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Finite Element Analysis

Frame Equations
Example

by

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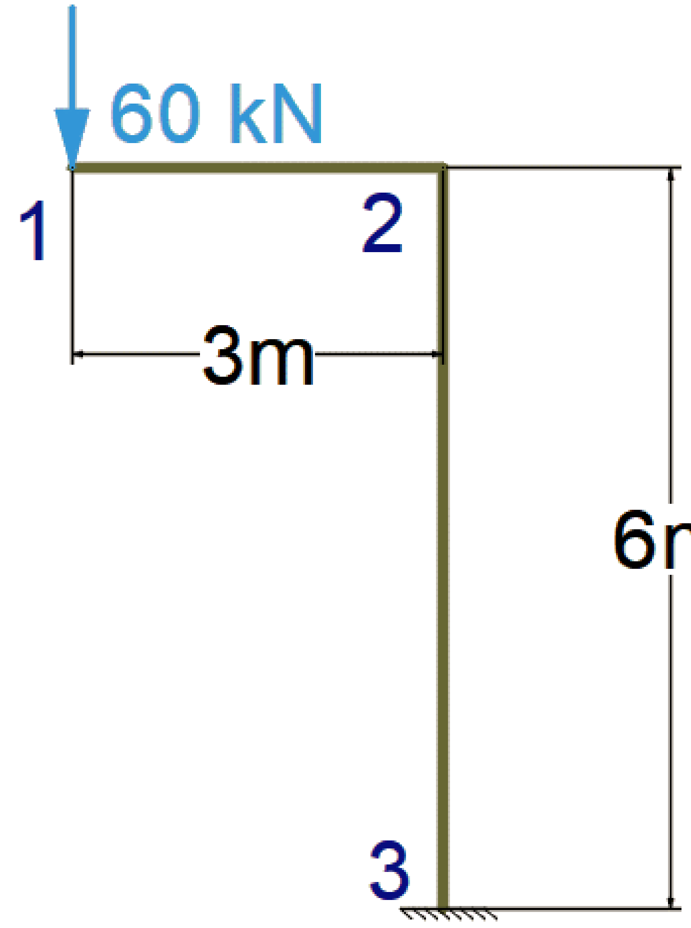
Lesson Outcomes

- At the end of this lesson, the student should be able to:
  - Apply the element equations for beam-columns to a frame example
  - Evaluate the unknown deformations of a frame structure using Finite Element Analysis

Example Frame Structure

- Find the unknown deformations at nodes 1 and 2 for the frame
- Use:
- $E = 200GPa$
- $A = 0.0112m^2$
- $I = 2.39 \times 10^{-5}m^2$



Discretization

- The structure is discretized into 3 nodes and 2 elements
- Element 1 is connected to nodes 1 and 2 and element 2 is connected to nodes 2 and 3
- Node 1 has a 60kN load acting downwards
- Node 3 is fixed
- Length of element 1 is 3m and that of element 2 is 6m

Stiffness Matrix for Element 1

- $\theta = 0, C = 1, S = 0$
- $\frac{E}{L} = \frac{2\times10^{11}}{3} = 6.67 \times 10^{10}$
- $\frac{12I}{L^2} = \frac{12\times2.39\times10^{-5}}{3^2} = 3.19 \times 10^{-5}$
- $\frac{6I}{L} = 4.78 \times 10^{-5}$
- $4I = 9.56 \times 10^{-5}$
- $2I = 4.78 \times 10^{-5}$

Stiffness Matrix for Element 1 (Continued)

- $[k] = 6.67 \times 10^{10} \begin{bmatrix} 0.0112 & 0 & 0 & -0.0112 & 0 & 0 \\ 0 & 3.19 \times 10^{-5} & 4.78 \times 10^{-5} & 0 & -3.19 \times 10^{-5} & 4.78 \times 10^{-5} \\ 0 & 4.78 \times 10^{-5} & 9.56 \times 10^{-5} & 0 & -4.78 \times 10^{-5} & 4.78 \times 10^{-5} \\ -0.0112 & 0 & 0 & 0.0112 & 0 & 0 \\ 0 & -3.19 \times 10^{-5} & 4.78 \times 10^{-5} & 0 & 3.19 \times 10^{-5} & 4.78 \times 10^{-5} \\ 0 & -4.78 \times 10^{-5} & 4.78 \times 10^{-5} & 0 & 4.78 \times 10^{-5} & 9.56 \times 10^{-5} \end{bmatrix}$

