Universiti Malaysia PAHANG
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Electricity, Magnetism & Optics

Magnetic Forces and Fields

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Chapter Description

- Aims

  Students will understand that the magnetic fields are actually generated by moving charges, and how to calculate them

- Expected Outcomes
  - Able to understand the nature of magnetic field produced by a single moving charge
  - Able to describe magnetic field produced by current-carrying conductor
  - Able to calculate magnetic field produced by any moving charge

Content

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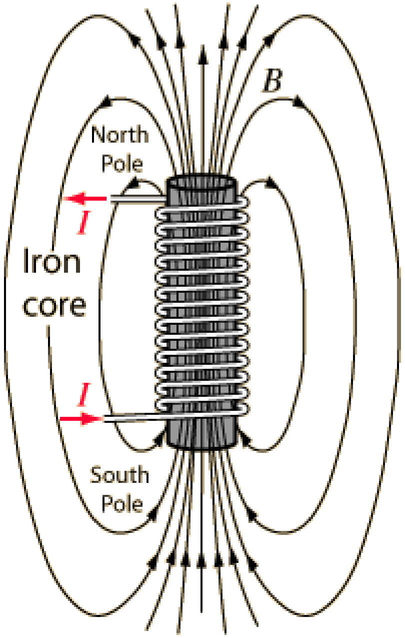

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8.1 Magnetic Field of a Moving Charge

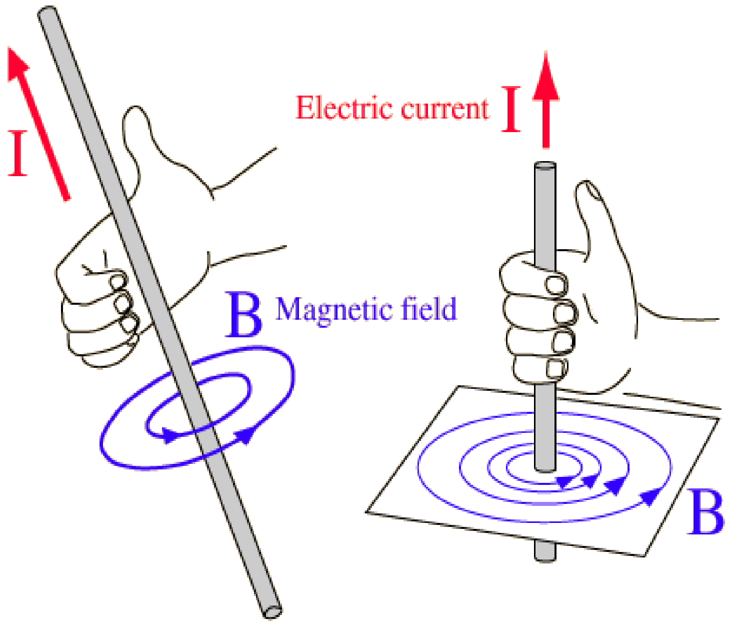
Electricity	Magnetism
1. A distribution of electric charge at rest creates an electric field $\vec{E}$ in the surrounding space 2. The electric field exerts a force $\vec{F} = q\vec{E}$ on any other charges	1. A moving charge or current creates a magnetic field (in addition to its electric field) 2. The magnetic field exerts a force $\vec{F}$ on any other moving charges or current

- **Electric field** exerts force on *STATIC* electric charge (also on moving charge)
- But, **magnetic field** exerts force only on *MOVING* charge.
- Thus, only moving charges can create/produce magnetic field (?)

- This chapter will begin with magnetic field $\vec{B}$ from a single moving charge to a straight current carrying conductor (bottom up approach)

Magnetic Field: Moving charge

- Just like electric field, experiments show that $\vec{B}$ is proportional to $|q|$ and $1/r^2$
- However! The direction of $\vec{B}$ is different from $\vec{E}$
- $\vec{B}$ is perpendicular to the velocity of the moving charge, and also perpendicular to the plane containing the line from the charge to field point
- The magnetic field is also proportional to the speed of the moving charge.



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Magnetic Field: Moving charge

- Thus, from experiments,

$$B = \frac{\mu_0}{4\pi}\frac{|q| v \sin\phi}{r^2}$$ (magnetic field from a moving charged particle)

- $\frac{\mu_0}{4\pi}$ is the proportionality factor, and μ_0 is a constant called permeability of free space (or vacuum permeability or magnetic constant)

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}$$

- μ_0 and ε_0 are related to the speed of light as $\frac{1}{\varepsilon_0\mu_0} = c^2$

- Using vector notation, the magnetic field from a moving charge can also be written as

$$\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\boldsymbol{r}}}{r^2}$$

8.2 Magnetic Field of a Straight Current-Carrying Conductor

- Just as electric field, there is a principle of superposition of magnetic fields:

  The **total magnetic field** caused by **several moving charges** is the **vector sum** of the fields caused by the individual charges

- The magnetic field due to current-carrying conductor can be calculated as follows.

