# BMA4723 VEHICLE DYNAMICS

# Ch4 Vehicle Equation of Motions

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# Chapter Description

- Aims
  - Explain the steer characteristics of the vehicle (understeer, oversteer and normal steer)
- Expected Outcomes
  - Students are able to determine the steer characteristics of the vehicle.
- References
  - M.Abe, Vehicle Handling Dynamics Theory and Application, Second Edition, Published by Elsevier Ltd, 2015
  - Thomas D.Gillespie, Fundamental of Vehicle Dynamics, Published by Society of Automotive Engineers

# Outlines

- 4.7 Steer characteristics of the vehicle
- 4.7.1 Effect of the fixed steering angle to the steer characteristics
- 4.7.2 Effect of the fixed turning radius to the steer characteristics
- 4.7.3 Effect of the yaw velocity to the steer characteristics
- 4.7.4 Effect of the side slip angle to the steer characteristics

# 4.7 Steer Characteristics of the Vehicle

- Steer characteristics of the vehicle can be divided into three categories; neutral steer (NS), over steer (OS) and under steer (US).
- If the vehicle is turning with the fixed steering angle, $\delta_o$, the velocity of the vehicle, $V$ is related to the turning radius, $\rho$.
- And if the turning radius is fixed, $\rho_o$ the velocity of the vehicle, $V$ is related to the steering angle, $\delta$.

# 4.7.1 Effect of the fixed steering angle to the steer characteristics

- From the previous subsection, the steady-state cornering have been discussed.
- The steady-state cornering is the condition when the vehicle is turning at the fixed steer angle, $\delta$ and constant speed, $V$.
- However, if the steer angle, $\delta$ and speed, $V$ is not constant, the turning radius either will increase or decrease.
- To analyse the steer characteristics of the vehicle, the equation of turning radius, $\rho$ that have been derived in the previous section will be used.

