BMA4723 VEHICLE DYNAMICS

Ch4 Vehicle Equation of Motions

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Chapter Description

- Aims
  - Explain the coordinates axes of the vehicle
  - Explain the velocity and acceleration vector of the vehicle from the vector unit
  - Explain the side-slip angle of the vehicle

- Expected Outcomes
  - Students are able to derive the equations of motion based on the unit vector.
  - Students are able to determine the acceleration and deceleration of the vehicle by using the equations of motion.
  - Students are able to know how side-slip angle of the vehicle can be generated.

- References

  - M.Abe, Vehicle Handling Dynamics Theory and Application, Second Edition, Published by Elsevier Ltd, 2015
  - Thomas D.Gillespie, Fundamental of Vehicle Dynamics, Published by Society of Automotive Engineers

Outlines

- 4.1 Velocity and Acceleration of the Vehicle
- 4.2 Side-slip Angle of the Vehicle

4.1 Velocity and Acceleration of the Vehicle

- In the analysis of the velocity and acceleration of the vehicle, the fixed coordinate axes at the vehicle is used.
- Fig.1 shows the fixed coordinate axes at the vehicle.

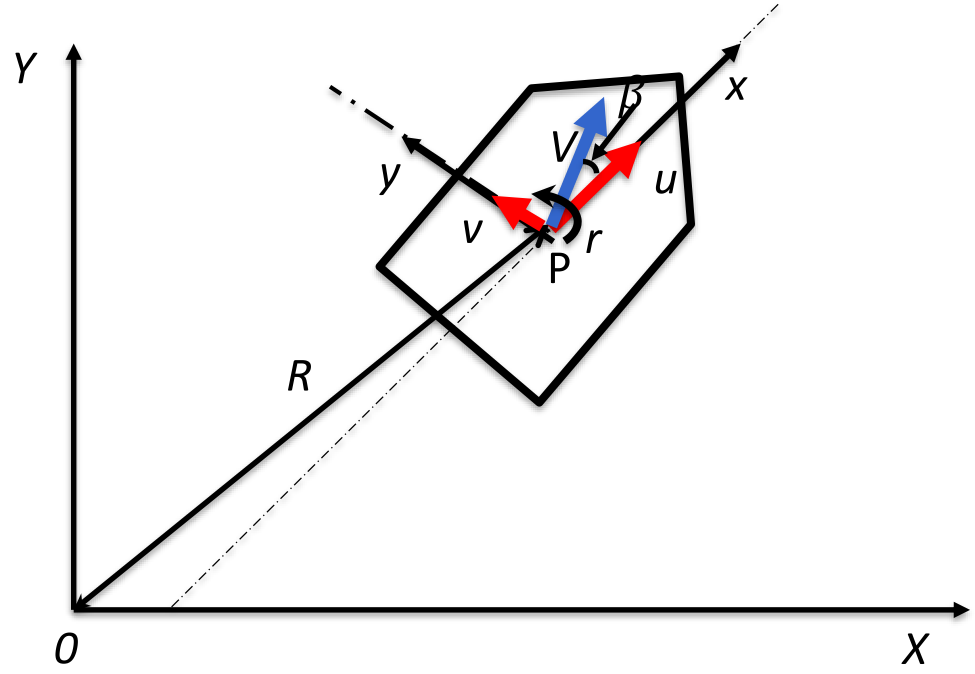

Figure 1 Fixed coordinate at the vehicle

4.1 Velocity and Acceleration of the Vehicle

From Fig.1:

P: center gravity of the vehicle.

R: position vector from the fixed coordinate on the ground to the fixed coordinate at the vehicle.

u: the longitudinal velocity of point Pat the *x*-axis

v: the lateral velocity of point P at the *y*-axis

β: side slip angles of the vehicle

r: yaw rate of the vehicle

4.1 Velocity and Acceleration of the Vehicle

- Based on Fig.1, the position vector of the vehicle from the reference on the ground (coordinate system $X - Y$), is defined as R.
- Then, the velocity vector, $\dot{R}$ can be written as:

$$\dot{R} = ui + vj \qquad \text{(Eq.1)}$$

- where *i* and *j* are the unit vectors in x and y directions respectively.

4.1 Velocity and Acceleration of the Vehicle

- Differentiating the velocity $\dot{R}$ in Eq.1 with time, the acceleration of the vehicle $\ddot{R}$ can be written as:

$$\ddot{R} = \dot{u}i + u\dot{i} + \dot{v}j + v\dot{j} \qquad \text{(Eq.2)}$$

4.1 Velocity and Acceleration of the Vehicle

- Now, considered that the unit vectors at the fixed coordinate on the ground as i_F and j_F.

