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BCN 1043

COMPUTER ARCHITECTURE & ORGANIZATION

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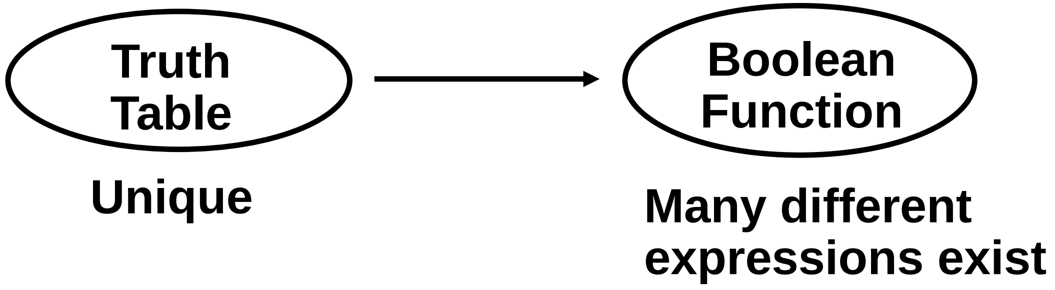
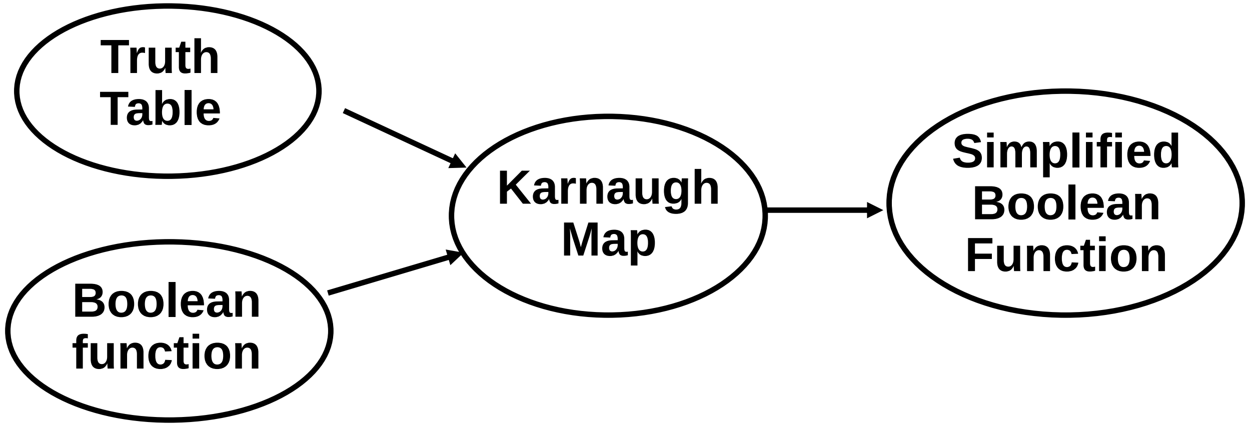
Chapter 3 continues…

SIMPLIFICATION

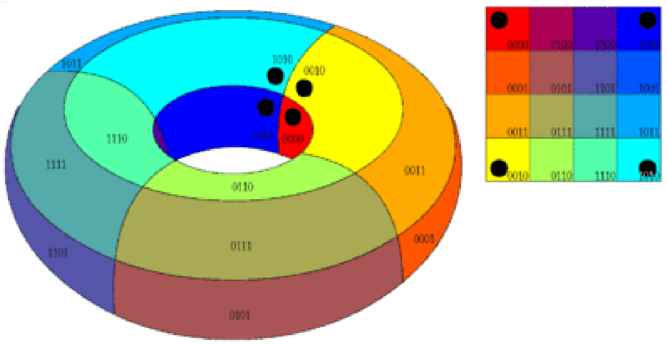
Simplification of Boolean function

- Reducing to lesser number of Boolean literals
- for least cost implementation
- Karnaugh Map (K-map) is a tabular method to reduce Boolean expressions.