# 4.7.1 Effect of the fixed steering angle to the steer characteristics

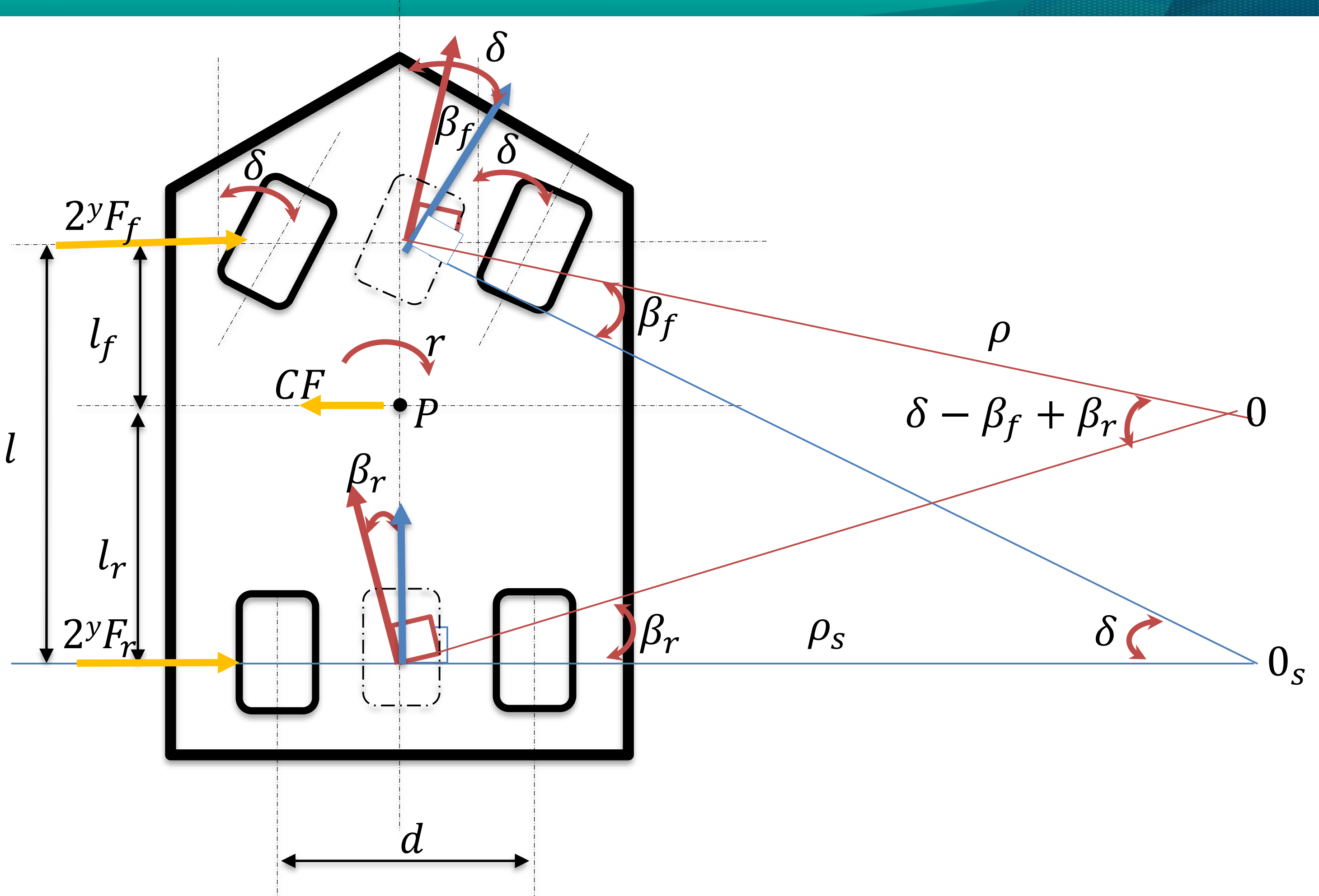


Figure 1 Steady-state cornering at high speed

# 4.7.1 Effect of the fixed steering angle to the steer characteristics

- In the previous subsection, Fig.1 have been used to derive the equation of turning radius, $\rho$.
- The turning radius, $\rho$ is:

$$\rho = \frac{V}{r} = \left(1 - \frac{m}{2l^2} \frac{l_f K_f - l_r K_r}{K_f K_r} V^2\right) \frac{l}{\delta} \qquad \text{(Eq.1)}$$

- Considered that the steering angle is fixed, $\delta_o$, then the turning radius, $\rho$ can be described as:

$$\rho = \left(1 - \frac{m}{2l^2} \frac{l_f K_f - l_r K_r}{K_f K_r} V^2\right) \frac{l}{\delta_o} \qquad \text{(Eq.2)}$$

- From this equation, if the velocity, $V$ is changes, the turning radius, $\rho$ also will changes.

# 4.7.1 Effect of the fixed steering angle to the steer characteristics

- By using Eq.1, the relation of the vehicle speed, $V$ to the turning radius, $\rho$ is illustrated in Fig.1.

