HYDRAULICS

NON - UNIFORM FLOW IN OPEN CHANNEL

TOPIC 3.1

by

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NON -UNIFORM FLOW IN OPEN CHANNEL

3.1

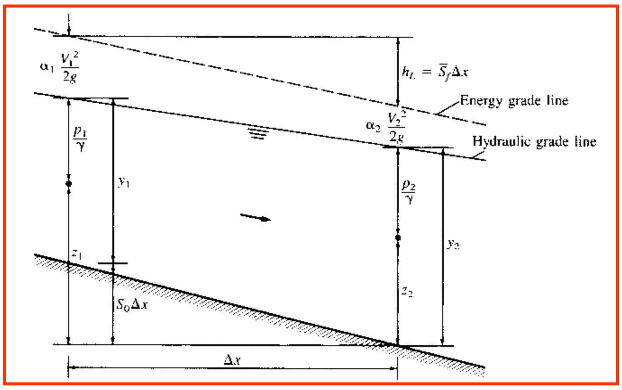
- Specific energy

Non – Uniform Flow

The energy grade line, water surface and channel bottom are not parallel;

Sf ≠ Sw ≠ So

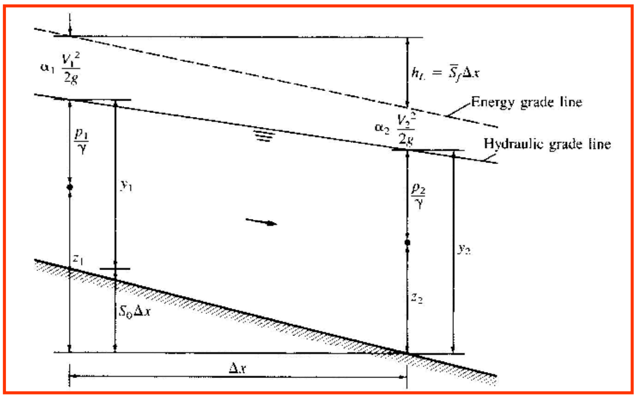
Sf = slope energy grade line , Sw= slope of the water surface , So= slope of the channel bed



Total Energy

The total energy of a channel flow referred to a datum is given by:

$$H = z + y + (v^2/2g)$$



3.1 : SPECIFIC ENERGY

- The concept of specific energy introduced by Bakhmeteff (1932)
- Specific energy is defined as the sum of depth and velocity head.
- A more formal definition of specific energy is the height of the energy grade line above the channel bottom.
- In uniform flow, for example, the energy grade line by definition is parallel to the channel bottom, so that the specific energy is constant in the flow direction.

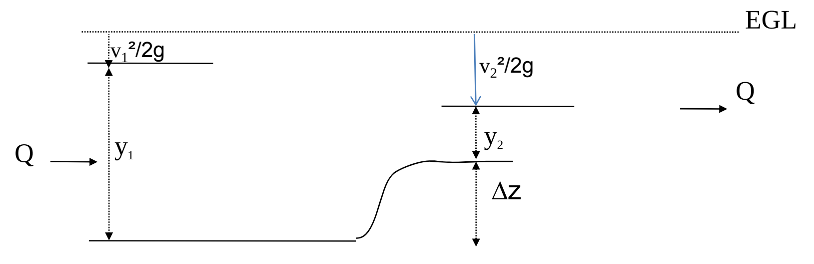


Figure 1 :Transition with bottom step

The **specific energy decreases in the flow direction**, but it would be equally possible for **the specific energy to increase** in the flow direction by **dropping** rather than raising the **channel bottom**.

❖ If for the moment we neglect the energy loss, the energy equation combined with continuity can be written as

$$E_1 = E_2 + \Delta z$$

$$y_1 + Q^2/2gA_1^2 = y_2 + Q^2/2gA_2^2 + \Delta z \qquad (3.1)$$

Where is

y	=	depth
Q	=	discharge
A	=	Cross-sectional area of flow
Δz	=	$z_2 - z_1$
	=	change in bottom elevation from cross section 1 to 2

EXAMPLE 3.1

Water flows in a rectangular channel with 5m width and 8m³/s flowrate. Depth of channel is 1m. Determine the specific energy for this channel.