Stiffness Matrix for Element 2

- $\theta = 270, C = 0, S = -1$
- $\frac{E}{L} = \frac{2\times10^{11}}{6} = 3.33 \times 10^{10}$
- $\frac{12I}{L^2} = \frac{12\times2.39\times10^{-5}}{6^2} = 7.97 \times 10^{-6}$
- $\frac{6I}{L} = 2.39 \times 10^{-5}$
- $4I = 9.56 \times 10^{-5}$
- $2I = 4.78 \times 10^{-5}$

Stiffness Matrix for Element 2 (Continued)

- $[k] = 3.33 \times 10^{10} \begin{bmatrix} 7.97\times10^{-6} & 0 & 2.39\times10^{-5} & -7.97\times10^{-6} & 0 & 2.39\times10^{-5} \\ 0 & 0.0112 & 0 & 0 & -0.0112 & 0 \\ 2.39\times10^{-5} & 0 & 9.56\times10^{-5} & -2.39\times10^{-5} & 0 & 4.78\times10^{-5} \\ -7.97\times10^{-6} & 0 & 2.39\times10^{-5} & 7.97\times10^{-6} & 0 & 2.39\times10^{-5} \\ 0 & -0.0112 & 0 & 0 & 0.0112 & 0 \\ -2.39\times10^{-5} & 0 & 4.78\times10^{-5} & 2.39\times10^{-5} & 0 & 9.56\times10^{-5} \end{bmatrix}$
- Since node 3 is fixed, we only need the first part of this matrix for assembly

- $[k] = 3.33 \times 10^{10} \begin{bmatrix} 7.97\times10^{-6} & 0 & 2.39\times10^{-5} \\ 0 & 0.0112 & 0 \\ 2.39\times10^{-5} & 0 & 9.56\times10^{-5} \end{bmatrix}$

Assembly of Stiffness Matrix (Including Boundary Conditions)

- $[k] = 6.67 \times 10^{10} \begin{bmatrix} 0.0112 & 0 & 0 & -0.0112 & 0 & 0 \\ 0 & 3.19 \times 10^{-5} & 4.78 \times 10^{-5} & 0 & -3.19 \times 10^{-5} & 4.78 \times 10^{-5} \\ 0 & 4.78 \times 10^{-5} & 9.56 \times 10^{-5} & 0 & -4.78 \times 10^{-5} & 4.78 \times 10^{-5} \\ -0.0112 & 0 & 0 & 0.0112 & 0 & 1.2 \times 10^{-5} \\ 0 & -3.19 \times 10^{-5} & 4.78 \times 10^{-5} & 0 & 0.0112 & 4.78 \times 10^{-5} \\ 0 & -4.78 \times 10^{-5} & 4.78 \times 10^{-5} & 1.2 \times 10^{-5} & 4.78 \times 10^{-5} & 1.43 \times 10^{-4} \end{bmatrix}$

System of Equations

- The system of Equations is given as:

- $$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ m_1 \\ f_{2x} \\ f_{2y} \\ m_2 \end{Bmatrix} = [K] \begin{Bmatrix} u_1 \\ v_1 \\ m_1 \\ u_2 \\ v_2 \\ m_2 \end{Bmatrix}$$

- $$\begin{Bmatrix} 0 \\ -60000 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = 6.67 \times 10^{10} \begin{bmatrix} 0.0112 & 0 & 0 & -0.0112 & 0 & 0 \\ 0 & 3.19 \times 10^{-5} & 4.78 \times 10^{-5} & 0 & -3.19 \times 10^{-5} & 4.78 \times 10^{-5} \\ 0 & 4.78 \times 10^{-5} & 9.56 \times 10^{-5} & 0 & -4.78 \times 10^{-5} & 4.78 \times 10^{-5} \\ -0.0112 & 0 & 0 & 0.0112 & 0 & 1.2 \times 10^{-5} \\ 0 & -3.19 \times 10^{-5} & 4.78 \times 10^{-5} & 0 & 0.0112 & 4.78 \times 10^{-5} \\ 0 & -4.78 \times 10^{-5} & 4.78 \times 10^{-5} & 1.2 \times 10^{-5} & 4.78 \times 10^{-5} & 1.43 \times 10^{-4} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \phi_1 \\ u_2 \\ v_2 \\ \phi_2 \end{Bmatrix}$$

Solution

- After Solution of the system of equations, we get:

- $$\begin{Bmatrix} u_1 \\ v_1 \\ \phi_1 \\ u_2 \\ v_2 \\ \phi_2 \end{Bmatrix} = \begin{Bmatrix} 0.081 \\ -0.032 \\ 0.03 \\ 0.081 \\ -0.0002 \\ -0.027 \end{Bmatrix}$$

- The translations deformations are in meters and the rotational deformations are in radians

Author Information

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