For updated version, please click on
http://ocw.ump.edu.my

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

NUMERICAL METHODS & OPTIMISATION

Part I: Roots of Equation
Bracketing Methods

Raihana Edros
Faculty of Engineering Technology
rzahirah@ump.edu.my

Chapter Description

- Aims
  - Apply numerical methods in solving engineering problem and optimisation

- Expected Outcomes
  - Calculate the root of equation by using bracketing methods
    - Graphical Method
    - Bisection Method
    - False-Position Method

- References
  - Steven C. Chapra and Raymond P. Canale (2009), Numerical Methods for Engineers, McGraw-Hill, 6th Edition

If you were given.....

$$f(x) = ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The value of *x* calculated are called the "roots" of equation.

The roots of equation is the value of *x* that makes *f*(*x*)=0.

But? $\sin x + x = 0 \Rightarrow x = ?$

How numerical methods work to solve the roots of equation?

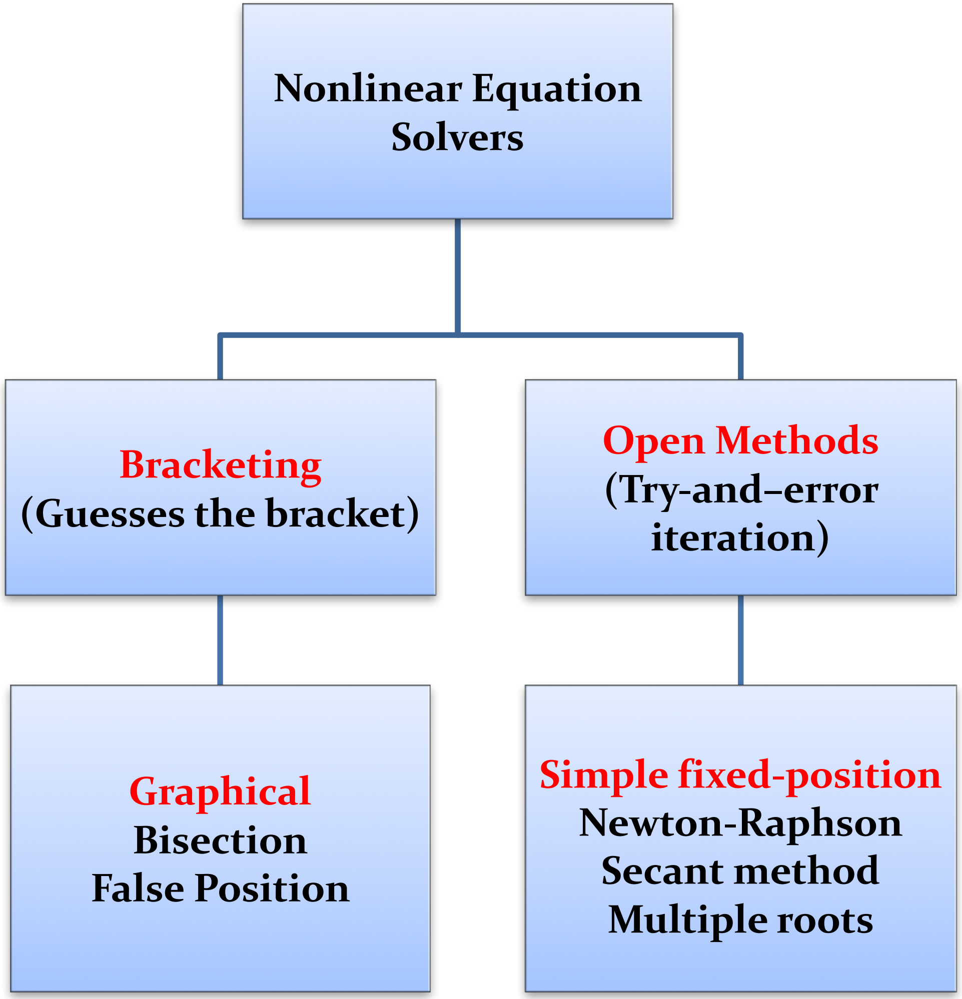
Hence, approximate solution can be one of the alternatives.

2 kinds of approaches:

1) Plot the function and determine where it crosses the x axis, ($f(x)=0$).
2) Guessing a value of x and evaluate whether $f(x)$ is zero.

