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HYDRAULICS

NON - UNIFORM FLOW IN OPEN CHANNEL

TOPIC 3.5

by

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NON -UNIFORM FLOW IN OPEN CHANNEL

3.5
- Gradually Varied Flow

3.5 : GRADUALLY VARIED FLOW (GVF)

steady flow whose depth varies gradually along the length of the channel

Conditions:

- Flow is steady;
  hydraulic characteristics of flow remain constant for the time interval under consideration

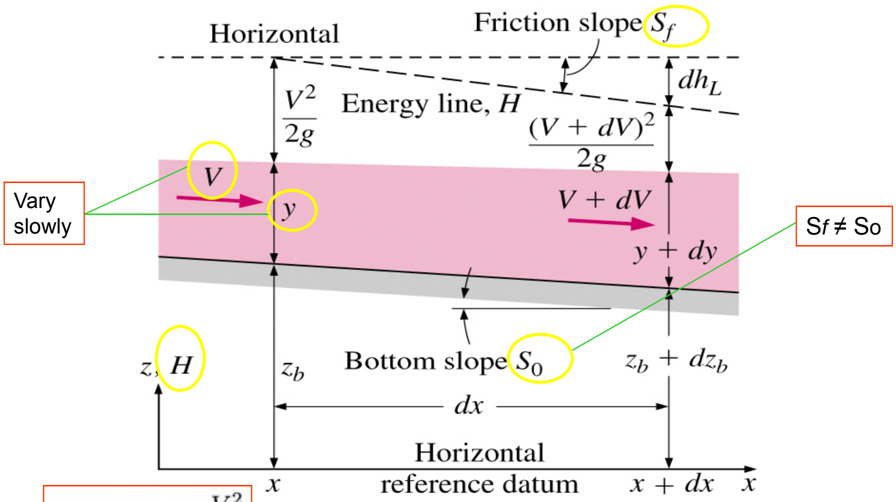
- Streamlines are practically parallel;
  that hydrostatic distribution of pressure prevails over the channel section

Assumption:

- Head loss along the section is same;
  Chezy @ Manning formula may be used

- Slope of the channel is small;
  - depth of flow is the same whether the vertical or normal (to a channel bottom) direction is used
  - the channel is prismatic; has constant alignment and shape
  - roughness coefficient constant along the channel

Governing Equations



$$H = z_b + y + \frac{V^2}{2g}$$

Differentiating H with respect to x gives :

$$\frac{dH}{dx} = \frac{d}{dx}\left(z_b + y + \frac{V^2}{2g}\right) = \frac{dz_b}{dx} + \frac{dy}{dx} + \frac{V}{g}\frac{dV}{dx} \quad \text{Eq: 3.5.1}$$

$$\frac{dH}{dx} = \textit{negative of the friction slope} = -\frac{dh_L}{dx} = -S_f \quad \text{Eq: 3.5.2}$$

$$\frac{dz_b}{dx} = \textit{negative of the bottom slope} = -So \quad \text{Eq: 3.5.3}$$

Substituting Eq 3.5.2 and Eq 3.5.3 into Eq 3.5.1, gives :

$$So - Sf = \frac{dy}{dx} + \frac{V}{g}\frac{dV}{dx} \quad \text{Eq: 3.5.4}$$

The continuity equation for steady flow in a rectangular channel is ybV = constant. Differentiating with respect to *x* gives;

$$0 = bV\frac{dy}{dx} + yb\frac{dV}{dx} \longrightarrow \frac{dV}{dx} = -\frac{V}{y}\frac{dy}{dx}$$ Eq: 3.5.5

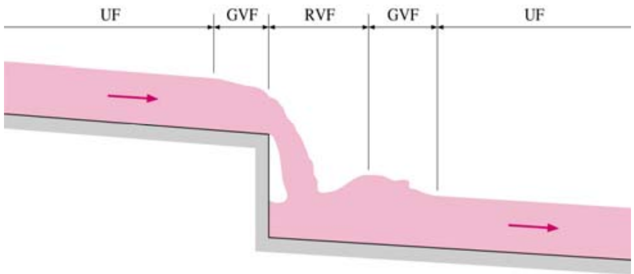
Substituting Eq 5.1.4 into Eq 5.1.3 and noting that V / √(gy) is Froude number,

$$So - Sf = \frac{dy}{dx} - \frac{V^2}{gy}\frac{dy}{dx} = \frac{dy}{dx} - Fr^2\frac{dy}{dx}$$ Eq: 3.5.6

