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Finite Element Analysis

Bar Element in 3D Space

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Lesson Outcomes

- At the end of this lesson, the student should be able to:
  - Transform bar element equations from local coordinates system to global 3D space coordinates systems
  - Analyze 3D space truss

Transformation in 3D Space

- In 3D spacy, we have 3 orthogonal axes
- Consider a vector in local coordinates system to be transformed into global space
- $u'\mathbf{i}' + v'\mathbf{j}' + w'\mathbf{k}' = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$
- Taking dot product with $\mathbf{i}'$
- $u' + 0 + 0 = u(\mathbf{i}'.\mathbf{i}) + v(\mathbf{i}'.\mathbf{j}) + w(\mathbf{i}'.\mathbf{k})$
- We know that
- $\mathbf{i}'.\mathbf{i} = \frac{x_2 - x_1}{L} = C_x$ where $C_x = \cos\theta_x$
- $\mathbf{i}'.\mathbf{j} = \frac{y_2 - y_1}{L} = C_y$ where $C_y = \cos\theta_y$
- $\mathbf{i}'.\mathbf{i} = \frac{z_2 - z_1}{L} = C_z$ where $C_z = \cos\theta_z$
- $L = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2 + (z_2 - z_2)^2}$

Transformation in 3D Space (Continued)

- Therefore:
- $u' = C_x u + C_y v + C_z w$
- For both ends:
- $u_1' = C_x u_1 + C_y v_1 + C_z w_1$
- $u_2' = C_x u_2 + C_y v_2 + C_z w_2$
- Matrix form
- $\begin{Bmatrix} u_1' \\ u_2' \end{Bmatrix} = \begin{bmatrix} C_x & C_y & C_z & 0 & 0 & 0 \\ 0 & 0 & 0 & C_x & C_y & C_z \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \end{Bmatrix}$

Transformation in 3D Space (Continued)

- The transformation matrix in 3D space is therefore obtained as:
- $[T] = \begin{bmatrix} C_x & C_y & C_z & 0 & 0 & 0 \\ 0 & 0 & 0 & C_x & C_y & C_z \end{bmatrix}$
- We can use this matrix to transform local deformations into global
- $\{d'\} = [T]\{d\}$
- $\{f'\} = [T]\{f\}$
- Now the basic FEA equation in local coordinates systems is given as:
- $\{f'\} = [k']\{d'\}$
- Using the transformations obtained above:
- $[T]\{f\} = [k'][T]\{d\}$
- $\{f\} = [T]^T[k'][T]\{d\}$
- $\{f\} = [k]\{d\}$
- $[k] = [T]^T[k'][T]$

Global Stiffness Matrix in 3D Space

- After performing the necessary matrix operations:
- $$[k] = \frac{AE}{L}\begin{bmatrix} C_x^2 & C_xC_y & C_xC_z & -C_x^2 & -C_xC_y & -C_xC_z \\ C_xC_y & C_y^2 & C_yC_z & -C_xC_y & -C_y^2 & -C_yC_z \\ C_xC_z & C_yC_z & C_z^2 & -C_xC_z & -C_yC_z & -C_y^2 \\ -C_x^2 & -C_xC_y & -C_xC_z & C_x^2 & C_xC_y & C_xC_z \\ -C_xC_y & -C_y^2 & -C_yC_z & C_xC_y & C_y^2 & C_yC_z \\ -C_xC_z & -C_yC_z & -C_y^2 & C_xC_z & C_yC_z & C_y^2 \end{bmatrix}$$
- This is the general form of the stiffness matrix for a bar element orientated arbitrarily in 3D space
- Generally, this matrix is divided into 4 portions for easy assembly

Global Stiffness Matrix in 3D Space (Continued)

- $[k] = \begin{bmatrix} [\lambda] & -[\lambda] \\ -[\lambda] & [\lambda] \end{bmatrix}$
- Where
- $[\lambda] = \begin{bmatrix} C_x^2 & C_x C_y & C_x C_z \\ C_x C_y & C_y^2 & C_y C_z \\ C_x C_z & C_y C_z & C_z^2 \end{bmatrix}$
- It can be observed that even this quarter of the stiffness matrix is symmetric in itself
- Therefore, one only needs to calculate 5 values that are then repeated

Author Information

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