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HYDRAULICS

DIMENSIONAL ANALYSIS AND HYDRAULIC SIMILARITY

TOPIC 4.1

by

Nadiatul Adilah Ahmad Abdul Ghani
Faculty of Civil Engineering and Earth Resources
nadiatul@ump.edu.my

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Chapter 4: Dimensional Analysis and Hydraulic Similarity by N Adilah A A Ghani

DIMENSIONAL ANALYSIS AND HYDRAULIC SIMILARITY

4.1

- Dimensional Analysis

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4.1 : DIMENSIONAL ANALYSIS

4.1.1 : Fundamental Dimensions, Systems of Units and Hydraulic Variables

MLT

- Mass, Length, Time

FLT

- Force, Length Time

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4.1.2: System Unit and Hydraulic Variables

No.	Quantity	MLT	FLT
1	Length, L	L	L
2	Area, A	L^2	L^2
3	Volume, V	L^3	L^3
4	Time , t	T	T
5	Velocity, v	LT^{-1}	LT^{-1}
6	Acceleration, a	LT^{-2}	LT^{-2}
7	Gravitational acceleration, g	LT^{-2}	LT^{-2}
8	Frequency, N	T^{-1}	T^{-1}
9	Discharge, Q	L^3T^{-1}	L^3T^{-1}
10	Force, F or Weight, W	MLT^{-2}	F
11	Power, P	ML^2T^{-3}	FLT^{-1}

	Quantity	MLT	FLT
12	Work or Energy, E	ML^2T^{-2}	FL
13	Pressure, p	$ML^{-1}T^{-2}$	FL^{-2}
14	Mass, m	M	FT^2L^{-1}
15	Mass density, ρ	ML^{-3}	FT^2L^{-4}
16	Specific weight, w	$ML^{-2}T^{-2}$	FL^{-3}
17	Dynamic viscosity, μ	$ML^{-1}T^{-1}$	FTL^{-2}
18	Kinematic viscosity, v	L^2T^{-1}	L^2T^{-1}
19	Surface Tension, σ	MT^{-2}	FL^{-1}
20	Shear stress, τ	$ML^{-1}T^{-2}$	FL^{-2}
21	Bulk Modulus, K	$ML^{-1}T^{-2}$	FL^{-2}

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EXAMPLE 4.1

Determine the dimensions of force, pressure, power, specific weight and surface tension in MLT system.

Solution

Force = MLT^{-2}

Pressure = $ML^{-1}T^{-2}$

Power = ML^2T^{-3}

Specific weight = $ML^{-2}T^{-2}$

Surface tension = MT^{-2}

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4.1.4 : Methods Of Dimensional Analysis

a) Rayleigh's Method

- If the number of dependent and independent variables in a physical phenomenon are known, a relationship can be established.
- If the variables are more than four or five this method becomes difficult to solve.

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Step in using Rayleigh's Method

1. Write the functional relationship with the given data
2. Write the equation in terms of a constant with exponents a, b, c…
3. Find out the values of a, b, c… by obtaining simultaneous equation.
4. Substitute the values of these exponents in the main equation and simplify it.

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EXAMPLE 4.2 (Rayleigh Method)

Using the Rayleigh method of dimensional analysis, develop an equation for the power delivered by a pump to lift a fluid of a specific weight γ with a rate of Q to a static level of H.

Solution:

Power is λQH

and from MLT/FLT table; $P = ML^2T^{-3}$

thus, the following equation can be write :

$$P = \varphi\left[(\lambda)^a (Q)^b (H)^c\right]$$

$$ML^2T^{-3} = \varphi\left[\left(\mathrm{ML^{-2}T^{-2}}\right)^a \left(\mathrm{L^3T^{-1}}\right)^b (L)^c\right]$$

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Fundamental equations are then written as:

$$M = (\mathrm{M})^a$$
$$a = 1$$

$$T^{-3} = \left(\mathrm{T}^{-2}\right)^a \left(\mathrm{T}^{-1}\right)^b$$
$$-3 = -2a - 1b$$
$$-3 = -2(1) - 1b$$
thus,
$$b = 3 - 2 = 1$$

$$L^2 = \left(\mathrm{L}^{-2}\right)^a \left(\mathrm{L}^3\right)^b (L)^c$$
$$2 = -2a + 3b + 1c$$
$$2 = -2(1) + 3(1) + c$$
thus,
$$c = 2 + 2 - 3 = 1$$

thus,

$P = \varphi\left[(\lambda)^a (Q)^b (H)^c\right]$ can be write :

$$P = \varphi\left[(\lambda QH)\right]$$

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b) Buckingham's Method

If there are n variables (dependent or independent) and if these variables contain m fundamental dimensions, these variables are arranged in (n-m), π terms

$$x_1 = f(x_2, x_3, x_4, \ldots x_n)$$
$$f_1(x_1, x_2, x_3, \ldots x_n) = 0$$
$$f(\pi_1, \pi_2, \pi_3, \ldots \pi_{n-m}) = 0$$

π term can be expressed as

$$\pi_1 = x_2^{a1} x_3^{b1} x_4^{c1}$$
$$\pi_2 = x_2^{a2} x_3^{b2} x_4^{c2}$$
$$\pi_{n-m} = x_2^{a(n-m)} x_3^{b(n-m)} x_4^{c(n-m)}$$

Repeating variables

- Geometrical variables – length, diameter
- Flow property – velocity, acceleration
- Fluid property – density, viscosity

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Step in using Buckingham's Method

1. List all the variables that are involved in the problem
2. Express each of the variable in term of basic dimension
3. Determine the required number of π term
4. Select a number of repeating variables, where the number required is equal to the number of reference dimensions.
5. Form a π term by multiplying one of the nonrepeating variables by the product of repeating variables each raised to an exponent that will make the contribution dimensionless.
6. Repeat Step 5 for each of the remaining nonrepeating variables.
7. Check all the resulting π terms to make sure they are dimensionless.
8. Express the final form as a relationship among the π terms and think about what it means.

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EXAMPLE 4.3 (Buckingham's Method)

Using the Buckingham-π method, derive an expression for the shear stress, τ, in fluid flowing in a pipe assuming that it is a function of the diameter, D pipe roughness e, fluid density, ρ, dynamic viscosity μ and fluid velocity v.

Solution:

Variables are : , τ, D, e, ρ, μ, v. (6 variables (n) contain M and F, the m=3; thus repeating π =6-3 =3).

Lecturer Information (Authors)

Pn. Nadiatul Adilah bt Ahmad Abdul Ghani
Dr Nor Azlina bt. Alias
Pn. Wafty bt. Abd Rahman
Dr. Jacqueline Isabella