Solving for *dy/dx* gives the desired relation for the rate of change of flow depth (or the surface profile) in gradually varied flow in a open channel,

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$ Eq: 3.5.7

3.5.1 Classification of Open-Channel Flows



- Obstructions cause the flow depth to vary.
- Rapidly varied flow (RVF) occurs over a short distance near the obstacle.
- Gradually varied flow (GVF) occurs over larger distances and usually connects UF and RVF.

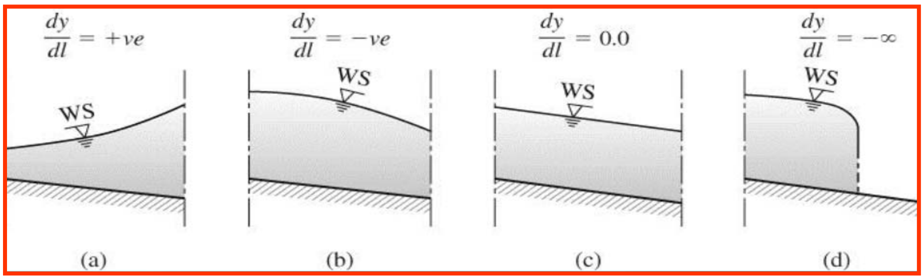
3.5.2 Classification of Flow Profile

The flow profile represent the **surface curve of the flow**:

a) **backwater** curve if the depth of flow increases in the direction of flow **(*dy/dx* = + ve)**

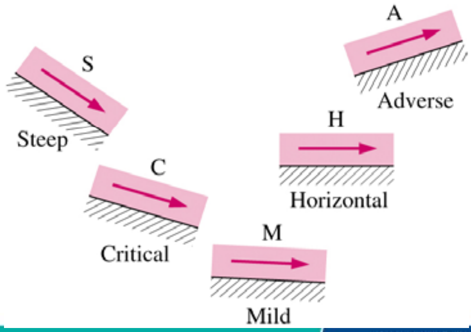
b) **drawdown** curve if the depth of flow decreases in the direction of flow **(*dy/dx* = - ve)**

Classification of profiles according to dy/dl or (dh/dx)

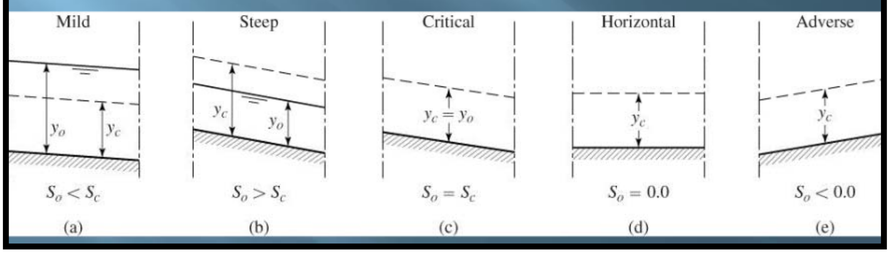


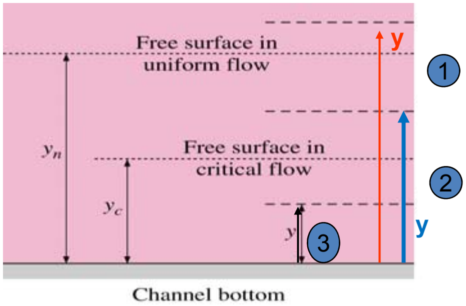
Bed slope S_0 is classified as:

- **Steep** : $y_o < y_c$ or $s_o > s_c$
- **Critical** : $y_o = y_c$ or $s_o = s_c$
- **Mild** : $y_o > y_c$ or $s_o < s_c$
- **Horizontal** : $S_0 = 0$
- **Adverse** : $S_0 < 0$



