# Electricity, Magnetism & Optics

# Inductance

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# Chapter Description

- Aims

  Students will understand the concept of mutual inductance, self-inductance and how they are used in daily lives

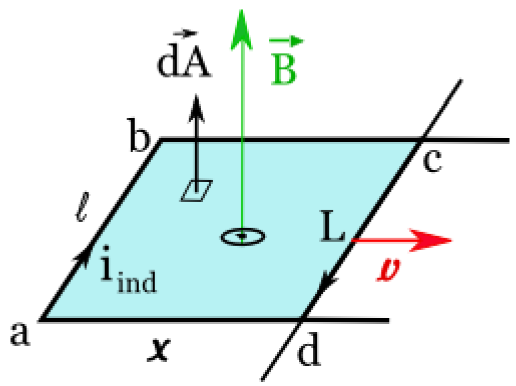
- Expected Outcomes
  - Able to apply Faraday's law to situations with two or more sets of coils and situations with changing current in one coil
  - Able to understand the concept of magnetic energy storage

# Content

10.1 Mutual Inductance

10.2 Self-Inductance and Inductors

10.3 Magnetic Energy Storage

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# 10.1 Mutual Inductance

- If two wires carry a *steady* current, there will be force exerted on both wires due to the magnetic field from both wires.

- However, consider a situation where one of the wire carries *changing* current. The changing current will produce a *changing magnetic field*, and will induce a current in the second wire (Faraday's law)

- This is called mutual inductance
- We will use lowercase letter to denote variables which are dependent on time

$$I \Rightarrow i(t)$$

$$B \Rightarrow b(t)$$

# Defining Mutual Inductance

- Consider two solenoid coils coils
- Coil 1 carries changing current, $i_1$ while there is no current in coil 2.
- Some of the magnetic field produced by coil 1 pass through coil 2.
- Thus, the magnetic flux that pass through coil 2 is written as $\Phi_{B2}$
- Therefore, according to Faraday's law, the induced emf in coil 2 is

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt}$$

- The changing current in coil 1, $i_1$ is proportional to the flux passing through coil 2, $\Phi_{B2}$

$$N_2 \Phi_{B2} = M_{21} i_1$$

- Here, $M_{21}$ is used as the proportionality constant, called **mutual inductance**

# Defining Mutual Inductance

- Deriving the previous equation yields

$$N_2 \frac{d\Phi_{B2}}{dt} = M_{21} \frac{di_1}{dt}$$

- Substituting in the first equation will give

$$\mathcal{E}_2 = -M_{21} \frac{di_1}{dt}$$

- This equation shows that a changing current in coil 1 will induce emf in coil 2, with proportionality constant $M_{21}$ between the two variables

- The mutual inductance can be written as $$M_{21} = \frac{N_2 \Phi_{B2}}{i_1}$$

# Defining Mutual Inductance

- Repeating the same discussion for the opposite case (changing current in coil 2 and zero current in coil 1) will yield the same result.

- It turns out that $M_{12}$ is always equal to $M_{21}$!
- Thus, the mutual inductance does not depend on the geometry of the coil, and is usually written without subscript, i.e: $M$

- Therefore, the relationship between changing current in one coil and induced emf in another coil is

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M \frac{di_2}{dt}$$

(mutually induced emfs in two coils)

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

(mutual inductance)

- The unit for mutual inductance is **henry** (H)
- 1 H = 1 Wb/A = 1 V·s/A = 1 Ω·s = 1 J/A$^2$

# 10.2 Self-Inductance and Inductors

- Now we consider the effect of Faraday's Law on a SINGLE isolated circuit.
- When a current is flowing in the circuit, it produces a magnetic field that causes a magnetic flux through *the same* circuit.
- This flux changes when the current changes.

- Thus, any circuit that carries a varying current has an emf induced in it by the variation of *its own* magnetic field
- Such an emf is called a **self-induced emf**

- By Lenz's law, a self-induced emf always opposes the change in the current that caused the emf to occur, and so tends to make it more **difficult** for variations in current to flow

# Defining Self-Inductance

- Self-induced emfs can occur in any circuit, but the effect is greatly enhanced if the circuit includes a coil with *N* turns of wire.
- Based on the discussion on mutual inductance, the self-inductance can be defined as

$$L = \frac{N\Phi_B}{i} \qquad \text{(self-inductance)}$$

- And based on Faraday's law, the induced emf will be

$$\mathcal{E} = -L\frac{di}{dt} \qquad \text{(self-induced emf)}$$

- The unit for self-induction is also **henry**. (Why?)

# Inductors

- A circuit device that is designed to have a particular inductance in called an **inductor**, or a **choke**.
- The usual symbol for an inductor is 
- Their purpose is to oppose any variations in the current through the circuit.

# 10.3 Magnetic Field Energy

- An inductor carrying a current has energy stored in it.
- This energy can be calculated as follows.
- Power delivered to the inductor is $P = V_{ab} i$
- Ignoring internal resistance, the power can be written as

$$P = V_{ab} i = Li \frac{di}{dt}$$

- The energy $dU$ supplied to the inductor during an infinitesimal time interval $dt$ is $dU = P\,dt$. So, $dU = Li\ di$
- Thus, the total energy $U$ supplied while current increase from zero to final value, $I$ is written as

$$U = L \int_0^I i\ di = \frac{1}{2} L I^2$$

(energy stored in an inductor)

# Conclusion

- Mutual Inductance
  - When an emf is produced in a coil because of the change in current in a coupled coil
- Self-Inductance
  - Self-inductance of the coil is defined as the property of the coil due to which it opposes the change of current flowing through it

# References


- **University Physics 14th Edition**, Hugh D. Young, Roger A. Freedman, IOP Publishing Ltd, 2015

- **Physics for Scientists & Engineers 4th Edition**, Douglas C. Giancoli, Pearson, 2008

- **Physics for Scientists & Engineers 9th Edition**, Raymond A. Serway & John W. Jewett, Cengage Learning, 2014

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# Thank you!

# Next chapter:
# The Nature and Propagation of Light

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