- First, we calculate the magnetic field from a short segment of the conductor, $d\ell$. The volume of the segment is $A\,d\ell$, where A is the cross-sectional area of the conductor.
- Thus, the total moving charge in the segment is $dQ = nqA\,d\ell$

Biot-Savart Law

- Therefore, the magnetic field produced by this segment is

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d \sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_d A\, d\ell \sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{I d\ell \sin\phi}{r^2}$$

- This is called **Biot-Savart law**
- In order to get the total magnetic field, we integrate both sides of the equation,

$$B = \frac{\mu_0}{4\pi}\int \frac{I d\ell \sin\phi}{r^2} = \frac{\mu_0}{4\pi}\int \frac{I d\vec{\ell} \times \hat{\boldsymbol{r}}}{r^2}$$

- This is valid for current-carrying conductors with any shapes (straight line, curved, loop, bent etc)!
- However, let's just consider a straight current-carrying conductor for this course.

Magnetic Field: Straight conductor

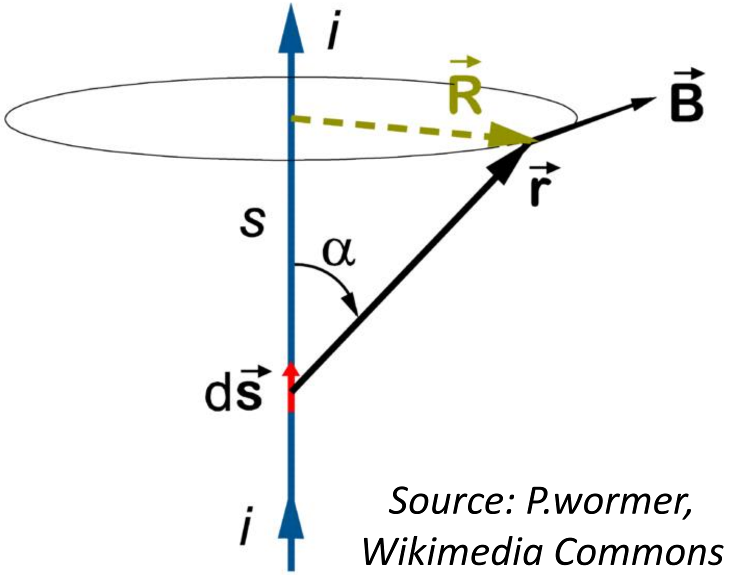
- Now, let's consider a straight wire placed along y-axis with length $2a$, and carrying current I towards +y-axis.
- The small segment $d\ell$ is shown to the left.
- Let's calculate the magnetic field at point P.
- First, $d\ell = dy \quad r = \sqrt{x^2 + y^2}$

and $\sin\phi = \sin(\pi - \phi) = \dfrac{x}{\sqrt{x^2 + y^2}}$

- Thus, from Biot-Savart's law,

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x dy}{\left(x^2 + y^2\right)^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}}$$



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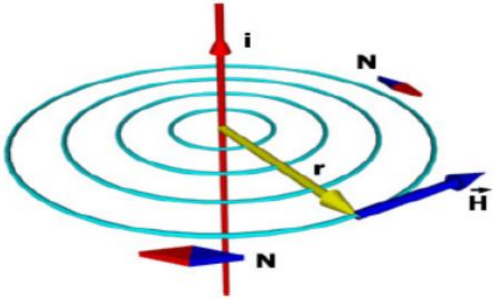
Magnetic Field: Straight conductor

- When the length $2a$ of the conductor is very large in comparison to its distance x to point P, we can consider it to be infinitely long.
- When a is much longer than x, $\sqrt{x^2 + a^2} \rightarrow a$
- Thus, the equation can be simplified as

$$B = \frac{\mu_0 I}{2\pi r}$$

(magnetic field near a long, straight current-carrying conductor)



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8.3 Force between Parallel Straight Conductors

- It is understood that current-carrying conductor produces magnetic field around it. If there are two conductors, the magnetic field from one wire will exert force on the other!
- Also, if there are more than one wires, the magnetic field from all the wires will exert force on each wire.

