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NUMERICAL METHODS & OPTIMISATION

Part II: Curve Fitting

Raihana Edros
Faculty of Engineering Technology
rzahirah@ump.edu.my

Chapter Description

- Aims
  - Apply numerical methods in solving engineering problem and optimisation

- Expected Outcomes
  - Estimate the first and higher-order of mathematical model that represents the experimental data by using different kinds of curve fitting methods
  - Estimate the regression coefficient, standard deviation and standard error of experimental data by using different kinds of curve fitting methods
  - Apply the curve fitting methods to solve engineering problems

- References
  - Steven C. Chapra and Raymond P. Canale (2009), Numerical Methods for Engineers, McGraw-Hill, 6th Edition

Interpolation

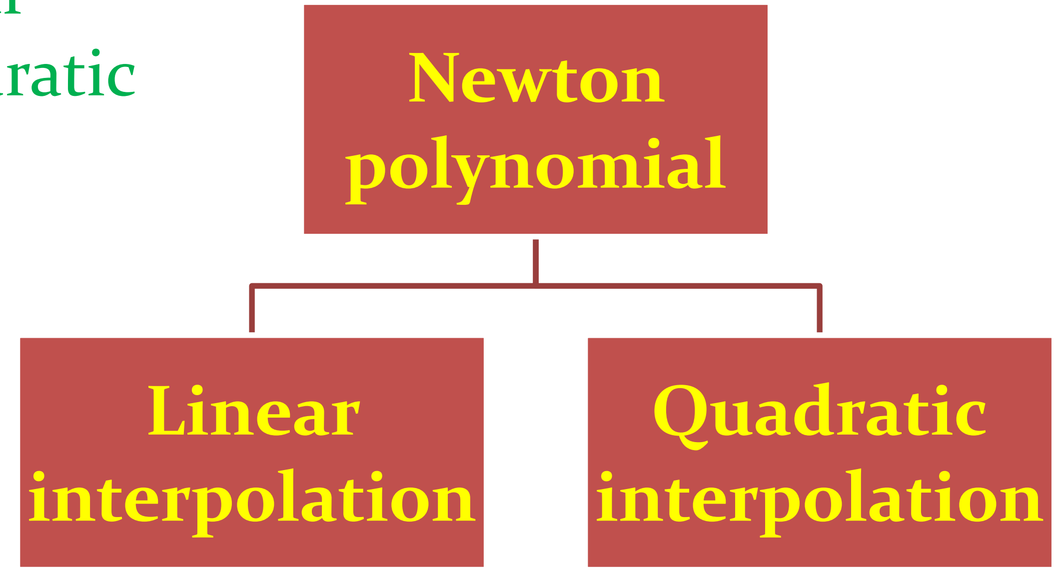
- There are times in which the intermediate values have to be estimated precisely
- Most common method is polynomial that can be represented by:

$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

- Polynomial interpolation consists of determining the unique nth-order polynomials that fits n+1 data points
- Three common methods are:
  - Newton's divided-difference interpolating polynomials
  - Langrange interpolating polynomial
  - Spline interpolation

Newton's divided-difference interpolating polynomials

- Common methods:
  - Linear
  - Quadratic



Newton's: Linear

- The simplest form of interpolation to connect two data point with straight line
- The shaded areas indicate the similar triangles used to derive the linear-interpolation formula
- The equation is given by:

$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

Newton's: Quadratic

- Error originated from linear interpolation is huge even at lower step size
- Quadratic interpolation improves the estimation by introducing some curvature into the line connecting the points
- For quadratic interpolation is given by:

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

- Quadratic interpolation requires some extra steps in which the coefficients b_o, b_1 & b_2 have to be firstly determined

Newton's: Quadratic - Exercise

Given the data:

x	1.6	2	2.5	3.2	4	4.5
f(x)	2	8	14	15	8	2

a) Calculate *f(2.8)* using Newton's interpolating polynomials of order 1 to 3. Choose the sequence of the points for your estimates to attain the best possible accuracy.

