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BMA4723 VEHICLE DYNAMICS

Ch4 Vehicle Equation of Motions

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Chapter Description

- Aims
  - Explain the steady-state cornering when the vehicle is travelling at a low speed and high speed.

- Expected Outcomes
  - Students are able to determine the steady-state cornering of the vehicle.

- References

  - M.Abe, Vehicle Handling Dynamics Theory and Application, Second Edition, Published by Elsevier Ltd, 2015
  - Thomas D.Gillespie, Fundamental of Vehicle Dynamics, Published by Society of Automotive Engineers

Outlines

- 4.5 Steady-state cornering at low speed (without centrifugal force)
- 4.6 Steady-state cornering with centrifugal force

4.5 Steady-state cornering at low speed (without centrifugal force)

- Steady-state cornering is a condition when the vehicle is travelling at a constant speed, V and fixed front steering angle, δ.
- In this condition, the vehicle will make a constant radius of curvature, ρ.

4.5 Steady-state cornering at low speed (without centrifugal force)

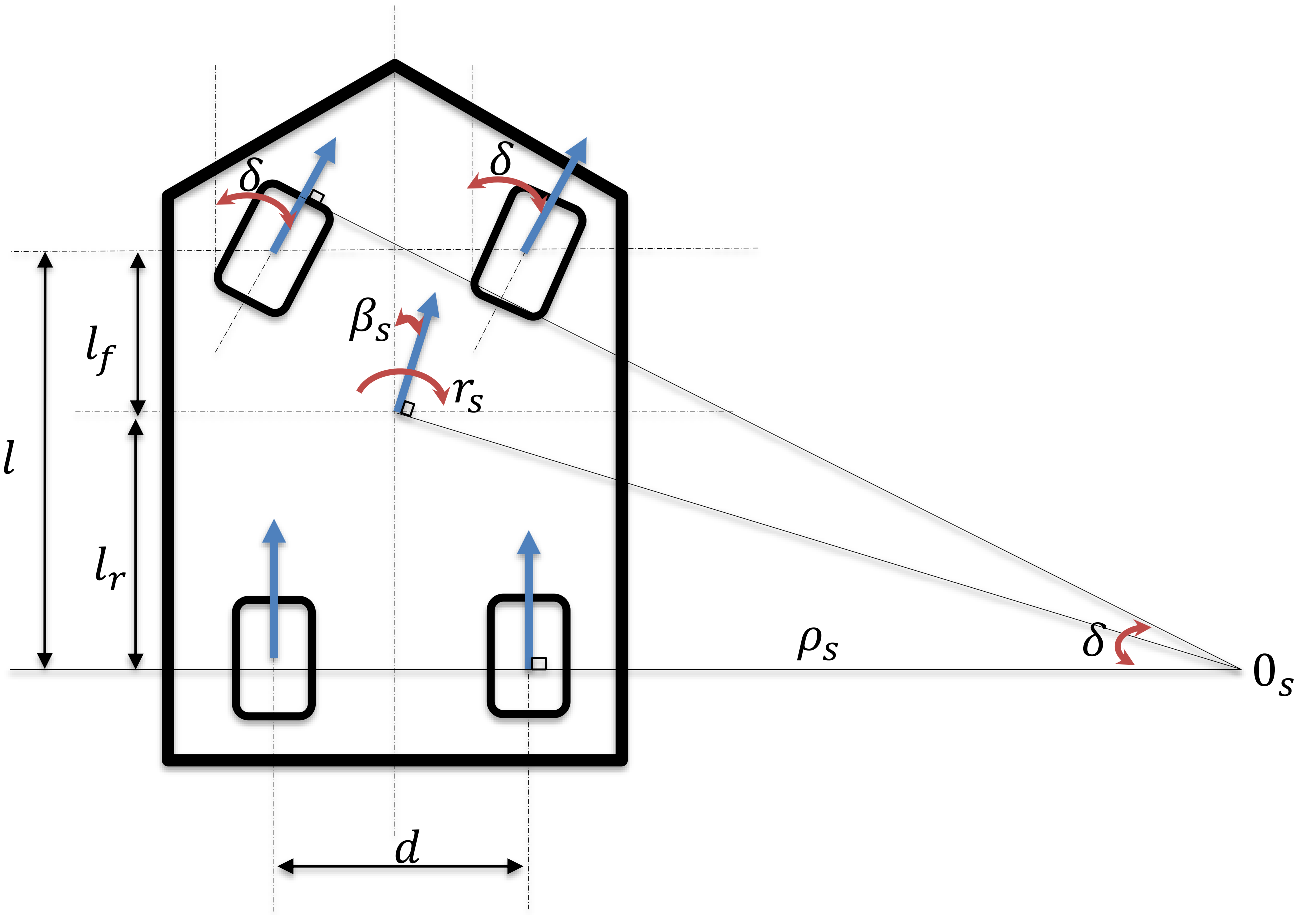

Figure 1 Steady-state cornering at low speed

4.5 Steady-state cornering at low speed (without centrifugal force)

- Fig.1 shows the steady-state cornering at low speed, which is $V \approx 0$.
- In this condition, no centrifugal force act on the vehicle.
- The lateral force are not generated at the tire, and at the same time, the side-slip angle also not created at the tire.
- The heading direction of the tire is same as the travelling direction.
- As shown in Fig.1, during steady-state cornering at low speed, the vehicle will make a circular motion around the point 0_s.
- Then, the geometric relations can be formulated as:

$$\rho_s = \frac{l}{\delta} \qquad \text{(Eq.1)}$$

$$r_s = \frac{V}{\rho_s} = \frac{V}{l}\delta \qquad \text{(Eq.2)}$$

$$\beta_s = \frac{l_r}{\rho_s} = \frac{l_r}{l}\delta \qquad \text{(Eq.3)}$$

whereby $0 < \delta \ll 1$ and $l \ll \rho$.

4.5 Steady-state cornering at low speed (without centrifugal force)

- The relation of Eq.1-Eq.3 also known as Ackermann steering geometry.
- From Eq.1, the Ackermann angle is described as:

$$\delta = \frac{l}{\rho_s} \qquad \text{(Eq.4)}$$

4.6 Steady-state cornering with centrifugal force

- When the vehicle make a circular motion at a larger speed, the centrifugal force will acts at the vehicle center of gravity.
- At this moment, the cornering forces at the front and rear tires will balance this centrifugal force .
- As a result, the side-slip angle will produce at the tires.