Methods to determine roots of equation

- Bracketing methods (two initial guesses for the root are required)
  a) Graphical method
  b) Bisection method
  c) False position method
- Open methods
  a) Simple Fixed-Iteration method
  b) Newton-Raphson Method
  c) The secant Method



Bracketing method

- Two initial guesses for the root are required.
- These guesses must be in "bracket" or be on either side of the root.

- Graphical method
  - Plot the function $f(x)$ and observe the value of x where $f(x)=0$.
  - Rough estimation

Graphical Method: Example 1

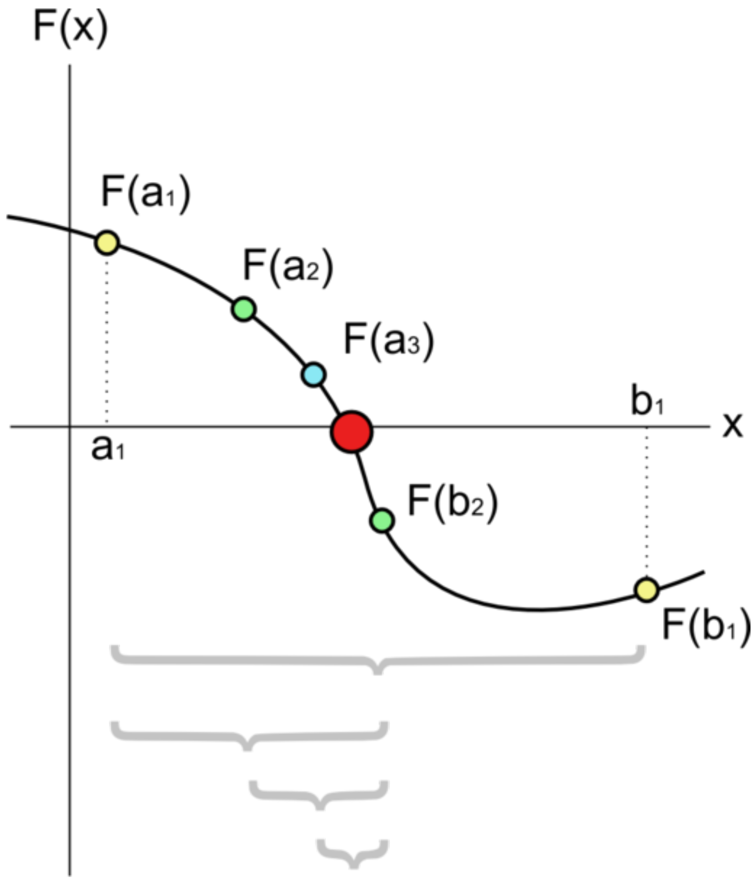
Use graphical approach to determine the drag coefficient *c* needed.

$$f(c) = \frac{667.38}{c}(1 - e^{-0.146843c}) - 40$$

Substitute various values of c into the right hand side of this eqn to compute *f(c)*

Bisection Method

- Bisection method is called as binary chopping where the interval is divided in half.
- The root is obtained by halving the initial guesses.
- This is then repeated to refine the estimates of the roots.
- If the $f(x)$ change sign (+ve, -ve), the function value at the midpoint is evaluated.



Source:
https://commons.wikimedia.org/

Simple algorithm for The Bisection Method

Step 1: Choose lower (x_l) and upper (x_u) for the root such that the function changes sign over the interval. This can be checked by ensuring that $f(x_l)f(x_u) < 0$.

Step 2: Estimate the root by evaluating

$$x_r = \frac{x_l + x_u}{2}$$

Step 3: make the following evaluation to determine in which subinterval the roots lies:

Find the pair:

a) If $f(x_l)f(x_r) < 0$, root lies in the lower interval, then $x_u = x_r$ and go to step 2.

b) If $f(x_l)f(x_r) > 0$, root lies in the upper interval, then $x_l = x_r$, go to step 2.

c) If $f(x_l)f(x_r) = 0$, then root is x_r and terminate.