Number	Channel category	Symbol	Characteristic condition	Remark
1	Mild slope	M	$y_0 > y_c$	Subcritical flow at normal depth
2	Steep slope	S	$y_c > y_0$	Supercritical flow at normal depth
3	Critical slope	C	$y_c = y_0$	Critical flow at normal depth
4	Horizontal bed	H	$S_0 = 0$	Cannot sustain uniform flow
5	Adverse slope	A	$S_0 < 0$	Cannot sustain uniform flow





Initial depth is given a zone:

Zone 1 : $y > y_n$: The space above both critical and normal depth

Zone 2 : $y_c < y < y_n$: The region lies between the normal and critical dept

Zone 3 : $y < y_c$: The lowest zone of space that lies above the channel bed but below both critical and normal depth lines

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \mathbf{F}_r^2}$$

Zone 1 (M1 Profile)

Since $y > y_n$ in Zone 1, $S_f < S_o$. Therefore, the numerator of Eq. 5-7 is positive. Similarly, $\mathbf{F}_r < 1$ since $y > y_c$. Therefore, the denominator of Eq. 5-7 is positive as well. Hence, it follows from Eq. 5-7 that

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \mathbf{F}_r^2} = \frac{+}{+} = +$$

This means that y increases with distance x. As discussed previously, $y \rightarrow y_n$ asymptotically in the upstream direction and the water surface becomes almost horizontal as y becomes large in the downstream direction.

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \mathbf{F}_r^2}$$

Zone 2 (M2 Profile)

In this case, $S_f > S_o$ since, $y < y_n$. Therefore, the numerator of Eq. 5-7 is negative. However, the denominator is positive, since $\mathbf{F}_r < 1$ because $y > y_c$. Hence, it follows from Eq. 5-7 that

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \mathbf{F}_r^2} = \frac{-}{+} = -$$

Thus, y decreases as x increases. As discussed previously, $y \to y_n$ asymptotically; and $y \to y_c$ almost vertically.

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$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \mathbf{F}_r^2}$$

Zone 3 (M3 Profile)

In Zone 3, $S_f > S_o$ since $y < y_n$. Therefore, the numerator of Eq. 5-7 is negative. The denominator is negative as well, since $\mathbf{F}_r > 1$ because $y < y_c$. Hence, it follows from Eq. 5-7 that

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \mathbf{F}_r^2} = \frac{-}{-} = +$$

Thus, y increases as x increases.

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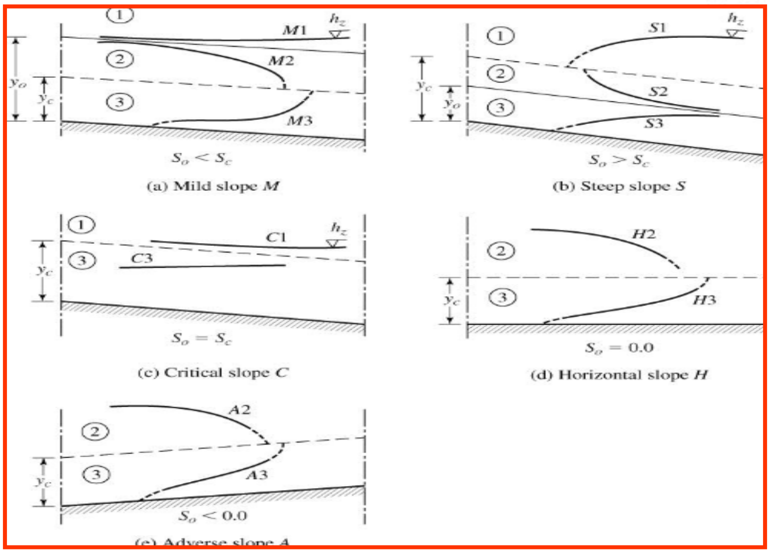

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OUTLINING WATER SURFACE PROFILES

1. Determine type of bed slope.
2. Plot y_c critical depth and y_n normal depth.
3. Select the corresponding profile(s) according to bed slope. Pay attention to zone.
4. Various type of curves should be allowed beyond their own category.
5. Before any gate or dam, the elevation of water must be above y_n & y_c .
6. Before hydraulic jump, M3,H3 orA3 should exist.
7. Subcritical flows are governed by downstream control
8. Supercritical flows are dominated by upstream control