4.6 Steady-state cornering with centrifugal force

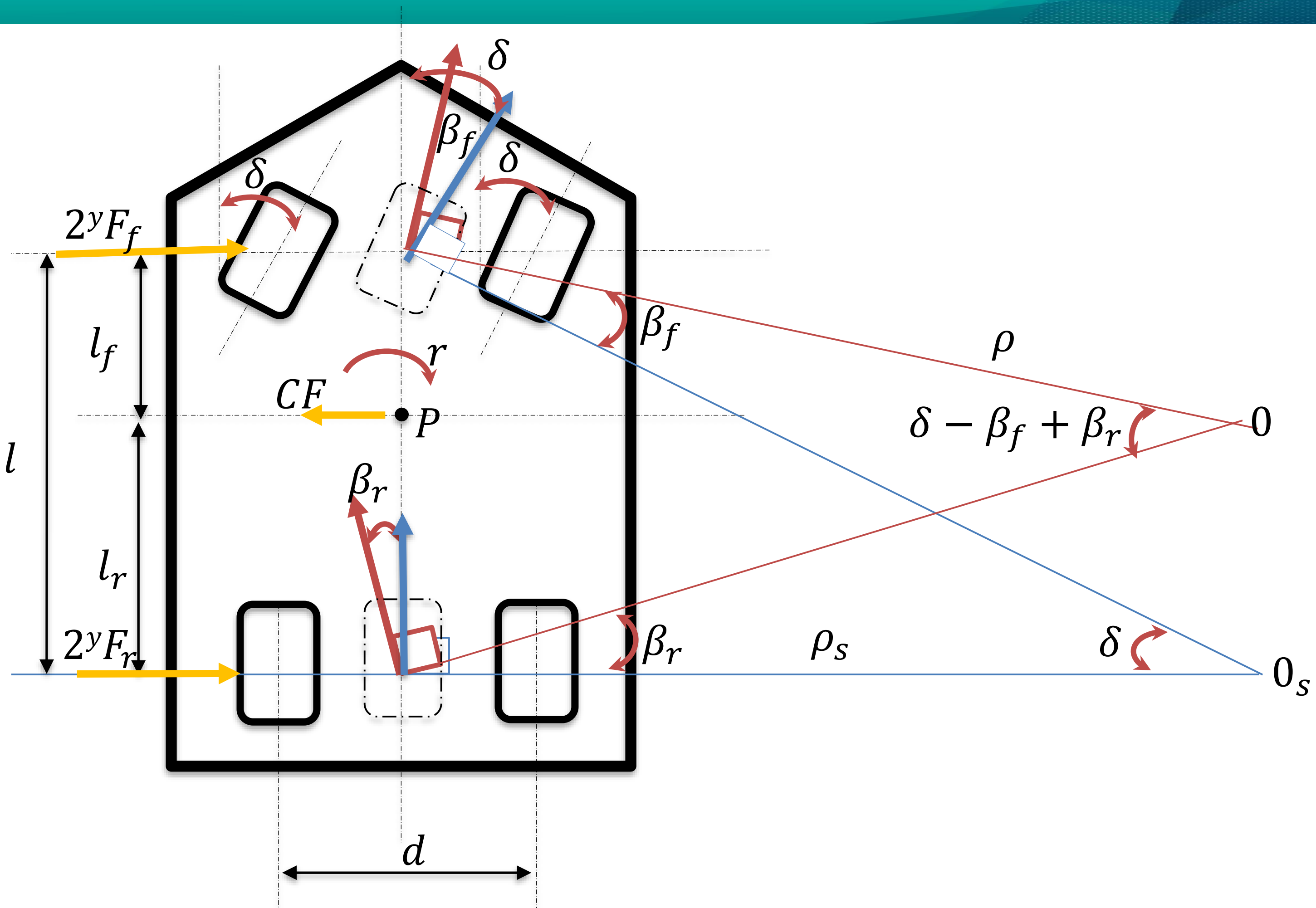

Figure 2 Steady-state cornering at high speed

4.6 Steady-state cornering with centrifugal force

- From Fig.2, the center of the circular motion, 0 is the intersecting of the two straight lines perpendicular to the travelling direction of the front and rear tires.
- Then, the turning radius, ρ, and yaw angular velocity at the center of the vehicle, r can be described as:

$$\rho = \frac{l}{\delta - \beta_f + \beta_r} \quad \text{(Eq.5)}$$

$$r = \frac{V}{\rho} = \frac{V(\delta - \beta_f + \beta_r)}{l} \quad \text{(Eq.6)}$$

4.6 Steady-state cornering with centrifugal force

- The side-slip angle at the vehicle center of gravity, β and at the front and rear tires can be illustrated as in Fig.3.