EXAMPLE 3.2

From Example 3.1, if the channel is a trapezoidal channel with side slope is 1.5(H):1(V) and width of channel is 2m. Determine the specific energy for this channel.

Chapter 3: Non - Uniform Flow in Open Channel by N Adilah A A Ghani

3.1 Specific Energy in Open Channel

- Considered square channel (**prismatic and straight**)

$$E = y + q^2/2gy^2 \qquad (3.2)$$

where, q = flowrate per unit width ($m^3/s/m$)

$q = Q/b$, and $A = by$

- Equation 3.2 (energy, **E**, depth of flow, **y** and flowrate, **q**) may be written/defined in 2 conditions as below;-

i. E and y if q is constant
ii. q and y if E is constant

i) E and y if q is constant

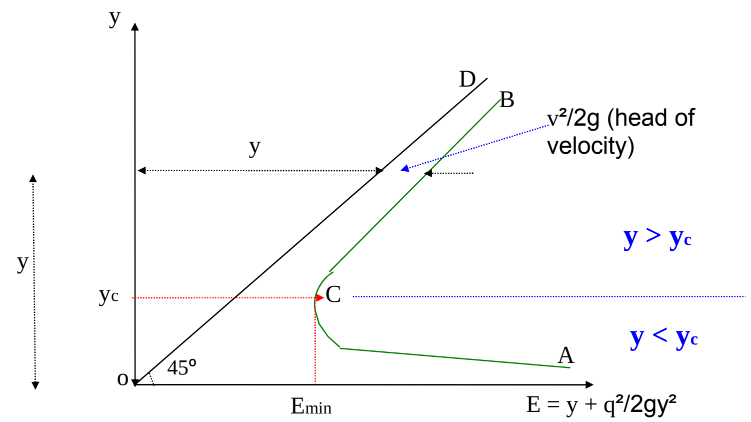
From Eq (3.2)

$$E = y + q^2/2gy^2$$

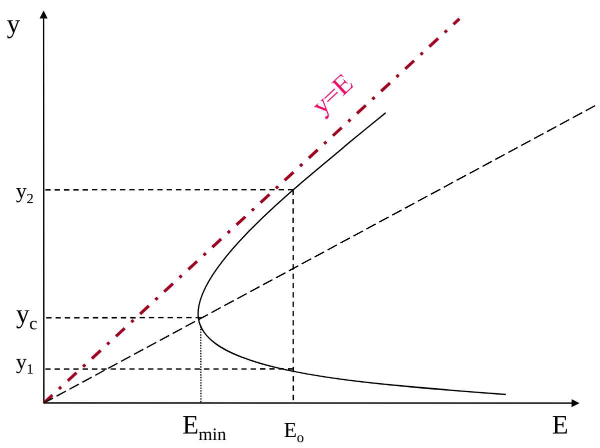
$$0 = y^3 - Ey^2 + q^2/2g \qquad (3.3)$$

- It is apparent from Equation (3.3) that there indeed is a unique functional variation between y and E for a constant value of q, and it is sketched as the specific energy diagram.

Specific Energy Diagrams (E-y)



Specific Energy Diagrams



*Note q is constant.

At C, specific energy is minimum and normal depth at this point is **'critical depth'**, y_c

If

$y > y_c$; $v < v_c$ ==> Subcritical flow (steady)

$y < y_c$; $v > v_c$ ==>Supercritical flow (turbulant)

Differentiation of Equation (3.2)

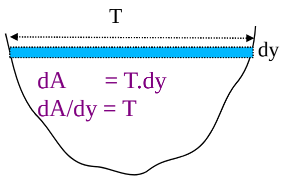
$E = y + Q^2/2gA^2$

$dE/dy = 1 - (Q^2/2g)(2/A^3)dA/dy$

$dE/dy = 1 - (Q^2/gA^3).T$

$= 1 - (v^2/gA).T$

$= 1 - v^2/gD$



At critical point, E is minimum i.e. dE/dy = 0. therefore;-

$v^2/gD = 1$; (Froude, Fr = 1)