Langrange interpolating polynomials

- A reformulation of Newton's polynomial that avoids the computation of divided difference

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq 0}}^{n} \frac{x - x_j}{x_i - x_j}$$

Spline Interpolation

- Using previous methods, interpolation makes curve fitting easier
- Eg: For 8 points data, 7^{th} order polynomial can be used to predict the curve
- However, this can lead to erroneous results due to round-off errors – especially for functions with the presence of abrupt changes
- Spline interpolation can deal with this case easily

Spline interpolation (cont'd)

- **Spline Interpolation**
  - Linear splines
  - Quadratic splines
  - Cubic splines

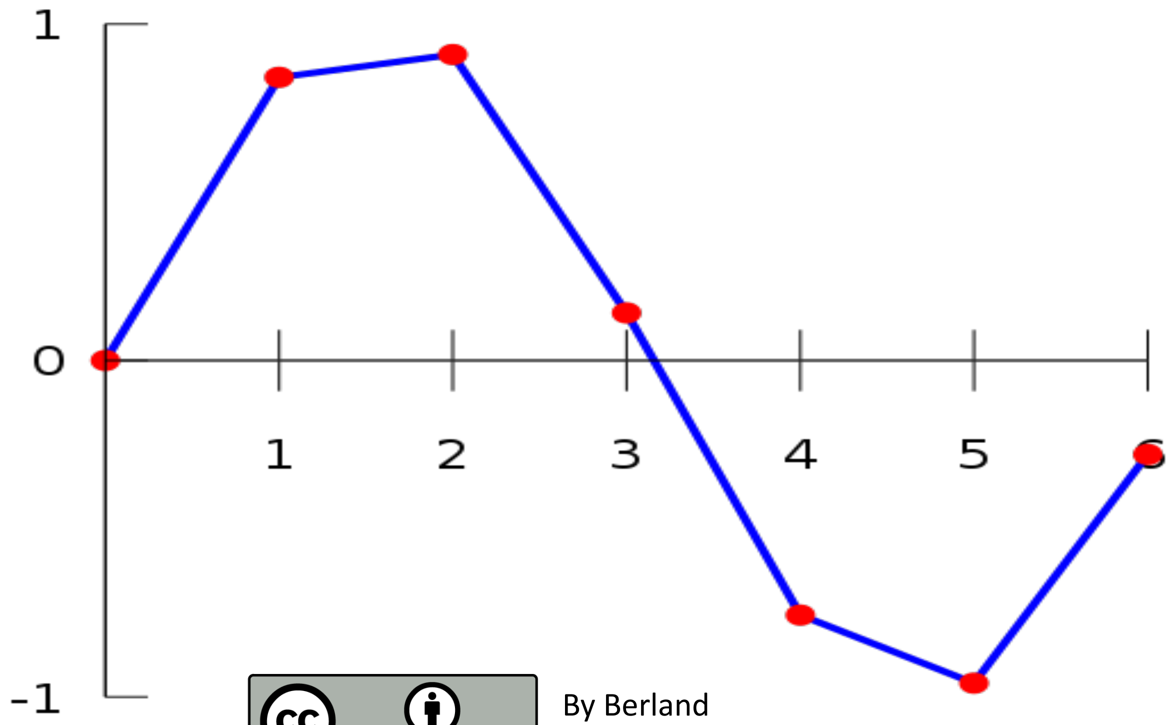
Spline interpolation: Linear

- Linear spline is regarded as first-order spline
- The simplest connection between 2 points & can be defined as:

$$
\begin{aligned}
f(x) &= f(x_0) + m_0(x - x_0) && x_0 \le x \le x_1 \\
f(x) &= f(x_1) + m_1(x - x_1) && x_1 \le x \le x_2 \\
&\vdots \qquad\qquad \vdots \\
f(x) &= f(x_{n-1}) + m_{n-1}(x - x_{n-1}) && x_{n-1} \le x \le x_n
\end{aligned}
$$

- Where m_i is the slope of the straight line

Spline interpolation: Linear (cont'd)



Conclusion

- First and higher-order of mathematical model that represents the experimental data can be estimated by using different kinds of curve fitting methods
- Regression coefficient, standard deviation and standard error of experimental data can be estimated by using different kinds of curve fitting methods

Main Reference

Steven C. Chapra and Raymond P. Canale (2009), Numerical Methods for Engineers, McGraw-Hill, 6th Edition

Any enquiries kindly contact:

Raihana Edros, PhD
rzahirah@ump.edu.my