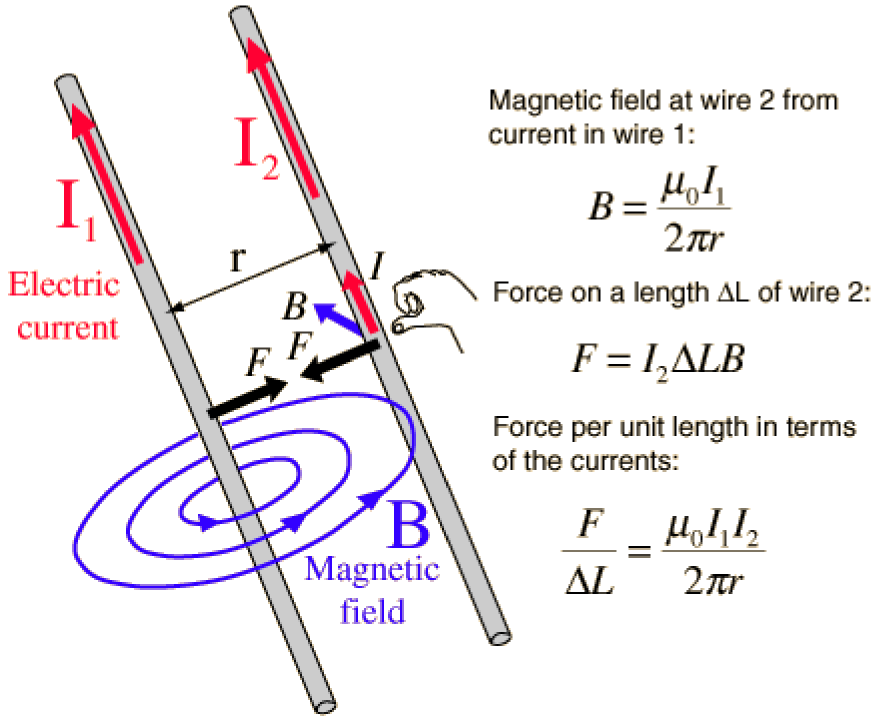
- From the previous chapter (7) and from sub-chapter 8.2, the force can be written as

$$\frac{F_{ij}}{L} = \frac{\mu_0 I_i I_j}{2\pi r_{ij}}$$

(*force per unit length* of wire *i* on wire *j*. Both *i* and *j* are long, parallel wires.)

Attractive or repulsive?

- From calculation, it can easily be seen that for two conductors carrying current in the same direction, both conductors will be attracted to each other.
- And if the current is in opposite direction, they will repel each other



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8.4 Ampere's law

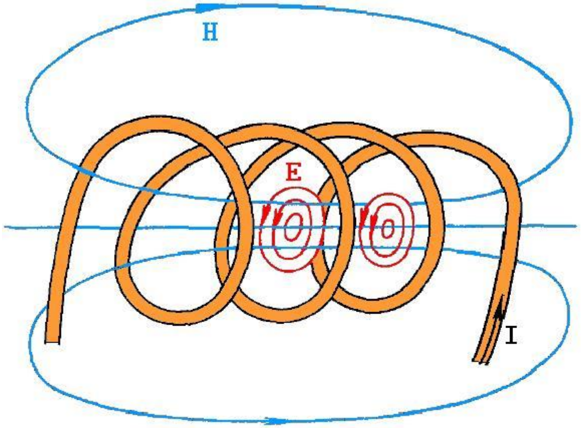
- Just like Gaussian law for electricity, magnetic field has something called Ampere's law.

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} \qquad \text{(Ampere's law)}$$

- Ampere's law states that the integral of magnetic field around a closed loop, is proportional to the total current passing through the loop

Field of a Solenoid

- A solenoid consists of a helical winding of wire on a cylinder, usually circular in cross section. There can be thousands of closely spaced turns, each of which can be regarded as a circular loop.



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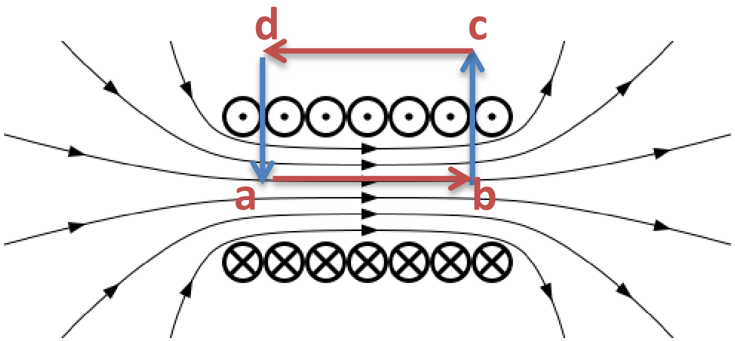
- All turns carry the same current *I*, and the total field at every point is the vector sum of the fields caused by the individual turns.
- The magnetic field can be calculated by totalling all the magnetic fields from each loop.
- However, Ampere's law can be used as a simpler way to calculate the field at the centre of the solenoid

Calculating field inside a solenoid

- Consider a solenoid with *N* number of turns with length ℓ. The solenoid carries current *I*. The magnetic field outside the solenoid is zero.



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- Taking the integration along the rectangle *abcd* will yield $\oint \vec{\boldsymbol{B}} \cdot d\vec{\ell} = B\ell$
- While on the right side, $\mu_0 I_{\text{encl}} = \mu_0 NI$
- Therefore, taking the left side and right side together, $B\ell = \mu_0 NI$

$$B = \frac{\mu_0 NI}{\ell}$$

(Magnetic field inside a solenoid)

Conclusion

- Sources of Magnetic Field
  - Magnetic field is generated whenever charged particles are moving
- Force between parallel straight conductors
  - Straight conductors carrying current will generate magnetic field, and in turn, will influence the other conductor either by attracting or repelling, depending on the direction of the current flowing in them

References

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- **Physics for Scientists & Engineers 4th Edition**, Douglas C. Giancoli, Pearson, 2008
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Thank you!

Next chapter: Electromagnetic Induction

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