The Bisection Method: Example

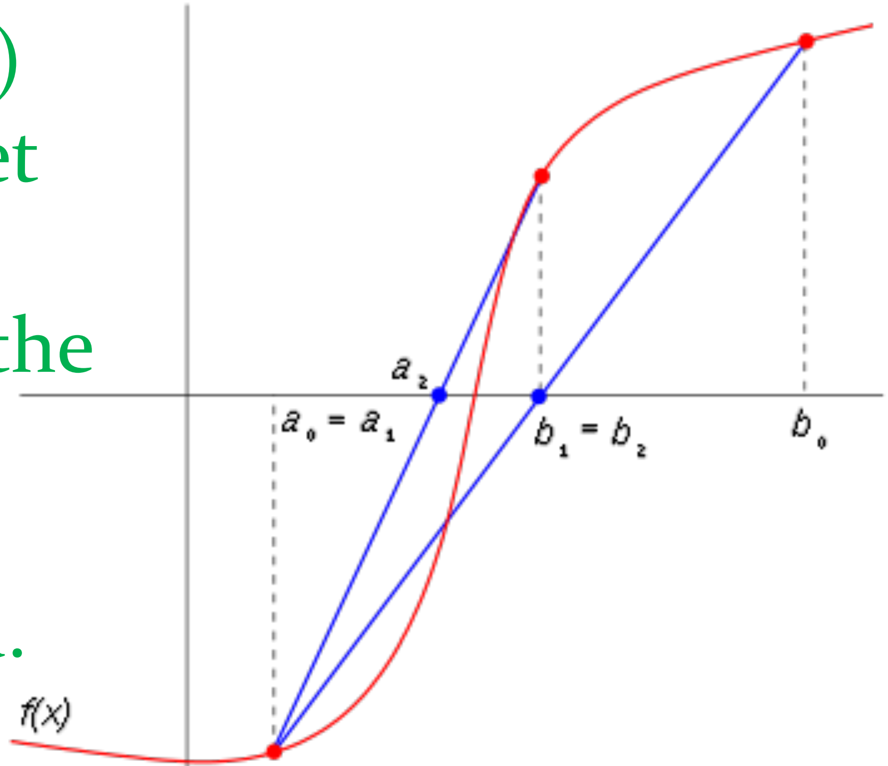
- Use bisection method to solve the same problem as in the previous example (Graphical Method) with ε_s=0.5%.
- The first step in bisection method is to guess two values of the unknown (x_l and x_u) where the function changes.
- From the graph we can see that, f(c) changes sign between 12 and 16.
- Step 1: x_l=12, x_u =16. Estimate the root xr:
- Step 2: 1st iteration for x_r=14
- >> Continue calculation till termination criteria is met.

The Bisection Method: Example

Iteration	x_l	x_u	x_r	ε_a (%)	ε_t (%)
1	12	16	14		5.279
2	14	16	15	6.667	1.487
3	14	15	14.5	3.448	1.896
4	14.5	15	14.75	1.695	0.204
5	14.75	15	14.875	0.840	0.641
6	14.75	14.875	14.8125	0.422	0.219

The False-Position Method

- If $f(x_l)$ is much close to zero than $f(x_u)$, it is likely that the root is closer to x_l than x_u.
- Join this $f(x_l)$ and $f(x_u)$ by a straight line to get the intersection on x-axis which represent the x_r value.
- Also called as linear interpolation method.



Source:https://de.wikipedia.org/wiki/Datei:False_position_method.svg

Steps of The False-Position Method

1. Find the values of x_l and x_u These values are evaluated using:

2. Compute the x_r using:

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)} \qquad f(x_l)f(x_u) < 0$$

3. a) If $f(x_l)f(x_r)<0$, root lies in the lower interval, then determine $x_u=x_r$ and go to step 2.
   b) If $f(x_l)f(x_r)>0$, root lies in the upper interval, then $x_l=x_r$, go to step 2.
   c) If $f(x_l)f(x_r)=0$, then root is x_r and terminate.

Steps of The False-Position Method

4. Compute ε_a

$$\varepsilon_a = \left| \frac{x_r^{new} - x_r^{old}}{x_r^{new}} \right| \times 100\%$$

5. Calculation can be terminated when ε_a is lower than the given ε_s.

The False-Position Method: Example

Use the false-position method to determine the root of the same equation investigated in Example 1.

(Initiate the calculation with guesses of $x_l = 12$ and $x_u = 16$).

The False-Position Method: Example

Given

$$f(x) = -25 + 82x - 90x^2 + 44x^3 - 8x^4 + 0.7x^5$$

Using initial guesses of x_l=0.5 and x_u=1.0, determine the root using:

a) Bisection method to ε_s =10%

b) False-position method to ε_s =0.2%

Conclusion

- The root of equation can be estimated by using bracketing methods such as graphical, bisection and false-position methods.

Universiti Malaysia PAHANG
Engineering • Technology • Creativity

Main Reference

Steven C. Chapra and Raymond P. Canale (2009), Numerical Methods for Engineers, McGraw-Hill, 6th Edition

Any enquiries kindly contact:
Raihana Edros, PhD
rzahirah@ump.edu.my