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NUMERICAL METHODS & OPTIMISATION

Part II: Optimisation

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Chapter Description

- Aims
  - Apply numerical methods in solving engineering problem and optimisation

- Expected Outcomes
  - Calculate the maximum, minimum and optimal values of an equation by using the following methods for optimisation:
    - Newton's Method
    - Direct Method
  - Solve engineering problems by using methods for optimisation

- References
  - Steven C. Chapra and Raymond P. Canale (2009), Numerical Methods for Engineers, McGraw-Hill, 6th Edition

Newton's Method

- This method is similar to Newton-Raphson method – can be used to find an optimum of f(x) by using the following equation:

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

- The equation can be used to find the minimum and maximum of f(x).
- Unlike, golden-section search, this method does not need two initial guesses that bracket the optimum. However, it has possibility to diverge compared to golden-section search method.

Newton's Method: Example

Use Newton's method to find the maximum of

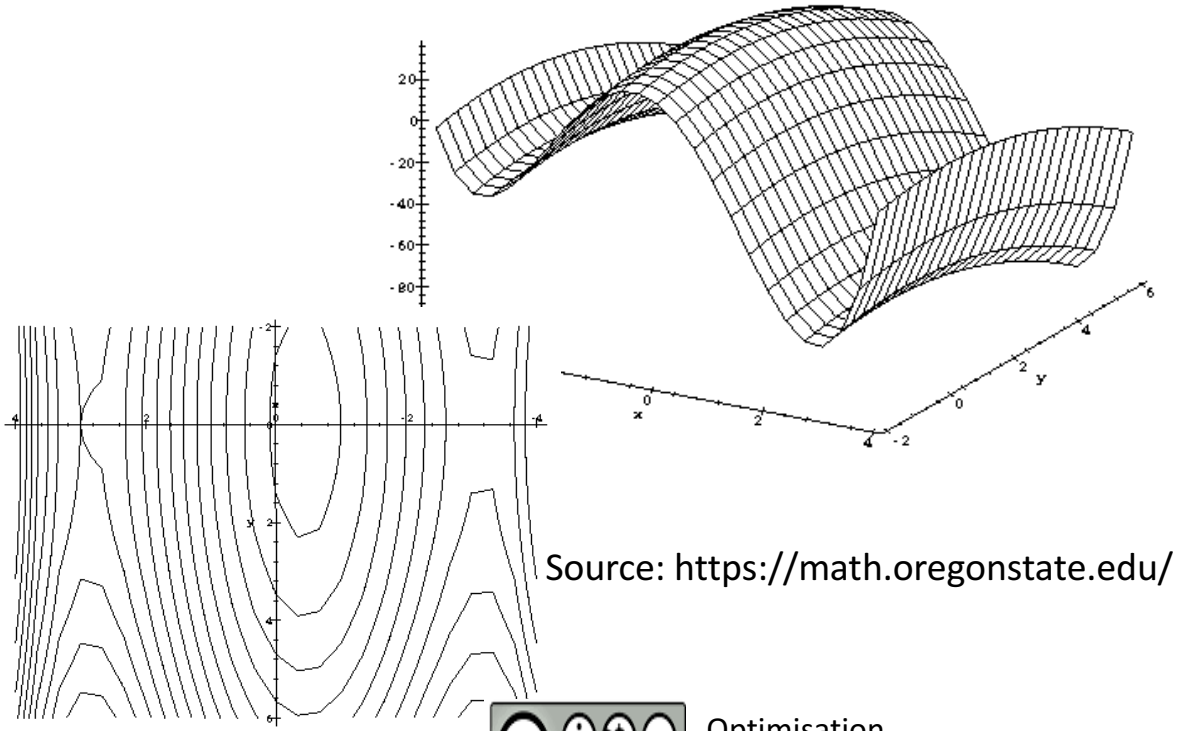
$$f(x) = 2\sin x - \frac{x^2}{10}$$

with an initial guess of $x_0 = 2.5$ and **perform three iterations**.

Multidimensional Unconstrained Optimization

- Describes techniques to find the minimum or maximum of a function of several variables
- One dimensional optimization – one dimension visual image – higher dimension:

- Two types:
  - Non Gradient method
  - Gradient method
- These techniques are evaluated based on whether the functions require derivative evaluation



Source: https://math.oregonstate.edu/

Direct Method

- Method is based on evaluation of the function randomly at selected values of the independent variables.
- Two direct methods:
  - Random search approach: evaluates the function at randomly selected values
  - Univariate & pattern searches: more efficient & does not require derivative evaluation

$$x = x_l + (x_u - x_l)r$$

$$y = y_l + (y_u - y_l)r$$

- If a sufficient number of samples are conducted, the optimum will be eventually located, by using:

where *r* is between 0 to 1, and Δr = 0.1.

Direct Method (cont'd)

r	x	y	$f(x,y)$
0	x_l	y_l	
0.1			
0.2			
0.3			
0.4			
0.5			
0.6			
0.7			
0.8			
0.9			
1	x_u	y_u	

Direct Method: Example

Use a random number generator to locate the maximum of

$$f(x,y) = 2x + 4y^2 - 7x^2$$

in the domain bounded by $x = -2$ *to* 2 *and* $y = 1$ *to* 3
(*r* is between 0 to 1, and $\Delta r = 0.1$)

Direct Method: Example (cont'd)

r	x	y	$f(x,y)$
0	-2	1	-28
0.1	-1.6	1.2	-15.36
0.2	-1.2	1.4	-4.64
0.3	-0.8	1.6	4.16
0.4	-0.4	1.8	11.04
0.5	0	2	16
0.6	0.4	2.2	19.04
0.7	**0.8**	**2.4**	**20.16**
0.8	1.2	2.6	19.36
0.9	1.6	2.8	16.64
1	2	3	12

Tutorial

a) Solve for the value of x that maximizes $f(x)$ using golden section search. Employ initial guesses of $x_l = 0$ **and** $x_u = 2$ and **perform three iterations**.

$$f(x) = -1.5x^6 - 2x^4 + 12x$$

b) Repeat (a), except use quadratic interpolation. Employ initial guesses of $x_0 = 0$, $x_1 = 1$ ***and*** $x_2 = 2$, and **perform three iterations**.

Conclusion

- The maximum, minimum and optimal values of an equation can be estimated by using Newton's and Direct Methods for optimization

Main Reference

Steven C. Chapra and Raymond P. Canale (2009), Numerical Methods for Engineers, McGraw-Hill, 6th Edition

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