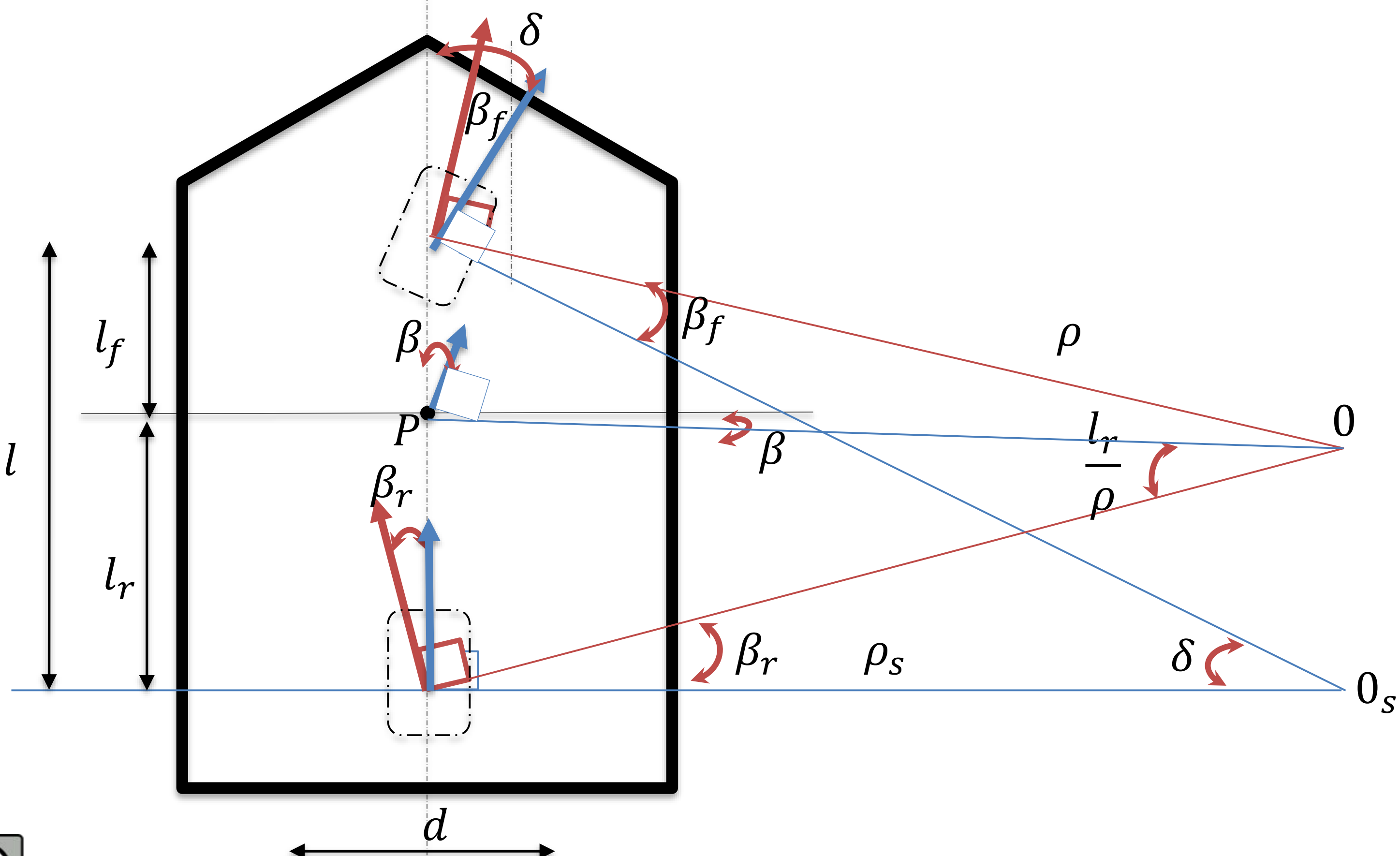

Figure 3 Side-slip angle during steady-state cornering

4.6 Steady-state cornering with centrifugal force

- The side-slip angle at the vehicle center of gravity, β and at the front and rear tires can be illustrated as in Fig.3.

$$\beta + \beta_r = \frac{l_r}{\rho} \qquad \text{(Eq.7)}$$

Then,

$$\beta = \frac{l_r}{\rho} - \beta_r = \frac{l_r}{l}\delta - \frac{l_r\beta_f + l_f\beta_r}{l} \qquad \text{(Eq.8)}$$

4.6 Steady-state cornering with centrifugal force

- During cornering, the lateral force at the front and rear tires, ${}^{y}F_{f}$ and ${}^{y}F_{r}$ are proportional to the side-slip angle of the tire, β_f and β_r.
- The equation of lateral force at the front and rear tires are:

$${}^{y}F_{f} = -2K_f\beta_f \qquad \text{(Eq.9)}$$

$${}^{y}F_{r} = -2K_r\beta_r \qquad \text{(Eq.10)}$$

- In this situation, the centrifugal force is generated at the center of the vehicle, and the equation of centrifugal force is

$$C_f = \frac{mv^2}{\rho} \qquad \text{(Eq.11)}$$

4.6 Steady-state cornering with centrifugal force

- When the vehicle is travelling in a steady-state turning, the equilibrium equation can be described as:

$$\frac{mv^2}{\rho} - 2K_f\beta_f - 2K_r\beta_r = 0 \qquad \text{(Eq.12)}$$

$$-2l_fK_f + 2l_rK_r = 0 \qquad \text{(Eq.13)}$$