SIMPLIFICATION: **KARNAUGH MAP**



K-map Terminology

- K-map is a tabular method derived from output values of Boolean function.
- *minterm* is a product term with all possible combinations of input variables

- E.g
- minterms of an expression with inputs *x* and *y* :

$$\bar{x}\bar{y},\ \bar{x}y,\ x\bar{y},\ \text{and}\ xy$$

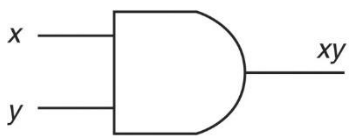
Minterm	X	Y
$\bar{X}\bar{Y}$	0	0
$\bar{X}Y$	0	1
$X\bar{Y}$	1	0
XY	1	1

- Minterms with three inputs

Minterm	X	Y	Z
$\bar{X}\bar{Y}\bar{Z}$	0	0	0
$\bar{X}\bar{Y}Z$	0	0	1
$\bar{X}Y\bar{Z}$	0	1	0
$\bar{X}YZ$	0	1	1
$X\bar{Y}\bar{Z}$	1	0	0
$X\bar{Y}Z$	1	0	1
$XY\bar{Z}$	1	1	0
XYZ	1	1	1

K-map

- K-map is referred as a cell for each minterm.
- truth table and k-map of function *F(x,y)* = *xy* is shown below



AND Gate

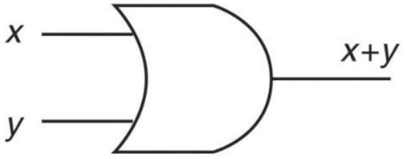
F(X,Y) = XY

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X \ Y	0	1
0	0	0
1	0	1

K-map

- E.g.2, *F(x,y) = x + y*
- Similar to **OR** gate



OR Gate

F(X,Y) = X+Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

X \ Y	0	1
0	0	1
1	1	1

X \ Y	0	1
0	0	1
1	1	1

$$F(x,y) = x + y = \bar{x}y + x\bar{y} + xy$$

$$F(x,y) = x + y$$

Kmap Simplification for Two Variables

rules for simplification :

- Group can contain only 1s; no 0s.
- Groups can occur only at right angles; no diagonal groups.
- In a group, number of 1s must be a power of 2
- Groups need to be as large as possible.
- Groups can overlap and wrap around the sides of the Kmap.

3-variable K-map Simplification

X \ YZ	00	01	11	10
0	$\overline{X}\,\overline{Y}\,\overline{Z}$	$\overline{X}\,\overline{Y}Z$	$\overline{X}YZ$	$\overline{X}Y\overline{Z}$
1	$X\overline{Y}\,\overline{Z}$	$X\overline{Y}Z$	XYZ	$XY\overline{Z}$

3-variable K-map Simplification

- E.g:

$$F(X,Y) = \overline{X}\,\overline{Y}Z + \overline{X}YZ + X\overline{Y}Z + XYZ$$

What could be the largest group of 1s?

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

3-variable K-map Simplification riables

- Simplified Boolean function, *F(x) = z.*

$$F(X,Y) = \bar{X}\bar{Y}Z + \bar{X}YZ + X\bar{Y}Z + XYZ$$

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

3-variable K-map Simplification

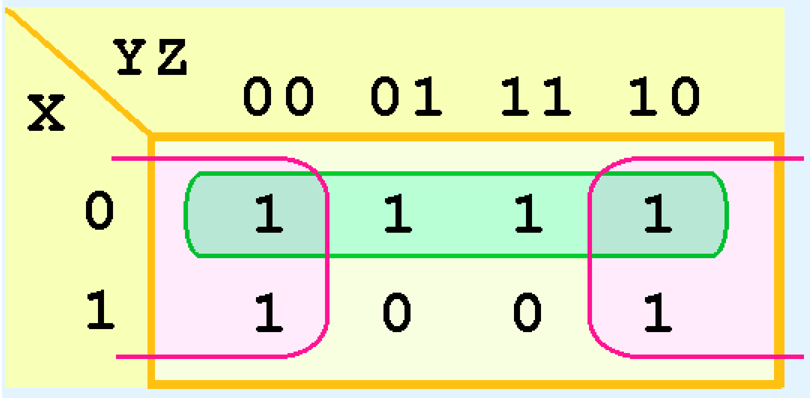
- E.g:

$$F(X,Y,Z) = \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}YZ + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z}$$

