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Finite Element Analysis

Structural Analysis Example using Spring Elements

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Lesson Outcomes

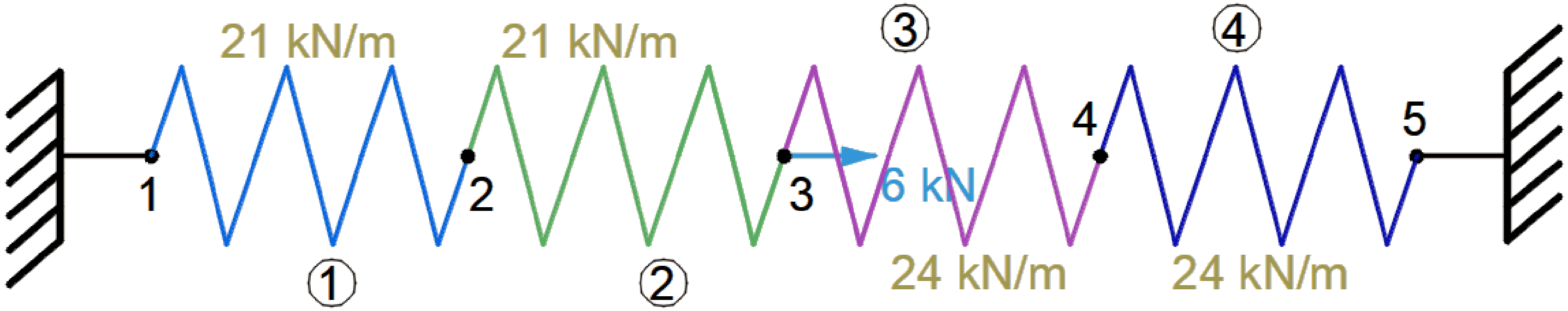
- At the end of this lecture, the student should be able to:
  - Understand the concept of Element Libraries
  - Use the previously developed element formulation for solving an example structure
  - Understand and interpret the Finite Element Analysis results
  - Draw the deformed shape based on FEA results

Element Libraries

- We do not have to derive element equations every time we want to analyze a structure
- A huge number of element formulations are available in the form of element libraries
- We can directly use these element libraries and go directly to the assembly step
- We will now use the spring element stiffness matrices developed in the previous lecture for the solution of a slightly more complicated problem by starting directly from the assembly step

Example Structure

- Solve for the unknown reactions at nodes 1 and 5; and unknown displacements at nodes 2, 3, and 4. Also find the deformations and forces in individual elements.



Stiffness Matrix

- Using the element equations developed previously:
- $\left[k^{(1)}\right] = \left[k^{(2)}\right] = \begin{bmatrix} 21 & -21 \\ -21 & 21 \end{bmatrix}$
- $\left[k^{(3)}\right] = \left[k^{(4)}\right] = \begin{bmatrix} 24 & -24 \\ -24 & 24 \end{bmatrix}$
- Assembling these matrices according to element connectivity
- $\left[k^{(1)}\right] = \begin{matrix} & u_1 & u_2 & \\ & \begin{bmatrix} 21 & -21 \\ -21 & 21 \end{bmatrix} & & \begin{matrix} u_1 \\ u_2 \end{matrix} \end{matrix}$
- $[K] = \begin{bmatrix} 21 & -21 & 0 & 0 & 0 \\ -21 & 21 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Stiffness Matrix (Continued)

- $$[k^{(2)}] = \begin{array}{c} \begin{matrix} u_2 & u_3 \end{matrix} \\ \begin{bmatrix} 21 & -21 \\ -21 & 21 \end{bmatrix} \end{array} \begin{matrix} u_2 \\ u_3 \end{matrix}$$

- $$[K] = \begin{bmatrix} 21 & -21 & 0 & 0 & 0 \\ -21 & 42 & -21 & 0 & 0 \\ 0 & -21 & 21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- $$[k^{(3)}] = \begin{array}{c} \begin{matrix} u_3 & u_4 \end{matrix} \\ \begin{bmatrix} 24 & -24 \\ -24 & 24 \end{bmatrix} \end{array} \begin{matrix} u_3 \\ u_4 \end{matrix}$$