$v^2/2g = D/2$ or **$v/\sqrt{(gD)} = 1$**

For a rectangular channel

Hydraulic Depth, D = A /T = by/b = y

Therefore, at critical condition ==>>

Fr	=	1	($y = y_c$, $v = v_c$)
$v_c /\sqrt{(g\, y_c)}$	=	1	(a)
$v_c^2/2g$	=	$y_c/2$	(b)

From the schematic diagram;- (E = min, $y = y_c$)

E	=	$y + q^2/2gy$	
dE/dy	=	$1 - q^2/gy_c^3$	= 0
q^2	=	gy_c^3	
y_c	**=**	**$\sqrt[3]{(q^2/g)}$**	(c)

q^2	=	gy_c^3	but	$q = vy$	$= v_c y_c$	
$v_c^2 y_c^2$	=	gy_c^3				
v_c^2	=	gy_c				
v_c	**=**	**$\sqrt{(gy_c)}$**	**or**	**$v_c^2/2g$**	**$= ½ y_c$**	(d)

E_{min}	=	$y_c + q^2/2gy_c^2$		
	=	$y_c + (g\,y_c^3) / (gy_c^2)$		
	=	$y_c + y_c\sqrt{2}$		
E_{min}	**=**	**$1.5y_c$ or y_c**	**$=(2/3)E_{min}$**	(e)

- The point of minimum E is found by setting dE/dy equal to zero, and solving for y. The result is $y_c = (2/3)E$, which is called the critical depth y_c. The corresponding velocity V is called the critical velocity v_c. The critical depth divides the energy curve into two branches. On the upper branch, y increases with E, while on the lower branch y decreases with E.

ii) q and y if E is constant

E	=	$y + q^2/2gy^2$	
q^2	=	$2gy^2(E - y)$	(3.4)

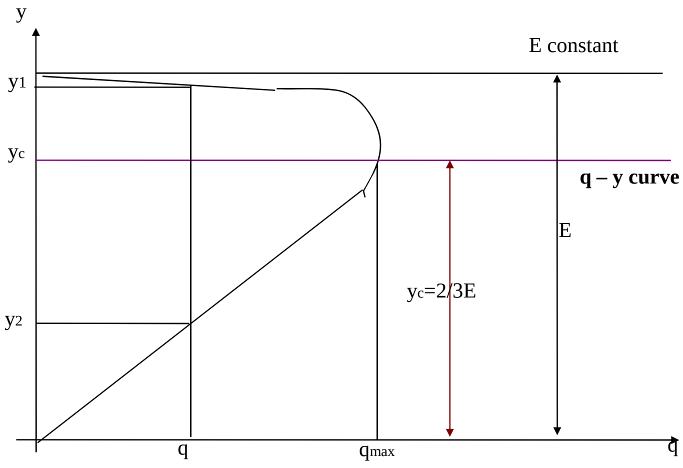
At critical point, $dq/dy = 0$

Differentiation of Equation (3.4):

q^2	=	$2gy^2$	
$(E - y)$	=	$2g(Ey^2 – y^3)$	
$2q dq/dy$	=	$2g(E\,2y_c – 3y_c^2) = 0$	
$2y_cE$	=	$3y_c^2$	
E	=	$1.5y_c$	
y_c	**=**	**$(2/3)E$**	(f)
q_{max}^2	=	$2gy^2(E – y)$ (from Eq. 3.4)	
	=	$2gy^2(1.5y_c – y_c)$	
	=	gy_c^3	
q_{max}	**=**	**$\sqrt{(gy_c^3)}$**	(g)

- Note;
- Subcritical and supercritical flow normally depend on the channel slope, S. Therefore, for the supercritical flow, value of S is high.
- *Critical Slope = slope at critical depth.

Specific Energy Diagrams (q-y)



Critical flow criteria (square/rectangular channel)

Fr = 1.0

'E' is minimum for 'q' constant

$E_{min} = 1.5y_c$

$y_c = \sqrt[3]{(q^2/g)}$

'q' is maximum at E constant

$y_c = 2/3E_{min}$

$q_{max} = \sqrt{(gy_c^3)}$

Velocity head ($v_c^2/2g$) is one-half of critical depth, y_c

$v_c^2/2g = y_c/2$

Critical velocity (v_c);

$v_c = \sqrt{(gy_c)}$

Froude Number, Fr and Flow classification

$q^2/gy_c^3 = 1$

Then,

$v_c^2/gy_c = 1$ at critical conditions

So,

at critical conditions, the Froude number =1

$v/\sqrt{gy_c}$ is known as the Froude Number, F

If F = 1, $y = y_c$ and flow is critical.

