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Finite Element Analysis

Grid Equations

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Lesson Outcomes

- At the end of this lesson, the student should be able to:
  - Understand the development of grid equations
  - Include torsional effects into equations for beam-columns

Grid Structure

- A grid is a structure on which loads are applied perpendicular to the plane of the structure, as opposed to the plane frame, where loads are applied in the plane of the structure
- Typical grid structures are bridge decks, roofing frames, etc.

Torsional Effects

- Recall the basic beam element stiffness matrix
- $$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$
- This matrix accounts for the shear and bending effects but does not account for torsion effects
- A typical grid element is subjected to torsion in addition to shear and bending

Torsional Effects (Continued)

- Assuming a linear relationship for the angle of twist:

$$\hat{\phi} = a_1 + a_2\hat{x}$$

- Expressing the coefficients in terms of unknown nodal angles of twist:

$$\hat{\phi} = \left(\frac{\hat{\phi}_{2x} - \hat{\phi}_{1x}}{L}\right)\hat{x} + \hat{\phi}_{1x}$$

- In matrix form:
- .

$$\hat{\phi} = [N_1 \quad N_2]\begin{Bmatrix} \hat{\phi}_{1x} \\ \hat{\phi}_{2x} \end{Bmatrix}$$

Torsional Effects (Continued)

- Shear stress/shear strain relationship is given as
- $\tau = G\gamma$
- Shear stress is related to applied torque by:
- $m = \frac{\tau J}{R}$
- Where, *J* is the *polar moment of inertia*, a.k.a. *torsional constant*
- After making substitutions, for both ends:
- $m_{1x} = \frac{GJ}{L}(\phi_{1x} - \phi_{2x})$
- $m_{2x} = \frac{GJ}{L}(\phi_{2x} - \phi_{1x})$

Torsional Effects (Continued)

- In matrix form:
- $$\begin{Bmatrix} m_{1x} \\ m_{2x} \end{Bmatrix} = \frac{GJ}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \phi_{1x} \\ \phi_{2x} \end{Bmatrix}$$
- So the stiffness matrix for a torsion bar is:
- $$[k] = \frac{GJ}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$
- It is apparent that this is similar to an axially loaded bar except *AE* are replaced by *GJ*
- We can combine these torsional effects with the stiffness matrix of the beam element accounting for shear and flexure to get the stiffness matrix for a grid element
- The stiffness matrix for the grid element will then account for shear, flexure, and torsion

Stiffness Matrix for a Grid Element: System of Equations accounting for Torsional Effects

- $$\begin{Bmatrix} f_{1y} \\ m_{1x} \\ m_{1z} \\ f_{2y} \\ m_{2x} \\ m_{2z} \end{Bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} \\ 0 & \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 \\ \frac{6EI}{L^2} & 0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} \\ 0 & -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 \\ \frac{6EI}{L^2} & 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} \end{bmatrix}$$

Arbitrarily Orientated Grid Element

- The stiffness matrix for an arbitrarily orientated grid element can be obtained by transforming the stiffness matrix obtained above from local coordinates system to the global
- The transformation matrix is:
- $$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & C & S & 0 & 0 & 0 \\ 0 & -S & C & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & C & S \\ 0 & 0 & 0 & 0 & -S & C \end{bmatrix}$$
- Where, C and S retain their conventional meaning
- The global matrix is given as:
- $[k] = [T]^T[k'][T]$

Author Information

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