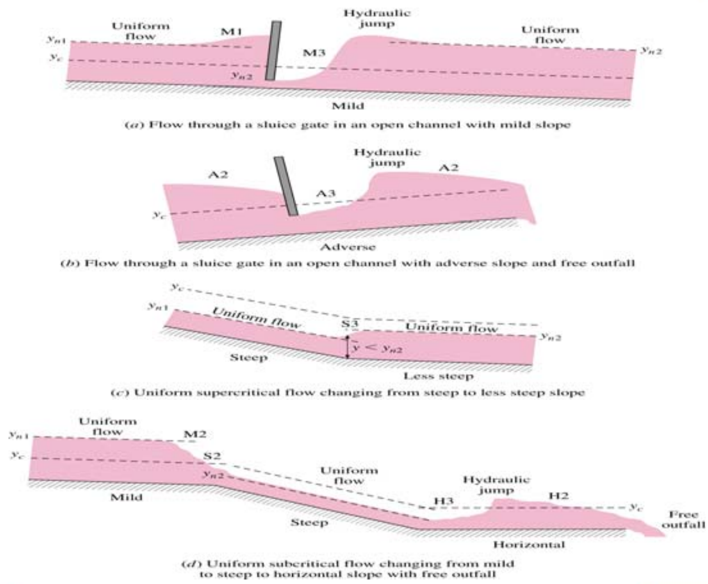

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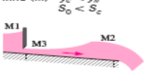
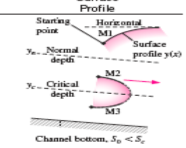
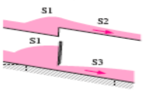
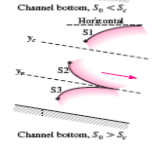
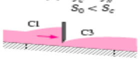
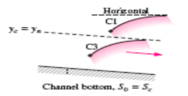
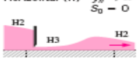
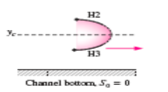
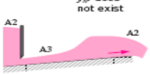
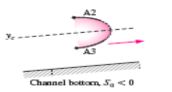
TABLE 13–3

Classification of surface profiles in gradually varied flow

Channel Slope	Profile Notation	Flow Depth	Froude Number	Profile Slope	Surface Profile
Mild (M) $y_c < y_n$, $S_0 < S_c$ 	M1	$y > y_n$	Fr < 1	$\frac{dy}{dx} > 0$	 Starting point; Horizontal; M1; Surface profile y(x); y_n Normal depth; M2; y_c Critical depth; M3; Channel bottom, $S_0 < S_c$
	M2	$y_c < y < y_n$	Fr < 1	$\frac{dy}{dx} < 0$	
	M3	$y < y_c$	Fr > 1	$\frac{dy}{dx} > 0$	
Steep (S) $y_c > y_n$, $S_0 < S_c$ 	S1	$y > y_c$	Fr < 1	$\frac{dy}{dx} > 0$	 Horizontal; S1; y_c; y_n; S2; S3; Channel bottom, $S_0 > S_c$
	S2	$y_n < y < y_c$	Fr > 1	$\frac{dy}{dx} < 0$	
	S3	$y < y_n$	Fr > 1	$\frac{dy}{dx} > 0$	
Critical (C) $y_c = y_n$, $S_0 < S_c$ 	C1	$y > y_c$	Fr < 1	$\frac{dy}{dx} > 0$	 Horizontal; C1; $y_c = y_n$; C3; Channel bottom, $S_0 = S_c$
	C3	$y < y_c$	Fr > 1	$\frac{dy}{dx} > 0$	
Horizontal (H) $y_n \to \infty$, $S_0 = 0$ 	H2	$y > y_c$	Fr < 1	$\frac{dy}{dx} < 0$	 H2; y_c; H3; Channel bottom, $S_0 = 0$
	H3	$y < y_c$	Fr > 1	$\frac{dy}{dx} > 0$	
Adverse (A) $S_0 < 0$, y_n: does not exist 	A2	$y > y_c$	Fr < 1	$\frac{dy}{dx} < 0$	 A2; y_c; A3; Channel bottom, $S_0 < 0$
	A3	$y < y_c$	Fr > 1	$\frac{dy}{dx} > 0$	

