Finite Element Analysis

3D Stress Analysis

by
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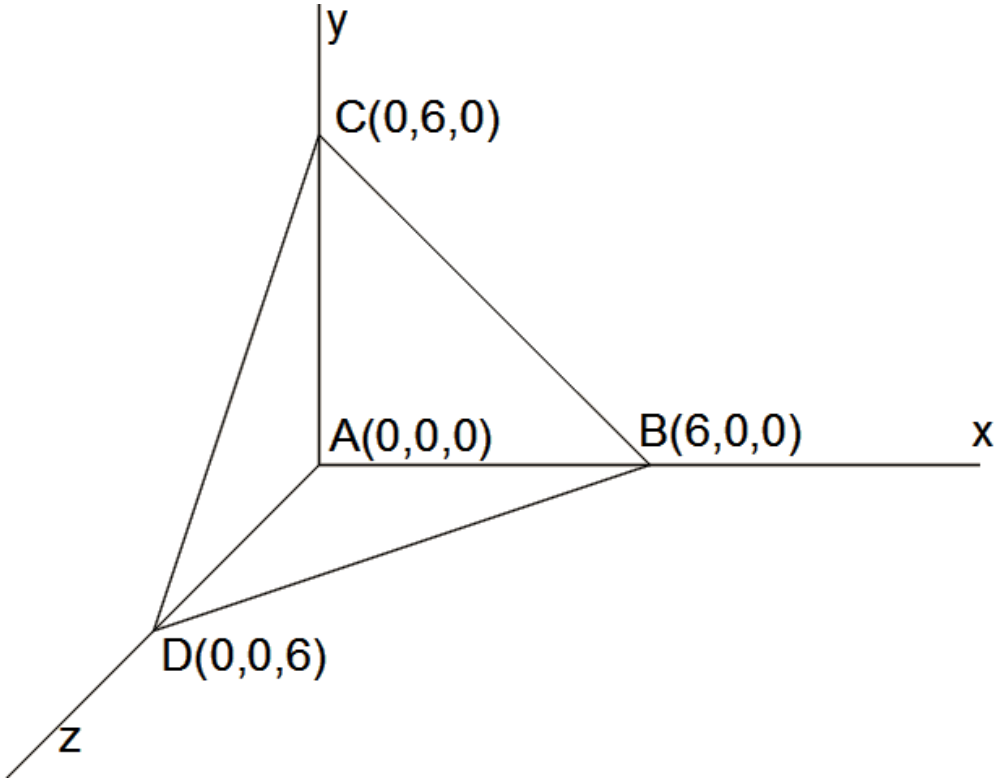
Lesson Outcomes

- At the end of this lesson, the student should be able to:
  - Understand the use of stiffness equations for 3D stress analysis
  - Develop the gradient matrix for 3D stress analysis

3D Stress Analysis Example

- Construct the gradient matrix for the tetrahedral solid element shown. The coordinates are in the units of centimetres and the modulus of elasticity of the material is 110GPa.



Solution

Chosing node numbering such that nodes 1, 2, and 3 are counterclockwise when viewed from node 4, points A, B, C, and D are denoted as node 1, 2, 3, and 4, respectively. Therefore;

$$x_1 = 0, \quad y_1 = 0, \quad z_1 = 0$$

$$x_2 = 6, \quad y_2 = 0, \quad z_2 = 0$$

$$x_3 = 0, \quad y_3 = 6, \quad z_3 = 0$$

$$x_4 = 0, \quad y_4 = 0, \quad z_4 = 6$$

$$6V = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 6 & 0 & 0 \\ 1 & 0 & 6 & 0 \\ 1 & 0 & 0 & 6 \end{vmatrix}$$

Solution (Continued)

$$6V = 1\begin{vmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{vmatrix} + 0 + 0 + 0$$

$$6V = 6\begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix} + 0 + 0$$

$$6V = 216\ cm^3$$

$$\beta_1 = -\begin{vmatrix} 1 & 0 & 0 \\ 1 & 6 & 0 \\ 1 & 0 & 6 \end{vmatrix} = -36\ cm^3$$

Solution (Continued)

$$\gamma_1 = \begin{vmatrix} 1 & 6 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 6 \end{vmatrix} = 36\ cm^3$$

$$\delta_2 = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 6 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\delta_1 = -\begin{vmatrix} 1 & 6 & 0 \\ 1 & 0 & 6 \\ 1 & 0 & 0 \end{vmatrix} = -36\ cm^3$$

$$\beta_3 = -\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 6 \end{vmatrix} = 0$$

$$\beta_2 = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 6 & 0 \\ 1 & 0 & 6 \end{vmatrix} = 36\ cm^3$$

$$\gamma_3 = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 6 & 0 \\ 1 & 0 & 6 \end{vmatrix} = 36\ cm^3$$

$$\gamma_2 = -\begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 6 \end{vmatrix} = 0$$

$$\delta_3 = -\begin{vmatrix} 1 & 0 & 0 \\ 1 & 6 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

Solution (Continued)

$$\beta_4 = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 6 & 0 \end{vmatrix} = 0$$

$$\gamma_4 = -\begin{vmatrix} 1 & 0 & 0 \\ 1 & 6 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\delta_4 = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 6 & 0 \\ 1 & 0 & 6 \end{vmatrix} = 36\ cm^3$$

The gradient matrix is given as:

Solution (Continued)

$$[B] = 0.167 \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Author Information

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