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Finite Element Analysis

Formulation of a Bar Element

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Lesson Outcomes

- At the end of this lecture, the student should be able to:
  – Describe a Bar Element
  – Understand the basic assumptions made for a bar element formulation
  – Formulate the stiffness matrix and element equations for a bar element

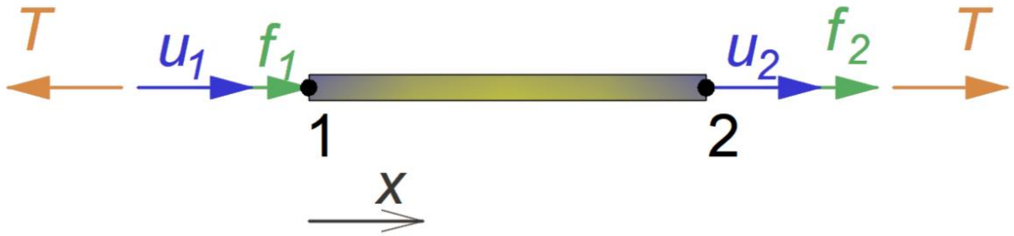
Truss Analysis

- Different structures have different types of members
- The members of a truss structure can not take bending or shear
- These members, therefore, are only subjected to axial forces
- In FEA, these members are modelled using ‘bar elements’
- Bar elements are one dimensional line elements that can take only compression or tension

Description a Bar Element

- A schematic of a bar element is shown in Figure 1
- It has two nodes and the local x-axis is assumed to be directed from node 1 to node 2
- u_1 and u_2 are the nodal deformations of nodes 1 and 2 respectively. The blue arrows in Figure 1 only show the sense of direction and the magnitude of these deformations

Figure 1



Description a Bar Element (Continued)

- f_1 and f_2 are the nodal forces. The green arrows in Figure 1 show the positive direction for these forces.
- T is the internal force in the bar element. It is assumed to be constant throughout the element and its value is taken as positive if it creates tension in the element as indicated by the orange arrows in Figure 1.
- It is assumed that the cross-section of the bar element is constant and is denoted by A

Description a Bar Element (Continued)

- The material response of the bar element is assumed to be Linear-elastic
- $\sigma = E\varepsilon$
- $\varepsilon = \frac{du}{dx}$
- $\sigma = \frac{T}{A}$; since both internal force and area are assumed constant, stress and strain are also constant
- $A\sigma_x = T = constant$
- Differential equation governing the linear elastic bar behavior is:
- $\frac{d}{dx}\left(AE\frac{du}{dx}\right) = 0$

Stiffness Matrix for a Bar Element

- Step 1: Select element type (bar element with 2 nodes)
- Step 2: Select a displacement function
- $u = a_1 + a_2 x$
- $u = \left(\frac{u_2 - u_1}{L}\right) x + u_1$
- $u = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$
- $N_1 = 1 - \frac{x}{L};\ N_2 = \frac{x}{L}$

Stiffness Matrix for a Bar Element (Continued)

- Step 3: Strain/Displacement and Stress/Strain Relationships
- $\varepsilon_x = \frac{du}{dx} = \frac{u_2 - u_1}{L}$
- $\sigma_x = E\varepsilon_x$
- Step 4: Element stiffness matrix
- $T = A\sigma_x$
- $T = AE\left(\frac{u_2 - u_1}{L}\right)$

Stiffness Matrix for a Bar Element (Continued)

- From equilibrium, we have:
- $f_1 = -T = -\frac{AE}{L}(u_2 - u_1)$
- $f_2 = T = \frac{AE}{L}(u_2 - u_1)$
- $\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$
- $[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Summar

- The stiffness matrix for a bar element is given as:
- $[k] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
- Where, A is the area of cross-section, E is the Young's Modulus of Elasticity, and L is the length of the bar element

Author Information

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