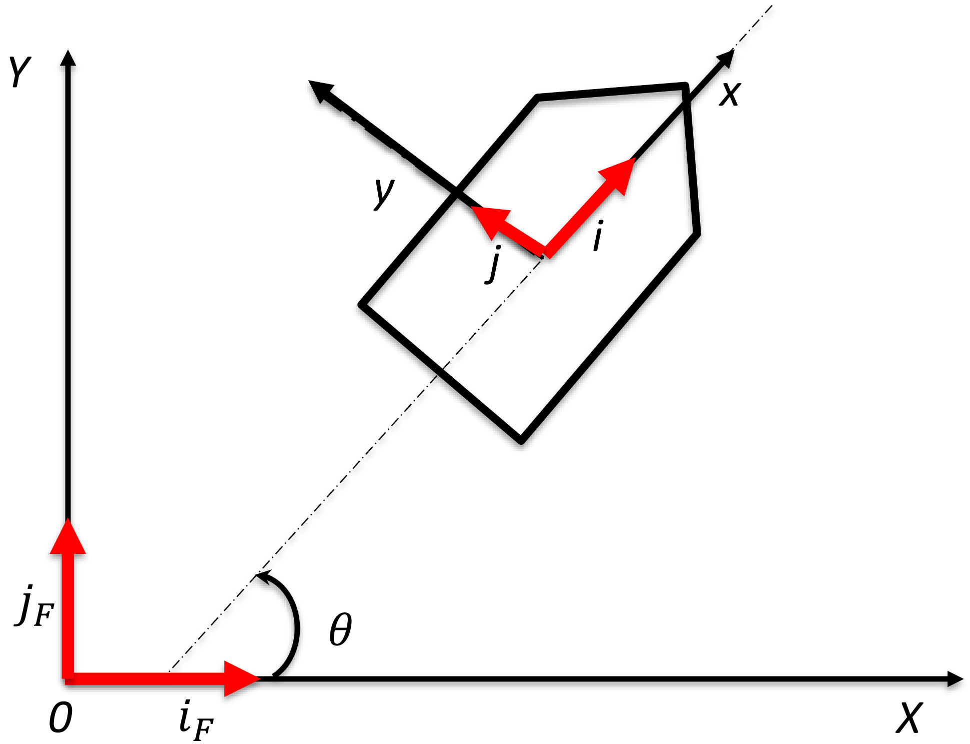

Figure 2 Unit vectors at the ground and at the vehicle

4.1 Velocity and Acceleration of the Vehicle

- From Fig.2, the relation of the unit vector at the x and y axis (i and j), to the reference X and Y axis on the ground (i_F and j_F) can be expressed as:

$$i = cos\theta\ i_f + sin\theta\ j_f \qquad \text{(Eq.3)}$$

$$\mathrm{j} = -sin\theta\ i_f + cos\theta\ j_f \qquad \text{(Eq.4)}$$

4.1 Velocity and Acceleration of the Vehicle

Then, differentiate i and j with the time:

$\dot{i} = -\dot{\theta}\ sin\theta\ i_F + \dot{\theta}\ cos\theta\ j_F = r(-sin\theta\ i_F + cos\theta\ j_{F)} = rj$ (Eq.5)

$\dot{j} = -\dot{\theta}\ cos\theta\ i_F - \dot{\theta}\ \sin\theta\ j_F = -r(cos\theta\ i_F + sin\theta\ j_{F)} = -ri$ (Eq.6)

- Substitute Eq.5 and Eq.6 into Eq.2:

$$\ddot{R} = \dot{u}i + u\dot{i} + \dot{v}j + v\dot{j}$$

$$\ddot{R} = \dot{u}i + u(rj) + \dot{v}j + v(-ri)$$

$$\ddot{R} = (\dot{u} - vr)i + (\dot{v} + ur)j \quad \text{(Eq.7)}$$

- From Eq.7, the longitudinal and lateral accelerations of the vehicle at point P can be expressed as:

$$a_x = \dot{u} - vr \quad \text{(Eq.8)}$$

$$a_y = \dot{v} - ur \quad \text{(Eq.9)}$$

4.1 Velocity and Acceleration of the Vehicle

The longitudinal and lateral acceleration of the vehicle are illustrated in Fig.3.