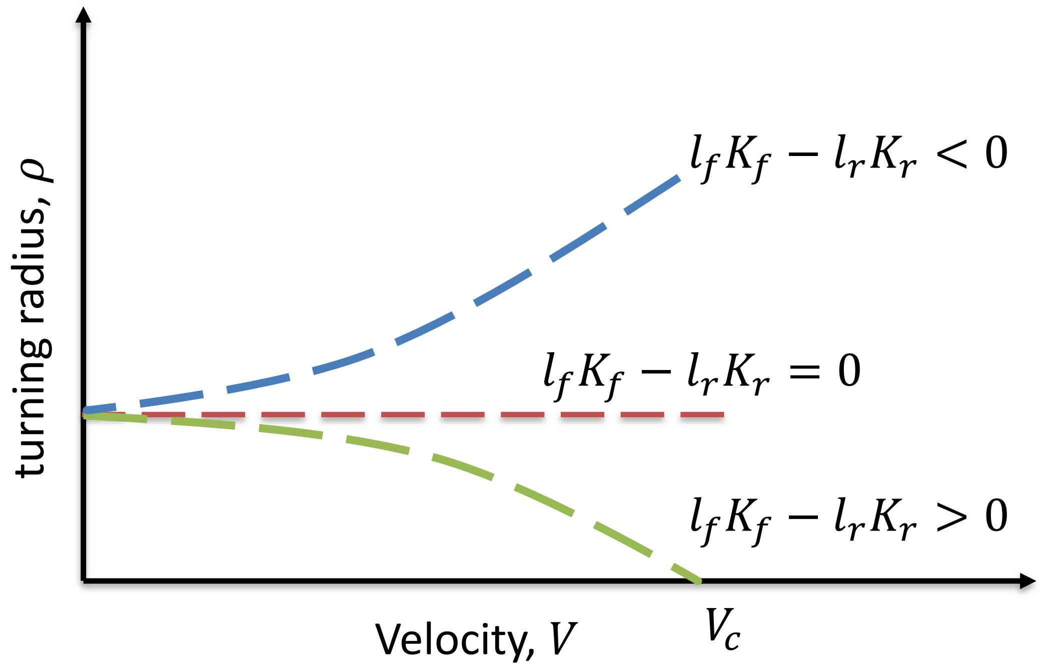


Figure 1 Relation of turning radius to the velocity at the fixed steering angle

# 4.7.1 Effect of the fixed steering angle to the steer characteristics

- From Fig.1, it can be seen that if $l_f K_f - l_r K_r = 0$, the changes of the velocity, $V$ is not affect the turning radius, $\rho$.
- At any velocity, the equation of the constant turning radius, $\rho_s$ is:

$$\rho_s = \frac{l}{\delta_o} \qquad \text{(Eq.3)}$$

- Eq.3 means that the turning radius is not dependent on velocity, $V$ and this steer characteristics is call neutral steer (NS).

# 4.7.1 Effect of the fixed steering angle to the steer characteristics

- From Fig.1, it also can be seen that if $l_f K_f - l_r K_r < 0$, the turning radius, $\rho$ will increase if the velocity of the vehicle, $V$ is increase.
- It means that the vehicle with $l_f K_f - l_r K_r < 0$ will make a larger turning radius, $\rho$.
- The vehicle will turn out from the original circular path and this steer characteristics is called under steer (US).
- On the other hand, if $l_f K_f - l_r K_r > 0$, the turning radius, $\rho$ will decrease if the velocity of the vehicle, $V$ is increase.
- In this situation, the vehicle will make a smaller turning radius, $\rho$.
- The vehicle will turn to the inner side of the original circular path, and this steer characteristics is called over steer (OS).

# 4.7.2 Effect of the fixed turning radius to the steer characteristics

By using Eq.2, we can analyse the effect of the velocity, $V$ to the steering angle, $\delta$ when the turning radius is fixed, $\rho_o$.

$$\delta = \left(1 - \frac{m}{2l^2}\frac{l_f K_f - l_r K_r}{K_f K_r} V^2\right)\frac{l}{\rho_o} \qquad \text{(Eq.4)}$$

- The relation between steering angle, $\delta$ and the velocity of the vehicle, $V$ is shown in Fig.2.