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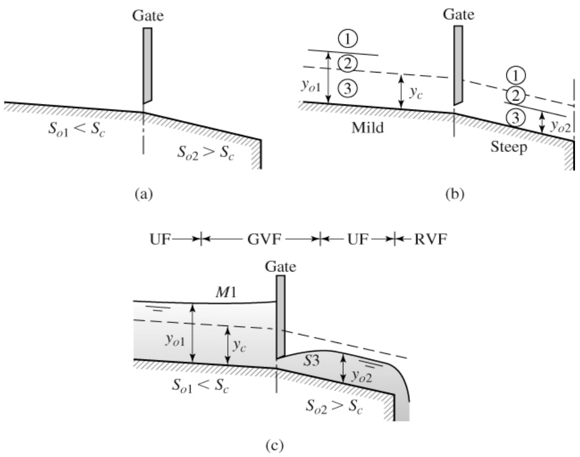
Example 3.11

Draw water surface profile for two reaches of the open channel given in Figure below. A gate is located between the two reaches and the second reach ends with a sudden fall.



Solution:

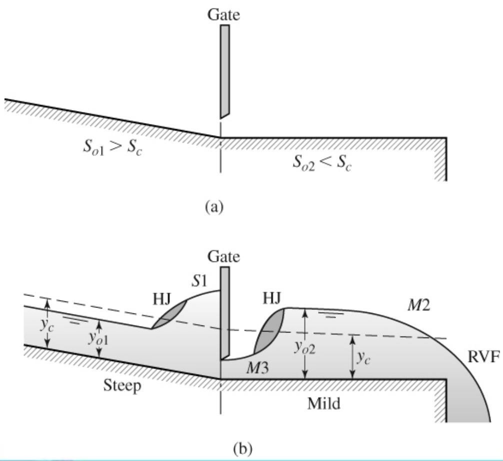
(a) The open channel and gate location.

(b) Critical and normal depths.

(c) Water surface profile.

Example 3.12

Draw water surface profile for two reaches of the open channel given in Figure below. A gate is located between the two reaches and the second reach ends with a sudden fall.



Solution:

(a) The open channel and gate location.

(b) Water surface profile.

3.5.3 Numerical Analysis of Water Surface Profile

There are several method to obtain surface water profile:

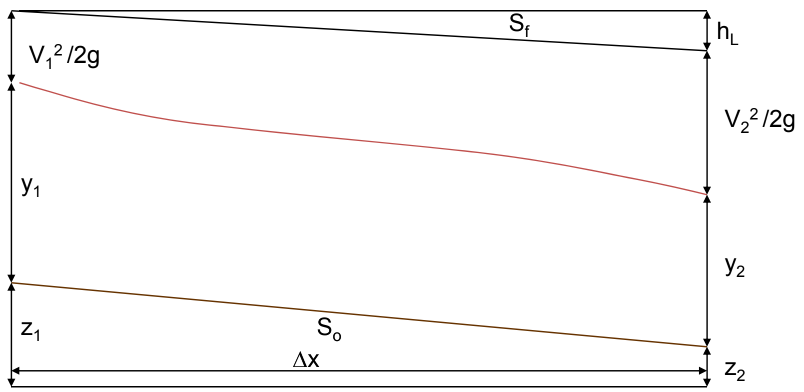
Prismatic channel
- a) Numerical Integration
- b) Direct Step Method

Non Prismatic channel
- a) Standard Step Method

Those method can identified :

- a) Depth (y) at some distances/lengths (L/*x*)
- b) Distances/lengths from one point to one point when both depth are known

a) Direct Step Method (Prismatic Channel)



Equating the total head at the two end section 1 and 2, the following may be written;

$$y_1 + v_1^2/2g + z_1 = y_2 + v_2^2/2g + z_2 + h_L$$
$$E_1 + (z_1 - z_2) = E_2 + h_L$$
$$E_1 + S_o \Delta x = E_2 + S_f \Delta x$$
$$\Delta x \ (S_o - S_f) = E_2 - E_1$$
$$\Delta x = \frac{E_2 - E_1}{(S_o - S_f)}$$

Where:

E = specific energy at one point $= y + v^2/2g$

s_f = slope energy grade line $= \frac{n^2v^2}{R^{4/3}} = \frac{v^2}{C^2R}$

	A/P			1 + 4 y+ ($v^2/2g$)	E_2 -E_1	(n^2v^2)/$R^{4/3}$			6/9	
1	2	3	4	5	6	7	8	9	10	11
y	R	v	$v^2/2g$	E	ΔE	S_f	S_{fbar}	$S_o - S_{fbar}$	Δx	L
					-		-	-	-	-

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Column	Formula	
1	Y	water depth (m)
2	R	A/P = hydraulic radius or y for very wide rectangular
3	V	v = flow velocity
4	$v^2/2g$	kinetic energy
5	$y + v^2/2g$	E = specific energy
6	$E_2 - E_1$	ΔE = energy loss
7	S_f	slope energy grade line $= \frac{n^2v^2}{R^{4/3}} = \frac{v^2}{C^2R}$
8	$(S_{f1} + S_{f2})/2$	EGL slope average
9	$(S_o - S_f)$	slope different
10	Δx	reach = $\Delta E / (s_o - s_f)$
11	L	length of surface water profile which is calculate from dam

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Example 3.13 (Using Direct Step Method)

The very wide rectangular channel carry the water at 2.5 $m^3/s/m$ with channel bed slope, 0.001 and n= 0.025.

Find the length of back water which is happened from one dam and obtained the 2 m water depth at the dam's back.

The calculation must from the dam to upstream until the water surface is 1% higher than normal depth.

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Manning for the very wide rectangular channel :

$$q = \frac{y_o^{5/3} S_o^{1/2}}{n}$$

$$2.5 = \frac{y_o^{5/3} (0.001)^{1/2}}{(0.025)}$$

$$y_o^{5/3} = \frac{2.5 (0.025)}{(0.001)^{1/2}}$$

$$y_o^{5/3} = 1.98$$

$$y_o = \underline{1.50 \text{ m}}$$

Critical depth :

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_c = \left(\frac{(2.5)^2}{(9.81)}\right)^{1/3}$$

$$y_c = 0.86 \text{ m}$$

1% from y_o = 0.01 x 1.50
= 0.015 m
≈ 0.02 m

L from y = 2.0 m till y = (1.50+0.02) = 1.52m

Solution for Example 3.13 (Using Direct Step Method)

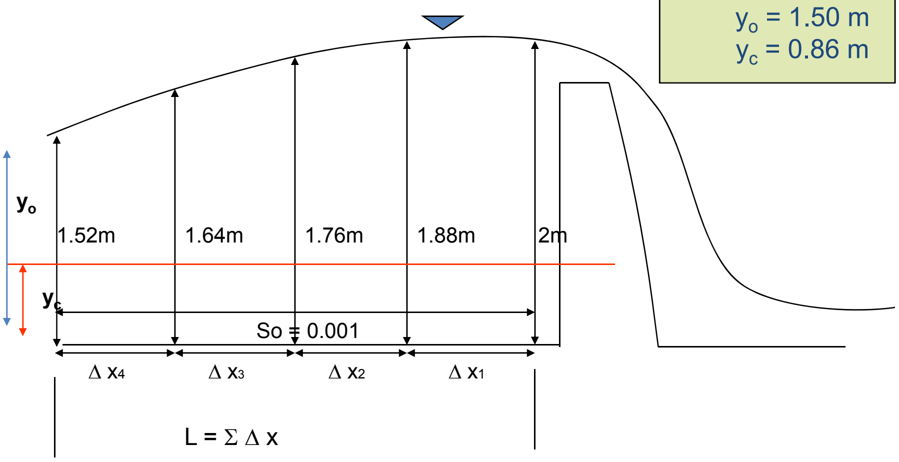
So = 0.001
q = 2.5
n = 0.025

	A/P			1 + 4 (y+v²/2g)	$E_2 - E_1$	$n^2v^2/R^{4/3}$			6/9	
1	2	3	4	5	6	7	8	9	10	11
y	R	v	$v^2/2g$	E	ΔE	S_f	S_{fbar}	$S_o - S_{fbar}$	Δx	L
2.00	2.00	1.25	0.08	2.08	-	0.00039	-	-	-	-
1.88	1.88	1.33	0.09	1.97	-0.11	0.00048	0.00043	0.00057	-192.776	-192.77
1.76	1.76	1.42	0.10	1.86	-0.11	0.00059	0.00053	0.00047	-230.676	-423.45
1.64	1.64	1.52	0.12	1.76	-0.10	0.00075	0.00067	0.00033	-318.489	-741.94
1.52	1.52	1.64	0.14	1.66	-0.10	0.00097	0.00086	0.00014	-714.146	-1456.09