Stiffness Matrix (Continued)

- $$[K]=\begin{bmatrix} 21 & -21 & 0 & 0 & 0 \\ -21 & 42 & -21 & 0 & 0 \\ 0 & -21 & 45 & -24 & 0 \\ 0 & 0 & -24 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
- $$\left[k^{(4)}\right]=\begin{array}{c} \begin{matrix} u_4 & u_5 \end{matrix} \\ \begin{bmatrix} 24 & -24 \\ -24 & 24 \end{bmatrix} \end{array} \begin{matrix} u_4 \\ u_5 \end{matrix}$$
- $$[K]=\begin{bmatrix} 21 & -21 & 0 & 0 & 0 \\ -21 & 42 & -21 & 0 & 0 \\ 0 & -21 & 45 & -24 & 0 \\ 0 & 0 & -24 & 48 & -24 \\ 0 & 0 & 0 & -24 & 24 \end{bmatrix}$$

Equations and Boundary Conditions

- $\{F\} = [K]\{d\}$
- $$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{Bmatrix} = \begin{bmatrix} 21 & -21 & 0 & 0 & 0 \\ -21 & 42 & -21 & 0 & 0 \\ 0 & -21 & 45 & -24 & 0 \\ 0 & 0 & -24 & 48 & -24 \\ 0 & 0 & 0 & -24 & 24 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}$$
- Nodes 1 and 5 are fixed
- It means $u_1 = u_5 = 0$
- We can remove the relevant rows and columns
- $$\begin{Bmatrix} 0 \\ 6 \\ 0 \end{Bmatrix} = \begin{bmatrix} 42 & -21 & 0 \\ -21 & 45 & -24 \\ 0 & -24 & 48 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

Solution: Forward Elimination

- The system of equations to be solved is
- $$\begin{Bmatrix} 0 \\ 6 \\ 0 \end{Bmatrix} = \begin{bmatrix} 42 & -21 & 0 \\ -21 & 45 & -24 \\ 0 & -24 & 48 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$
- First step is to make the augmented matrix
- $$\left[\begin{array}{cccc} 42 & -21 & 0 & \vdots & 0 \\ -21 & 45 & -24 & \vdots & 6 \\ 0 & -24 & 48 & \vdots & 0 \end{array}\right]$$

Solution: Forward Elimination (Continued)

- Apply 1st row operation: $R_2 + \frac{R_1}{2}$
- $$\begin{bmatrix} 42 & -21 & 0 & \vdots & 0 \\ 0 & 34.5 & -24 & \vdots & 6 \\ 0 & -24 & 48 & \vdots & 0 \end{bmatrix}$$
- Apply 2nd row operation: $R_3 + \frac{24R_2}{34.5}$
- $$\begin{bmatrix} 42 & -21 & 0 & \vdots & 0 \\ 0 & 34.5 & -24 & \vdots & 6 \\ 0 & 0 & 31.3 & \vdots & 4.174 \end{bmatrix}$$

Solution: Back Substitution

- $\begin{bmatrix} 42 & -21 & 0 & \vdots & 0 \\ 0 & 34.5 & -24 & \vdots & 6 \\ 0 & 0 & 31.3 & \vdots & 4.174 \end{bmatrix}$
- $u_4 = \frac{4.174}{31.3} = 0.1334m = 13.34cm$
- $34.5u_3 - 24u_4 = 6$
- $u_3 = 0.2667m = 26.67cm$
- $42u_2 - 21u_3 = 0$
- $u_2 = 0.1334m = 13.34cm$

Solution: Back Substitution (Continued)

- Going back to the 2 equations we removed earlier
- $F_1 = 21u_1 - 21u_2 = -21 \times 0.1334 = -2.8kN$
- $F_5 = -24u_4 + 24u_5 = -24 \times 0.1334 = -3.2kN$

