excel – Key in data

	Reagent 1	Reagent 2	Reagent 3
Catalyst A	86.8	93.4	77.9
	82.4	85.2	89.6
Catalyst B	71.9	74.5	87.5
	72.1	87.1	82.7
Catalyst C	65.5	66.7	72.7
	72.4	77.1	77.8
Catalyst D	63.9	73.7	79.8
	70.4	81.6	75.7

2. Follow the steps below:

Data-Data Analysis → ANOVA two factor with replication → enter the data range → set a value for α → OK



3. ANOVA table:

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Sample	683.49	3	227.83	7.716729	0.003903	3.490295
Columns	263.4775	2	131.7388	4.462065	0.03558	3.885294
Interaction	138.7825	6	23.13042	0.78344	0.598995	2.99612
Within	354.29	12	29.52417			

Reject H_0 if *P*-value ≤ α or $F > F\ crit$

H_{0AB} : There is no interaction effect between reagent and catalyst

H_{1AB} : There is an interaction effect between reagent and catalyst

Clearly that, $(Pvalue = 0.5990) > (\alpha = 0.05)$. Thus, do not reject H_0.

At $\alpha = 0.05$, there is no interaction effect between catalyst and reagent on the yield of a chemical process. Since there is no interaction effect, test of row effect and column effect should be conducted.

3. ANOVA table:

catalyst → Sample
reagents → Columns

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Sample	683.49	3	227.83	7.716729	0.003903	3.490295
Columns	263.4775	2	131.7388	4.462065	0.03558	3.885294
Interaction	138.7825	6	23.13042	0.78344	0.598995	2.99612
Within	354.29	12	29.52417			

H_{0A} : There is no effect of catalysts

H_{1A} : There is an effect of catalyst

H_{0B} : There is no effect of reagents

H_{1B} : There is an effect of reagents

ROW EFFECT:

$(P\text{-value} = 0.0039) < (\alpha = 0.05)$, the decision is reject H_0.

At $\alpha = 0.05$, there is an effect of catalyst on the yield of a chemical process .

COLUMN EFFECT:

$(P\text{-value} = 0.0356) < (\alpha = 0.05)$, the decision is reject H_0

At $\alpha = 0.05$, there is an effect of reagents on the yield of a chemical process.

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Thank You

NEXT: CHAPTER 5 LINEAR REGRESSION & CORRELATION