# 4.7.2 Effect of the fixed turning radius to the steer characteristics

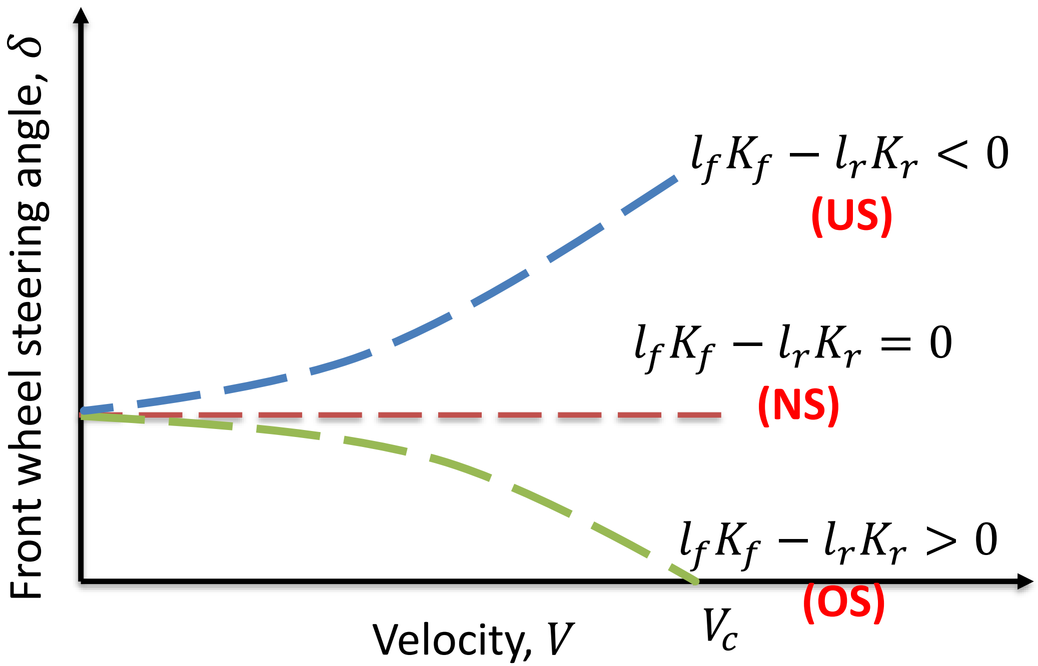


Figure 2 Relation of front wheel steering angle to the velocity at the fixed Turning radius

# 4.7.2 Effect of the fixed turning radius to the steer characteristics

- From Fig.2, it can be seen that if $l_f K_f - l_r K_r = 0$, the changes of the velocity, $V$ is not related to the steering angle, $\delta$ when the turning radius is fixed, $\rho_o$.
- It means that at any speed, if $l_f K_f - l_r K_r = 0$, the steering angle will constant.
- The equation of the constant steering angle, $\delta_s$ is:

$$\delta_s = \frac{l}{\rho_o} \qquad \text{(Eq.5)}$$

In this condition, the vehicle will turn at the original circular path, and this steer characteristics is call neutral steer (NS).

# 4.7.2 Effect of the fixed turning radius to the steer characteristics

- From Fig.2, when $l_f K_f - l_r K_r < 0$, the vehicle is under steer (US) and the steering angle must be added to maintain a vehicle at the original circular path.
- In contradict, when $l_f K_f - l_r K_r > 0$, the vehicle is over steer (OS) and the steering angle must be reduced to maintain a vehicle at the original circular path.

# 4.7.3 Effect of the yaw velocity to the steer characteristics

- The next part is to analyse the effect of the yaw velocity, $r$ to the steer characteristics when the velocity of the vehicle, $V$ is increase.
- In the previous subsection, the equation of yaw velocity has been derived as below:

$$r = \frac{1}{1 - \frac{m}{2l^2}\frac{l_f K_f - l_r K_r}{K_f K_r}V^2}\frac{V}{l}\delta \qquad \text{(Eq.6)}$$

- From this equation, when the steering angle is fixed, $\delta_o$, the equation of yaw velocity becomes:

$$r = \frac{1}{1 - \frac{m}{2l^2}\frac{l_f K_f - l_r K_r}{K_f K_r}V^2}\frac{V}{l}\delta_o \qquad \text{(Eq.7)}$$

# 4.7.3 Effect of the yaw velocity to the steer characteristics

- From Eq.7, the relation of the yaw velocity, $r$ to the velocity of the vehicle, $V$ can be illustrated as in Fig.3.