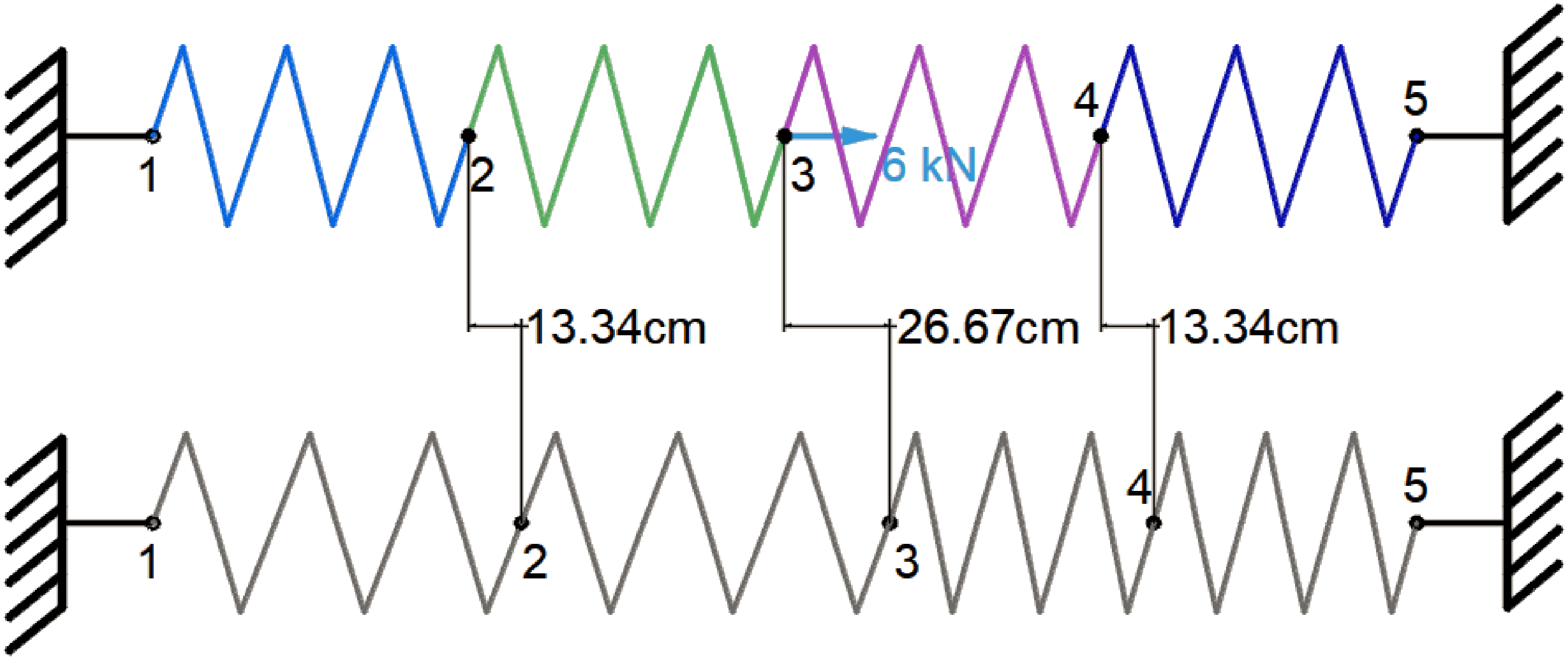
Element Deformations and Forces

- $\delta^{(1)} = u_2 - u_1 = 0.1334 - 0 = 0.1334m$
- $T^{(1)} = k^{(1)}\delta^{(1)} = 21 \times 0.1334 = 2.8kN$
- $\delta^{(2)} = u_3 - u_2 = 0.2667 - 0.1334 = 0.1334m$
- $T^{(2)} = k^{(2)}\delta^{(2)} = 21 \times 0.1334 = 2.8kN$
- $\delta^{(3)} = u_4 - u_3 = 0.1334 - 0.2667 = -0.1334m$
- $T^{(3)} = k^{(3)}\delta^{(3)} = 24 \times (-0.1334) = -3.2kN$
- $\delta^{(4)} = u_5 - u_4 = 0 - 0.1334 = -0.1334m$
- $T^{(4)} = k^{(4)}\delta^{(4)} = 24 \times (-0.1334) = -3.2kN$

Element Deformations and Forces

- Element deformations and forces are tensile if positive and compressive if negative
- It can be noted that elements 3 and 4 are taking the larger forces because these elements have higher stiffness

Deformed Shape



Author Information

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