4.6 Steady-state cornering with centrifugal force

- From these two equilibrium equations, the side slip angle at the front and left tires, β_f and β_r are:

$$\beta_f = \frac{mV^2 l_r}{2lK_f}\frac{1}{\rho} \qquad \text{(Eq.14)}$$

$$\beta_r = \frac{mV^2 l_f}{2lK_r}\frac{1}{\rho} \qquad \text{(Eq.15)}$$

4.6 Steady-state cornering with centrifugal force

- Substitutes β_f and β_r into Eq.5, Eq.6 and Eq.8 gives:

$$\beta = \left(\frac{1 - \frac{m}{2l} \frac{l_f}{l_r K_r} V^2}{1 - \frac{m}{2l^2} \frac{l_f K_f - l_r K_r}{K_f K_r} V^2} \right) \frac{l_r}{l} \delta \qquad \text{(Eq.16)}$$

$$r = \left(\frac{1}{1 - \frac{m}{2l^2} \frac{l_f K_f - l_r K_r}{K_f K_r} V^2} \right) \frac{V}{l} \delta \qquad \text{(Eq.17)}$$

$$\rho = \frac{V}{r} = \left(1 - \frac{m}{2l^2} \frac{l_f K_f - l_r K_r}{K_f K_r} V^2 \right) \frac{l}{\delta} \qquad \text{(Eq.18)}$$

Conclusion of the Chapter 4

- Conclusion #1
  - When the travelling speed is not inline with the longitudinal speed, the side slip angle will be created at the center of the vehicle.

- Conclusion #2
  - By using the geometry description, the steer performance of the vehicle at the low speed (without centrifugal force) and at the high speed (with centrifugal force) can be determined.

Vehicle Dynamics

Chapter 4

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