X \ YZ	00	01	11	10
0	1	1	1	1
1	1	0	0	1

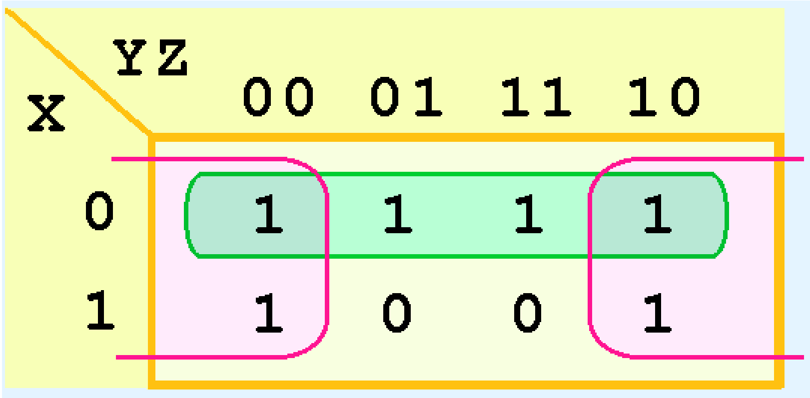
3-variable K-map Simplification

- E.g of side wrapping groups.



3-variable K-map Simplification

- Simplified function is: $F(X,Y,Z) = \overline{X} + \overline{Z}$



4-variable K-map Simplification

- With four variables, k-map can use 16 minterms

WX \ YZ	00	01	11	10
00	$\overline{W}\overline{X}\overline{Y}\overline{Z}$	$\overline{W}\overline{X}\overline{Y}Z$	$\overline{W}\overline{X}YZ$	$\overline{W}\overline{X}Y\overline{Z}$
01	$\overline{W}X\overline{Y}\overline{Z}$	$\overline{W}X\overline{Y}Z$	$\overline{W}XYZ$	$\overline{W}XY\overline{Z}$
11	$WX\overline{Y}\overline{Z}$	$WX\overline{Y}Z$	$WXYZ$	$WXY\overline{Z}$
10	$W\overline{X}\overline{Y}\overline{Z}$	$W\overline{X}\overline{Y}Z$	$W\overline{X}YZ$	$W\overline{X}Y\overline{Z}$

4-variable K-map Simplification

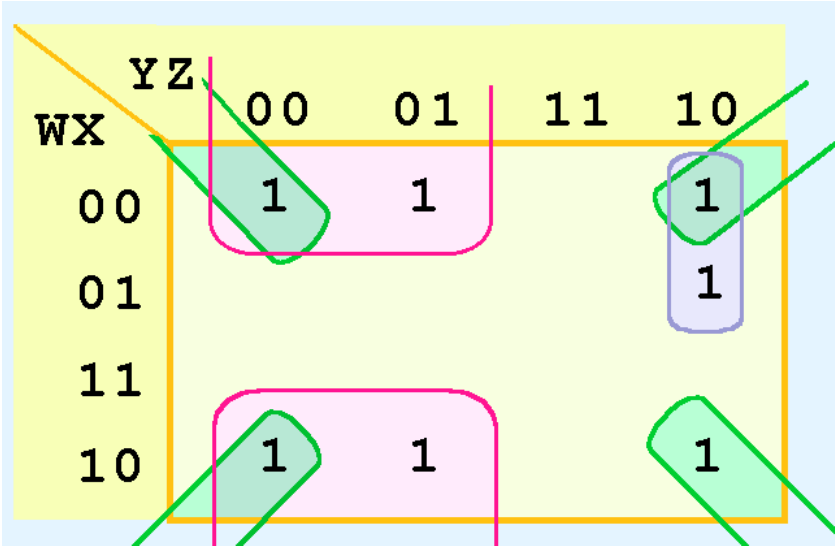
$$F(W,X,Y,Z) = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z} + \overline{W}\,\overline{X}\,\overline{Y}Z + \overline{W}\,\overline{X}Y\overline{Z}$$
$$+ \overline{W}XY\overline{Z} + W\overline{X}\,\overline{Y}\,\overline{Z} + W\overline{X}\,\overline{Y}Z + W\overline{X}Y\overline{Z}$$

