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Finite Element Analysis

Heat Transfer Example

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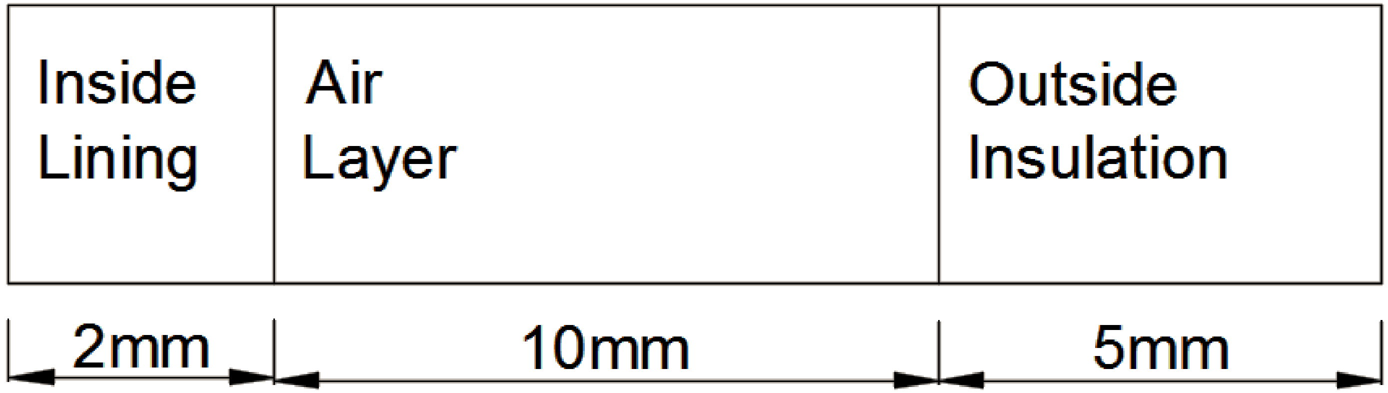
Lesson Outcomes

- At the end of this lesson, the student should be able to:
  - Understand the use of finite element heat transfer analysis
  - Evaluate the loss or gain of heat through a linear system

Heat Transfer Example

- Part of a cross-section of the wall of a water dispenser is shown on the next slide. The water in the dispenser has a temperature of 3oC and the outside air temperature is 35oC. The wall consists three layers; an inside lining, a layer of air and an outside insulation. The thermal conductivity of air can be assumed to be 0.5 W/moC. The inside lining has a thermal conductivity of 0.2 W/moC and the outside insulation has a thermal conductivity of 1.5 W/moC. The heat transfer coefficients for the inner, middle and outer layers are 5 W/m2-oC, 20 W/m2-oC, and 10 W/m2-oC, respectively. Evaluate the heat transferred through the wall of the water dispenser.

Heat Transfer Example (Continued)



Solution

Assuming it a one dimensional finite elment model with 3 elements each resp-resenting a wall layer of unit area, we have:

Element 1

$$k_{xx} = 0.2\ W/m^oC$$

$$h = 5\ W/m^2 -^o C$$

$$[k]^{(1)} = \frac{Ak_{xx}}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + hA\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[k]^{(1)} = \frac{0.2}{0.002}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[k]^{(1)} = \begin{bmatrix} 105 & -100 \\ -100 & 100 \end{bmatrix}$$

Solution (Continued)

Element 2

$$k_{xx} = 0.5\ W/m^oC$$

$$h = 20\ W/m^2 -^o C$$

$$[k]^{(2)} = \frac{Ak_{xx}}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k]^{(2)} = \frac{0.5}{0.01}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k]^{(2)} = \begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix}$$

Element 3

$$k_{xx} = 1.5\ W/m^oC$$

Solution (Continued)

$$h = 10\ W/m^2 - ^o C$$

$$[k]^{(3)} = \frac{Ak_{xx}}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + hA\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[k]^{(3)} = \frac{1.5}{0.005}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[k]^{(3)} = \begin{bmatrix} 300 & -300 \\ -300 & 310 \end{bmatrix}$$

The force terms are:

Left End (node 1)

$$h_L = 5\ W/m^2 - ^o C$$

$$T_{\infty,L} = 3^o C$$

Solution (Continued)

$$\{f_h\}_L = h_L T_{\infty,L} A = 5 \times 3 \times 1 = 15$$

Right End (node 4)

$$h_R = 10\ W/m^2 -^o C$$

$$T_{\infty,R} = 35^o C$$

$$\{f_h\}_R = h_R T_{\infty,R} A = 10 \times 35 \times 1 = 350$$

By direct assembly, we get:

$$\begin{bmatrix} 105 & -100 & 0 & 0 \\ -100 & 150 & -50 & 0 \\ 0 & -50 & 350 & -300 \\ 0 & 0 & -300 & 310 \end{bmatrix} \begin{Bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 0 \\ 0 \\ 350 \end{Bmatrix}$$

The augmented matrix is:

Solution (Continued)

$$\left|\begin{array}{cccccc} 105 & -100 & 0 & 0 & \vdots & 15 \\ -100 & 150 & -50 & 0 & \vdots & 0 \\ 0 & -50 & 350 & -300 & \vdots & 0 \\ 0 & 0 & -300 & 310 & \vdots & 350 \end{array}\right|$$

Multiply row 1 by 100/105 and add it to row 2

$$\left|\begin{array}{cccccc} 105 & -100 & 0 & 0 & \vdots & 15 \\ 0 & 54.7619 & -50 & 0 & \vdots & 14.2857 \\ 0 & -50 & 350 & -300 & \vdots & 0 \\ 0 & 0 & -300 & 310 & \vdots & 350 \end{array}\right|$$

Multiply row 2 by 50/54.7619 and add it to row 3

Solution (Continued)

$$\left[\begin{array}{cccccc} 105 & -100 & 0 & 0 & \vdots & 15 \\ 0 & 54.7619 & -50 & 0 & \vdots & 14.2857 \\ 0 & 0 & 304.3478 & -300 & \vdots & 13.0428 \\ 0 & 0 & -300 & 310 & \vdots & 350 \end{array}\right]$$

Multiply row 3 by 300/304.3478 and add it to row 4

$$\left[\begin{array}{cccccc} 105 & -100 & 0 & 0 & \vdots & 15 \\ 0 & 54.7619 & -50 & 0 & \vdots & 14.2857 \\ 0 & 0 & 304.3478 & -300 & \vdots & 13.0428 \\ 0 & 0 & 0 & 14.2857 & \vdots & 362.8565 \end{array}\right]$$

Solution (Continued)

Therefore:

$$t_4 = \frac{362.8565}{14.2857} = 25.4^oC$$

$$t_3 = \frac{13.0428 + 300 \times 25.4}{304.3478} = 25.08^oC$$

$$t_2 = \frac{14.2857 + 50 \times 25.08}{54.7619} = 23.16^oC$$

$$t_1 = \frac{15 + 100 \times 23.16}{105} = 22.2^oC$$

The heat flow is given as:

$$q = -\frac{k_{xx}A}{L}(t_2 - t_1) = -\frac{0.2}{0.002}(23.16 - 22.2) = -104.167\ W$$

104.167 W of heat is lost through the wall of the water dispenser.

Author Information

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