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NUMERICAL METHODS & OPTIMISATION

Part II: Roots of Equation
Open Methods

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Chapter Description

- Aims
  - Apply numerical methods in solving engineering problem and optimisation

- Expected Outcomes
  - Calculate the root of equation by using open methods
    - Simple-fixed Method
    - Secant Method
    - Newton Raphson Method
  - Apply bracketing method in finding the roots of equation for engineering problems

- References
  - Steven C. Chapra and Raymond P. Canale (2009), Numerical Methods for Engineers, McGraw-Hill, 6th Edition

Open Methods

- Bracketing methods: the root is located within an interval prescribed by lower & upper bound
- Repeated computation results in closer estimates of x_{root}
- These estimates are called as convergent

- Open methods required a single starting value of *x* or two starting values that do not necessarily bracket the root.

Bracketing vs. Open Methods

Bracketing Methods	Open Methods
• 2 initial guesses	• Can be either 1 or 2 initial guesses depending on the method used
• Root is located within lower and upper intervals • Closer estimates of the actual value can be calculated through repetition	• Root does not necessarily lie in bracket
• Convergence of values occurs as computation progresses	• Divergence of values are possible throughout calculation • If convergence occurs, the values move closer quickly

Simple-Fixed Point Iteration

1. Rearrange the function so that *x* is on the left side of the equation:

For example,

$x^2 - 2x + 3 = 0$ becomes $x = \dfrac{x^2 + 3}{2}$

$x = g(x)$ $\sin x = 0$ becomes $x = \sin x + x$

2. Begin with a given initial value, calculate a new value of *x* (x_{i+1}) by using an old value of *x* (x_i) and so on.

Simple-Fixed Point Iteration

4. Compute the ε_a

$$\varepsilon_a = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%$$

5. Iteration can be terminated until the ε_a is lower than the given ε_s.

Open Methods

Use simple fixed-point iteration to locate the root of $f(x) = e^{-x} - x$. Start your calculation with $x_o = 0$;

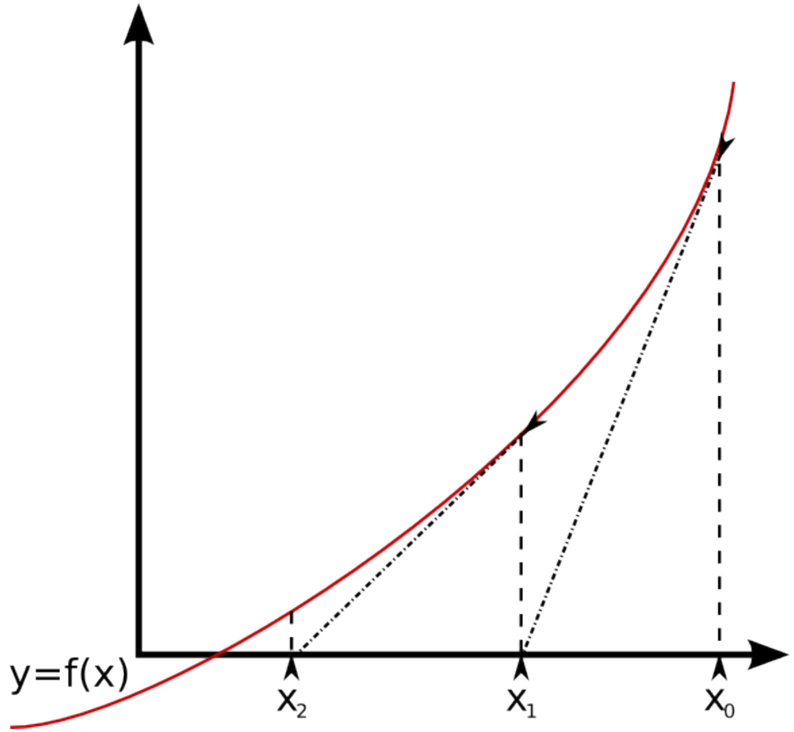
$$x_{i+1} = e^{-x_i}$$

Solution:

The equation can be rearranged and expressed as follows:

Newton-Raphson Method

- Most widely used method.
- If the initial guess at the root is x_i, a tangent can be extended to across the x axis.
- The point where a tangent across give an improved estimates of the root (x_{i+1}).



Source: https://commons.wikimedia.org/wiki/File:Newton–Raphson_method.png

Steps of Newton-Raphson

- Based on Taylor Series Expansion, the Newton-Raphson formula is given by:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

1. Begin the calculation with any initial value
2. Calculate a new value of x (x_{i+1}) by using an old value of x (x_i) and so on.

Steps of Newton-Raphson

- Compute the $\boldsymbol{\varepsilon}_a$

$$\varepsilon_a = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%$$

- Iteration can be terminated until the $\boldsymbol{\varepsilon}_a$ is lower than the given $\boldsymbol{\varepsilon}_s$

Newton-Raphson Method: Example

Use Newton-Raphson method to estimate the root of $f(x) = e^{-x} - x$, employing an initial guess of $x_o = 0$.

Solution:

The first derivative:

$$f'(x) = -e^{-x} - 1 \qquad x_{i+1} = x_i - \frac{e^{-x_i} - x_i}{-e^{-x_i} - 1}$$

Newton-Raphson Method: Solution

Iteration	x_i	ε_a (%)	ε_t (%)
0	0	-	100
1	0.5	100	11.8
2	0.566311	11.7	0.147
3	0.567143	0.15	0.000022
4	0.567143	2.2×10^{-5}	$< 10^{-8}$

ε_a and ε_T in Newton-Raphson decreases faster than simple – fixed point iteration

Newton-Raphson Method: Class activity

Use:

1. The fixed-point iteration and
2. Newton-Raphson method to determine the roots of

$$f(x) = -x^2 + 1.8x + 2.5$$

By using x_o=5. Perform the calculation until ε_a is less than $\varepsilon_{s=}$0.05%.

Secant Method

- This method uses the similar technique as Newton-Raphson method

- A potential problem in implementing Newton-Raphson is evaluation of the derivative, $f'(x_i)$.

- For Secant method, the derivative $f'(x_i)$ can be approximated by a backward finite divided difference and is given by:

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

Steps of Secant Method

1. Begin with a given initial value, calculate a new value of x (x_{i+1}) by using an old value of x (x_i) and so on.
2. Compute the $\boldsymbol{\varepsilon}_a$

$$\varepsilon_a = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%$$

3. Iteration can be terminated until the $\boldsymbol{\varepsilon}_a$ is lower than the given $\boldsymbol{\varepsilon}_s$.

Secant Method: Example

Use Secant method to estimate the root of $f(x) = e^{-x} - x$. Start with initial estimates of $x_{-1} = 0$ and $x_0 = 1.0$.

(Recall that the true root is 0.56714329)

Conclusion

- The root of equation can be estimated by using open methods such as simple-fixed iteration, Secant and Newton Raphson methods.
- The open methods can be applied to engineering problems in order to find the roots of equations

Main Reference

Steven C. Chapra and Raymond P. Canale (2009), Numerical Methods for Engineers, McGraw-Hill, 6th Edition

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