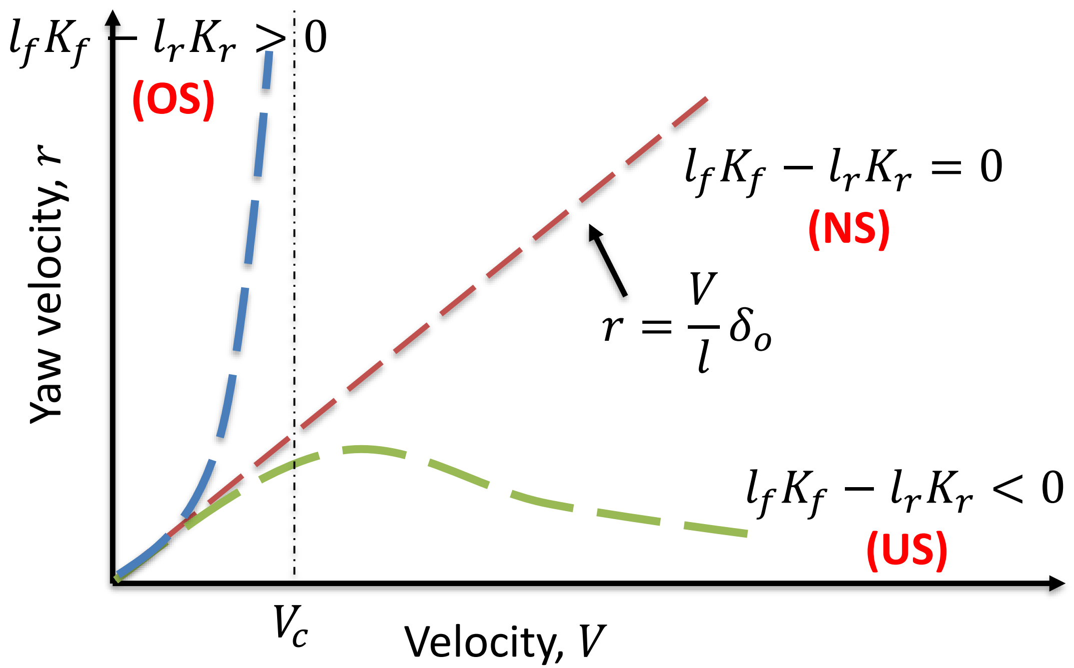


Figure 3 Relation of yaw velocity to the velocity of the vehicle at the fixed Steering angle

# 4.7.3 Effect of the yaw velocity to the steer characteristics

- From Eq.7 and Fig.3, the yaw velocity, $r$ is increase linearly with the velocity of the vehicle, $V$.
- In the under steer (US) characteristics, the yaw velocity, $r$ also increase with the velocity of the vehicle, $V$.
- However, at a certain point, the yaw velocity, $r$ becomes saturated.
- For over steer (OS) characteristics, the yaw velocity, $r$ increases rapidly with the velocity of the vehicle, $V$.
- At the velocity of the vehicle is $V_c$, the yaw velocity, $r$ becomes infinite.

# 4.7.4 Effect of the side slip angle to the steer characteristics

- The last part of this section is to analyse the effect of the side-slip angle, $\beta$ to the velocity of the vehicle, $V$ when the steering angle is fixed, $\delta_o$.
- The equation of side-slip angle, $\beta$ has been derived in the previous section, and the equation is:

$$\beta = \left( \frac{1 - \frac{m}{2l} \frac{l_f}{l_r K_r} V^2}{1 - \frac{m}{2l^2} \frac{l_f K_f - l_r K_r}{K_f K_r} V^2} \right) \frac{l_r}{l} \delta \qquad \text{(Eq.8)}$$

- When the steering angle, $\delta_o$ is fixed, the side-slip angle, $\beta$ is:

$$\beta = \left( \frac{1 - \frac{m}{2l} \frac{l_f}{l_r K_r} V^2}{1 - \frac{m}{2l^2} \frac{l_f K_f - l_r K_r}{K_f K_r} V^2} \right) \frac{l_r}{l} \delta_o \qquad \text{(Eq.9)}$$

# 4.7.4 Effect of the side slip angle to the steer characteristics

- From Eq.9 and Fig.4, the side-slip angle, $\beta$ decreases for all steer characteristics.
- From Fig.4, it also can be seen that at a certain velocity, the side-slip angle, $\beta$ becomes negative.
- For the under steer (US) characteristics, $\beta$ will reach a maximum value at larger velocity, $V$, and for the over steer (OS) characteristics, $\beta$ becomes negative infinity at $V_c$.

# Conclusion of the Chapter 4

- Conclusion #1
  - Steer characteristics can be divided into three categories, neutral steer (NS), under steer (US) and over steer (OS).

- Conclusion #2
  - By using the equation of turning radius, $\rho$, yaw velocity, $r$ and side-slip angle, $\beta$, the steer characteristics of the vehicle can be determined.

# Vehicle Dynamics

# Chapter 4

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