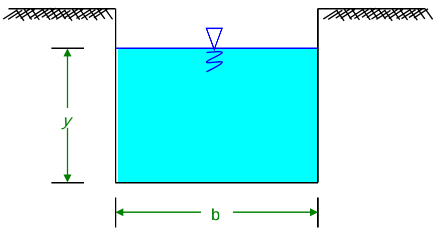
If F < 1, $y > y_c$ and flow is subcritical.

If F > 1, $y < y_c$ and flow is supercritical.

F is independent of the slope of the channel, y_c dependent only on Q.

EXAMPLE 3.3: Specific Energy Diagram (SED)

A rectangular channel of small slope has a channel width = 6.00 m;



a) Construct a family of specific-energy for Q_1 = 5 m^3/s, Q_2 = 10 m^3/s, Q_3 = 15 m^3/s.

b) Draw the locus of the critical depth point on these curves.

c) Plot a curve of critical depth against the discharge

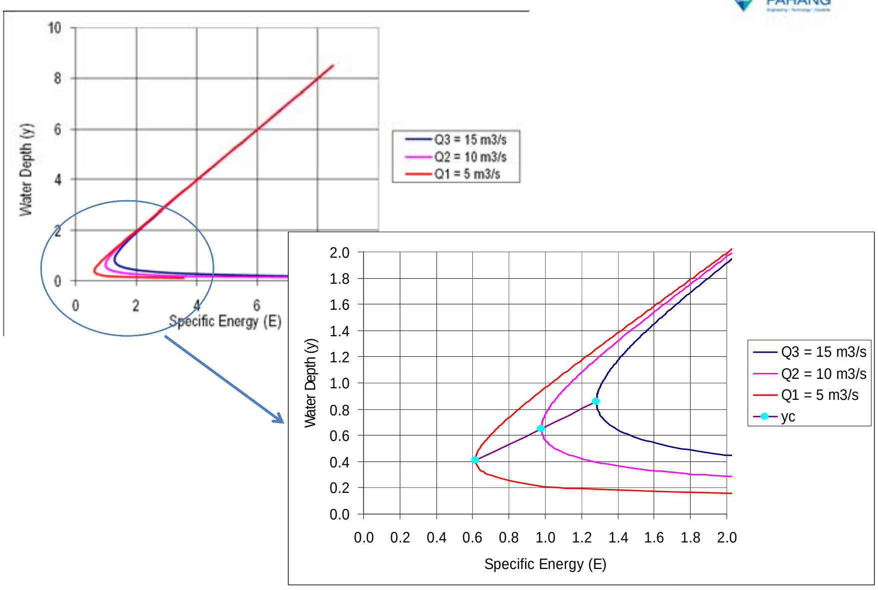
Solution 3.3:

a) Specific Energy (Table)