L = 1456.09 m

Solution: (SurfaceWater pofile)

$y_o > y_c$ = Mild
y_o = 1.50 m
y_c = 0.86 m



b) Standard Step Method (Non - Prismatic Channel)

- Applicable to non-prismatic channels and therefore to natural river

Objectives

- To calculate the surface elevations at the station with predetermined the station positions
- A trial and error method is employed

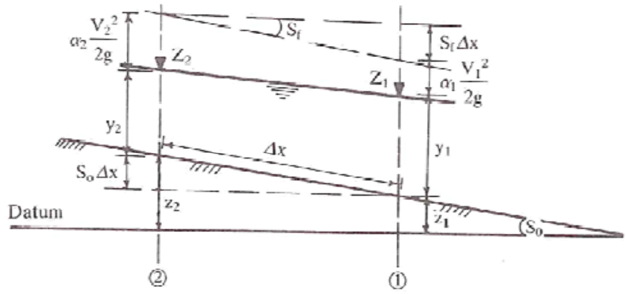
$$\frac{\Delta E}{dx} = S_o - S_f$$

This can be rewritten in finite difference form

$$\Delta E_s = \Delta X(S_o - \bar{S}_f)_{mean}$$

where 'mean' refers to the average values for the interval ΔX.

This form of the equation may be used to determine the depth given distance intervals. The solution method is an iterative procedure as follows;



$$y_1 + \alpha \frac{V_1^2}{2g} + h_f = S_o \Delta X + y_2 + \alpha \frac{V_2^2}{2g}$$

$$Z_1 = y_1 \qquad Z_2 = y_2 + S_o \Delta X$$

$$Z_1 + \alpha \frac{V_1^2}{2g} + h_f = Z_2 + \alpha \frac{V_2^2}{2g}$$

$$H_1 = Z_1 + \alpha \frac{V_1^2}{2g}; \quad H_2 = Z_2 + \alpha \frac{V_2^2}{2g}$$

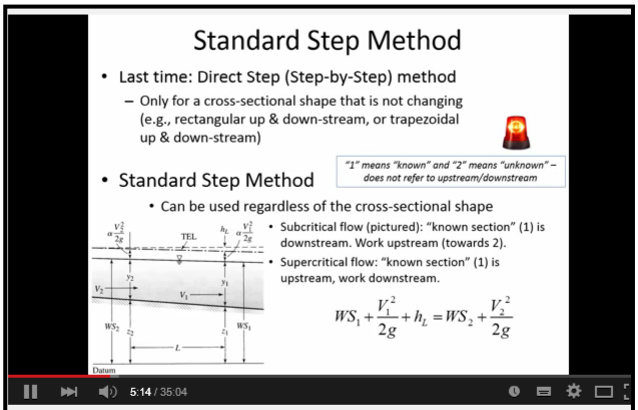
$$H_1 = h_f + H_2$$

H1 is known and ΔX predetermined.

1) Assume a value for depth (Z_2); simple add a small amount to Z_1
2) Calculate y_2 from $y_2 = Z_2 - So\Delta X$
3) Calculate the corresponding specific energy (E_2)
4) Calculate the corresponding friction slope S2
5) Calculate H_2
6) Calculate $H_1 = H_2 + S_f \Delta X$
7) Compare H2 and H_1 if the differences is not within the prescribed limit (e.g., 0.001m) re-estimate Z2 and repeat the procedure until the agreement is reached.

For detail explanation about standard step method :

https://www.youtube.com/watch?v=P4MhwS03KI0



CE 331 - Class 28 (4/24/2014) Water Surface Profile: Standard Step Method

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