WX \ YZ	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

4-variable K-map Simplification

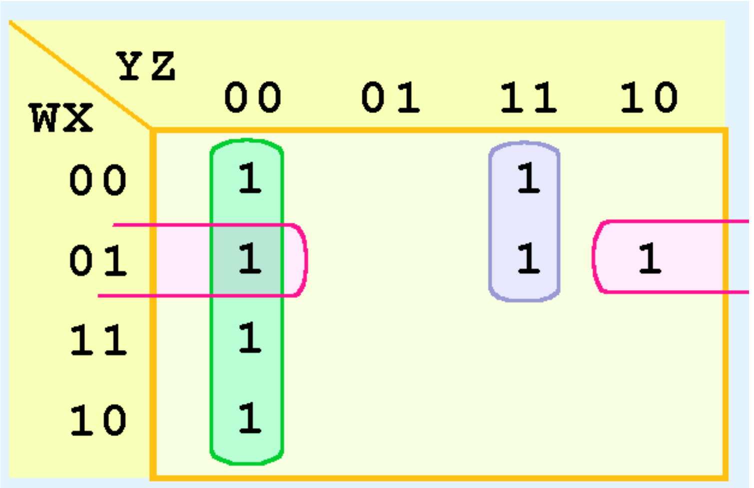
- three groups
- So we will have three terms in simplified function:

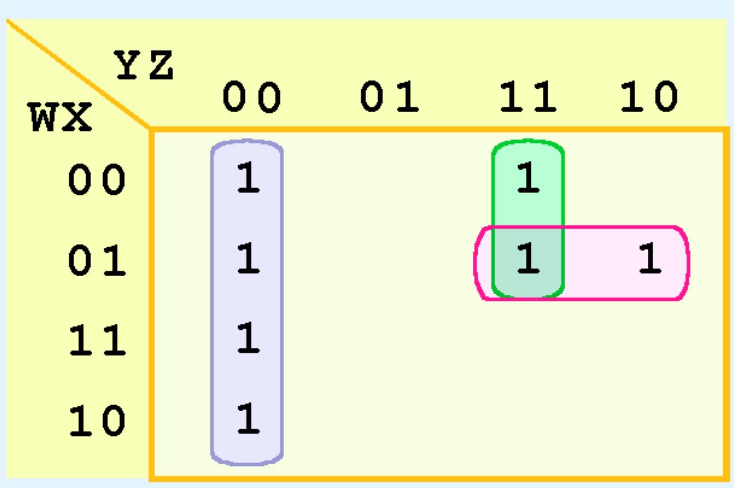
$$F(W,X,Y,Z) = \bar{W}\bar{Y} + \bar{X}\bar{Z} + \bar{W}Y\bar{Z}$$



4-variable K-map Simplification

- E.g of group formation





Don’t Care Conditions

- a circuit is designed in such a way that any particular input sets will never happen- *don’t care* condition.
- Used while grouping for simplification

Don’t Care Conditions

- Denoted by *X* of “d” in the K-map cell

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

Don’t Care Conditions

- E.g:

$$F(W,X,Y,Z) = \overline{W}\,\overline{Y} + YZ$$

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

Don’t Care Conditions

- E.g:

$$F(W,X,Y,Z) = \bar{W}Z + YZ$$

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

Don’t Care Conditions

- truth table of: $F(W,X,Y,Z) = \overline{W}\,\overline{Y} + YZ$

  differs from the truth table of: $F(W,X,Y,Z) = \overline{W}Z + YZ$

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

Will continue…