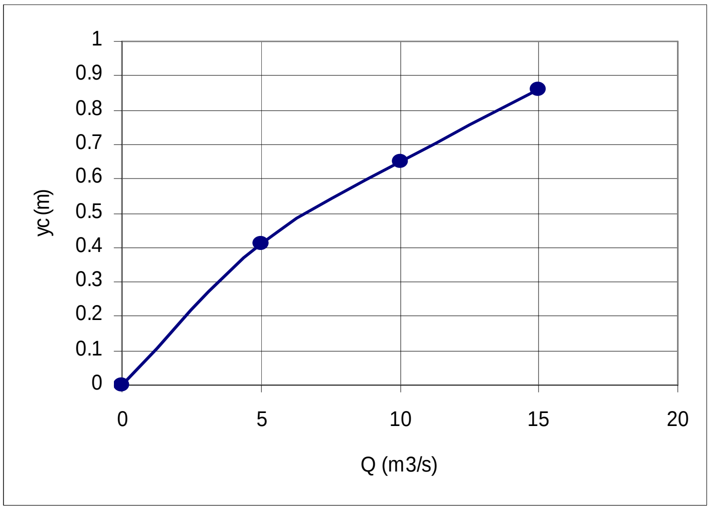
$$E = y + \frac{Q^2}{2gA^2}$$

Water Depth (y)	Area (A)	A Specific Energy (m)		
m	m²	Q1 = 5 m³/s	Q2 = 10 m³/s	Q3 = 15 m³/s
0.5	3	0.64	1.06	1.75
1	6	1.03	1.14	1.31
1.5	9	1.52	1.56	1.64
2	12	2.01	2.03	2.08
2.5	15	2.51	2.52	2.55
3	18	3.00	3.02	3.03
3.5	21	3.50	3.51	3.53
4	24	4.00	4.01	4.02
4.5	27	4.50	4.51	4.52
5	30	5.00	5.01	5.01
5.5	33	5.50	5.50	5.51
6	36	6.00	6.00	6.01
6.5	39	6.50	6.50	6.51
7	42	7.00	7.00	7.01
7.5	45	7.50	7.50	7.51
8	48	8.00	8.00	8.00
8.5	51	8.50	8.50	8.50

b) Critical point (Curve)



c) critical depth against discharge relationship (rating curve)



Flow characteristics of flow in rectangular channels

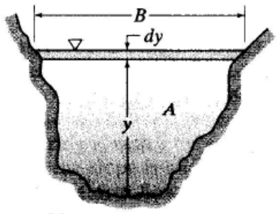
Characteristic	Subcritical	Critical	Supercritical
Depth of flow, y	$y > y_c$	$y = y_c = \left(\frac{q^2}{g}\right)^{1/3}$	$y < y_c$
Velocity of flow, V	$V < V_c$	$V = V_c = \sqrt{gy}$	$V > V_c$
Slope for uniform flow, S_0	Mild slope $S_0 < S_c$	Critical slope $S_0 = S_c$ [Eq. (10.30) if wide and shallow]	Steep slope $S_0 > S_c$
Froude number, $\mathbf{F} = \frac{V}{\sqrt{gy}} = \frac{q}{\sqrt{gy^3}}$	$\mathbf{F} < 1.0$	$\mathbf{F} = 1.0$	$\mathbf{F} > 1.0$
Disturbance waves (Sec. 10.20)	Will propagate in all directions	Will hold fast, not propagate upstream	Will form standing wa with $\sin\beta =$ downstrean
Velocity head compared with half-depth	$\frac{V^2}{2g} < \frac{y}{2}$	$\frac{V^2}{2g} = \frac{y}{2}$	$\frac{V^2}{2g} > \frac{y}{2}$
Can be followed by a hydraulic jump?	No	No	Yes

Critical Depth in non-rectangular channels

- For nun-rectangular channel, y_h in the definition of the Froude Number is different from the water depth.

$$\frac{dE}{dy} = 0 \Longrightarrow 1 - \frac{Q^2}{2g}\left(\frac{2}{A^3}\frac{dA}{dy}\right) = 0$$

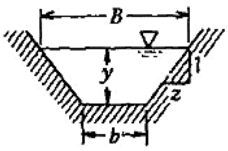
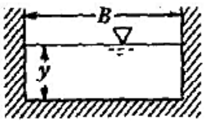
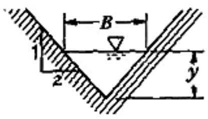
- dA is the increment of the flow area due to the change of water depth y, thus $dA/dy \approx B$, as shown in the accompany figure.



- The critical Froude number

$$1 - \frac{Q^2}{2g}\left(\frac{2}{A^3}B\right) = 0 \Rightarrow F_r = \frac{V}{\sqrt{g y_h}} = 1$$

Critical conditions for channels of various shape

			
A	$(B+zy)y$	By	zy^2
V_c	$\sqrt{\frac{B+zy_c}{B+2zy_c} g y_c}$	$\sqrt{g y_c}$	$\sqrt{\frac{g y_c}{2}}$
Q_c	$\frac{\sqrt{g}[(B+zy_c)y_c]^{1.5}}{(B+2zy_c)^{0.5}}$	$Q_c = B y_c^{1.5}\sqrt{g}$	$0.7 z y_c^{2.5}\sqrt{g}$
y_h	$\frac{(B+zy)y}{B+2zy}$	y	$0.5y$

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