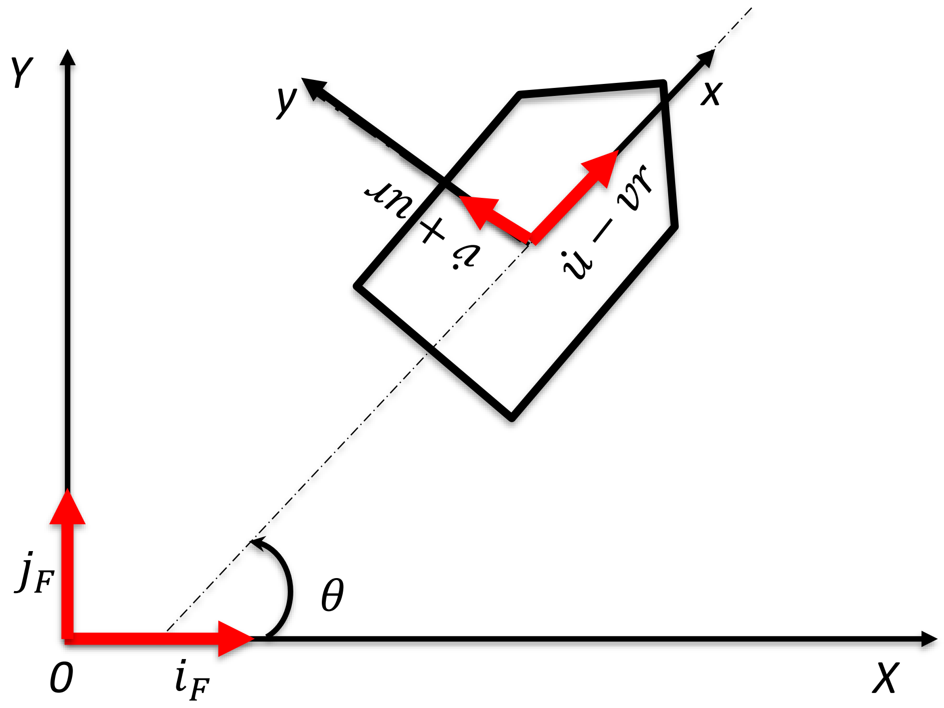

Figure 3 Longitudinal and lateral acceleration of the vehicle

4.1 Velocity and Acceleration of the Vehicle

- From the translational system of Newton's Second Law:

$$\sum F_x = ma_x \qquad \text{(Eq.10)}$$

$$\sum F_y = ma_y \qquad \text{(Eq.11)}$$

- The acceleration of the vehicle at the x and y axis can be expressed from the Eq.8 and Eq.9.
- Then, Eq.10 and Eq.11 can be written as:

$$m\left(\frac{du}{dt} - vr\right) = {}^{x}F_{FR} + {}^{x}F_{FL} + {}^{x}F_{RR} + {}^{x}F_{RL} \qquad \text{(Eq.12)}$$

$$m\left(\frac{dv}{dt} - vr\right) = {}^{y}F_{FR} + {}^{y}F_{FL} + {}^{y}F_{RR} + {}^{y}F_{RL} \qquad \text{(Eq.13)}$$

Where:

${}^{x}F$ is the traction or braking force and ${}^{y}F$ is the lateral force.

4.2 Side-slip Angle of the Vehicle

- In general, the vehicle travelling direction is same as the vehicle longitudinal direction.
- At the certain condition such as during cornering, the angle between the vehicle traveling direction and longitudinal direction is not same.
- The angle created between the vehicle travelling direction and longitudinal direction is called as side-slip angle, β.

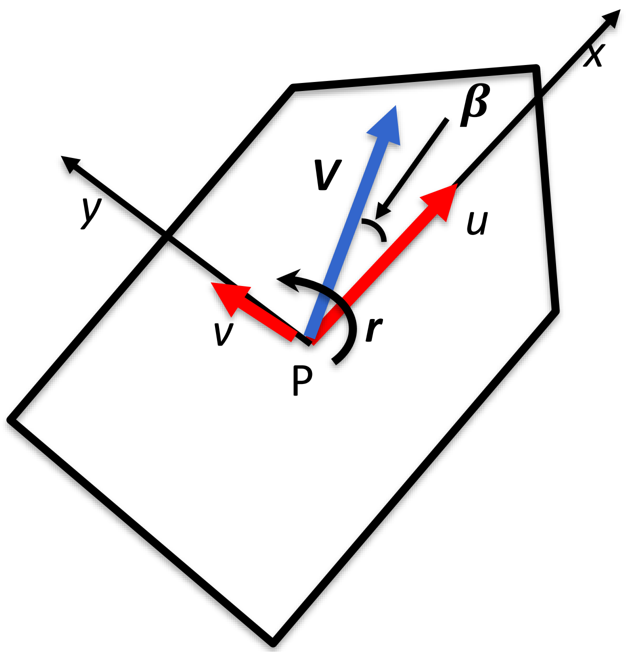

Figure 4 Side slip angle of the vehicle

4.2 Side-slip Angle of the Vehicle

- From Fig.4, the equation of side-slip angle, β is:

$$tan^{-1} = (^{v}/_{u}) \qquad \text{(Eq.14)}$$

- Then, the velocity and acceleration at the longitudinal and lateral axes are:

$$u = V\, cos\beta \qquad \text{(Eq.15)}$$

$$\dot{u} = -V\, sin\beta\dot{\beta} \qquad \text{(Eq.16)}$$

$$v = V\, sin\beta \qquad \text{(Eq.17)}$$

$$\dot{v} = V\, cos\beta\dot{\beta} \qquad \text{(Eq.18)}$$

4.2 Side-slip Angle of the Vehicle

- In the case of β is very small, $cos\beta \cong 1$ and $sin\beta \cong \beta$.
- At that moment, Eq.15 to Eq.18 can be expressed as:

$$u = V\, cos\beta \;\cong V \qquad \text{(Eq.19)}$$

$$\dot{u} = -V\, sin\beta\dot{\beta} \cong -V\beta\dot{\beta} \qquad \text{(Eq.20)}$$

$$v = V\, sin\beta \;\cong V\beta \qquad \text{(Eq21)}$$

$$\dot{v} = V\, cos\beta\dot{\beta} \cong V\dot{\beta} \qquad \text{(Eq22)}$$

4.2 Side-slip Angle of the Vehicle

- Substitutes Eq.19 to Eq.22 into Eq1 and Eq.7:

$$\dot{\boldsymbol{R}} = \boldsymbol{ui} + \boldsymbol{vj} \quad \textbf{(Eq.1)}$$

$$\dot{R} = Vi + V\beta j \quad \text{(Eq.23)}$$

$$\ddot{\boldsymbol{R}} = (\dot{\boldsymbol{u}} - \boldsymbol{vr})\boldsymbol{i} + (\dot{\boldsymbol{v}} + \boldsymbol{ur})\boldsymbol{j} \quad \textbf{(Eq.7)}$$

$$\ddot{R} = \left(-V\beta\dot{\beta} - V\beta r\right)i + \left(V\dot{\beta} + Vr\right)j$$

$$\ddot{R} = -V\beta\left(\dot{\beta} + r\right)i + V\left(\dot{\beta} + r\right)j \quad \text{(Eq.24)}$$

4.2 Side-slip Angle of the Vehicle

Fig.5 shows how the side-slip angle effect the velocity and acceleration at the longitudinal and lateral axes

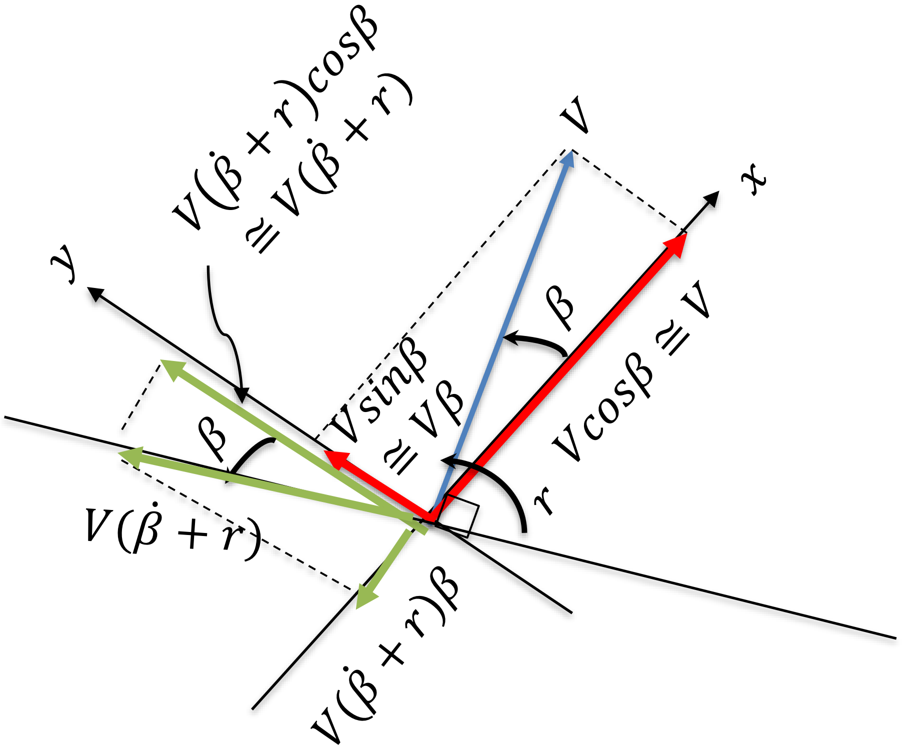

Figure 5 Side-slip angle effect the velocity and acceleration at the longitudinal and lateral axes

Conclusion of The Chapter 4

- Conclusion #1
  - The fixed coordinate of the vehicle can be used to analyse the velocity and acceleration of the vehicle.

- Conclusion #2
  - The side-slip angle of the vehicle is the angle between the vehicle travelling direction and longitudinal direction.

Vehicle Dynamics

Chapter 4

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