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HYDRAULICS

UNIFORM FLOW IN OPEN CHANNEL
TOPIC 2.4

by

Nadiatul Adilah Ahmad Abdul Ghani
Faculty of Civil Engineering and Earth Resources
nadiatul@ump.edu.my

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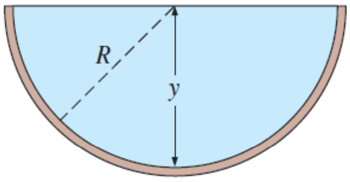
UNIFORM FLOW IN OPEN CHANNEL

2.4

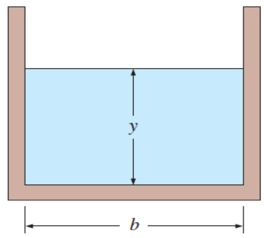
- Effectives Cross Sections (Circular, Rectangular, Trapezoidal)

2.4 : DESIGN OF OPEN CHANNEL (BEST HYDRAULIC CROSS SECTION)

- For a given channel length, the perimeter is representative of the system cost, and it should be kept to a minimum in order to minimize the size and cost.
- The best hydraulic cross-section is with the maximum hydraulic radius, meaning minimum wetted perimeter for a specified cross-sectional area.



The best hydraulic cross section is semicircular since it has the minimum wetted perimeter for a specified cross-sectional area, and thus the minimum flow resistance.



For a given cross-sectional area of rectangular open channel, the highest flow rate occurs when $y = b/2$.

Chart of Best Hydraulic Cross Sections

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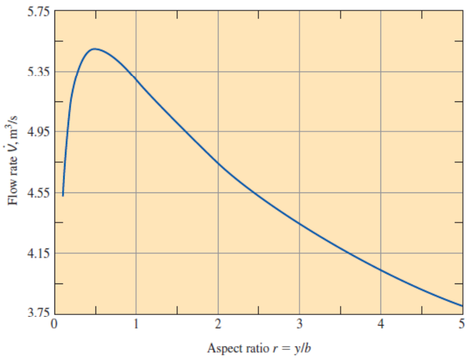
Variation of the hydraulic radius R_h and the flow rate $\dot{V}$ with aspect ratio y/b for a rectangular channel with $A_c = 1\ m^2$, $S_0 = \tan 1°$, and $n = 0.012$

Aspect Ratio y/b	Channel Width b, m	Flow Depth y, m	Perimeter p, m	Hydraulic Radius R_h, m	Flow Rate $\dot{V}$, m^3/s
0.1	3.162	0.316	3.795	0.264	4.53
0.2	2.236	0.447	3.130	0.319	5.14
0.3	1.826	0.548	2.921	0.342	5.39
0.4	1.581	0.632	2.846	0.351	5.48
0.5	1.414	0.707	2.828	0.354	5.50
0.6	1.291	0.775	2.840	0.352	5.49
0.7	1.195	0.837	2.869	0.349	5.45
0.8	1.118	0.894	2.907	0.344	5.41
0.9	1.054	0.949	2.951	0.339	5.35
1.0	1.000	1.000	3.000	0.333	5.29
1.5	0.816	1.225	3.266	0.306	5.00
2.0	0.707	1.414	3.536	0.283	4.74
3.0	0.577	1.732	4.041	0.247	4.34
4.0	0.500	2.000	4.500	0.222	4.04
5.0	0.447	2.236	4.919	0.203	3.81



The flow rate increases as the flow aspect ratio y/b increase, reaches a maximum at y/b=0.5, and then starts to decrease.

The trend is the same for the hydraulic radius, but the opposite trend for the wetted perimeter *P*.

Variation of the flow rate in a rectangular channel with aspect ratio $r = y/b$ for $A_c = 1\ m^2$ and $S_0 = \tan 1°$

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2.4.1 Best Hydraulic cross section (Rectangular Channel)

$$A = yb \quad \text{and} \quad P = b + 2y$$

$$P = \frac{A}{y} + 2y$$

$$\frac{dP}{dy} = 0$$

$$\frac{dP}{dy} = -\frac{A}{y^2} + 2 = -\frac{by}{y^2} + 2 = 0$$

Best hydraulic cross section (rectangular channel): $y = \frac{b}{2}$

Therefore, a rectangular open channel should be designed such that the **liquid height is half the channel width** to minimize flow resistance or to maximize the flow rate for a given cross-sectional area. This also minimizes the perimeter and thus the construction costs.

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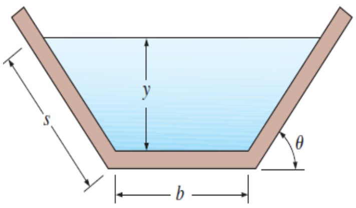
2.4.2 Best Hydraulic cross section (Trapezoidal Channel)

$$A_c = \left(b + \frac{y}{\tan\theta}\right)y \quad \text{and} \quad p = b + \frac{2y}{\sin\theta}$$

$$p = \frac{A_c}{y} - \frac{y}{\tan\theta} + \frac{2y}{\sin\theta}$$

$$R_h = \frac{A_c}{p} = \frac{y(b + y/\tan\theta)}{b + 2y/\sin\theta} = \frac{y(b\sin\theta + y\cos\theta)}{b\sin\theta + 2y}$$



$$dp/dy = 0$$

$$\frac{dp}{dy} = -\frac{A_c}{y^2} - \frac{1}{\tan\theta} + \frac{2}{\sin\theta} = -\frac{(b + y/\tan\theta)}{y} - \frac{1}{\tan\theta} + \frac{2}{\sin\theta}$$

Best hydraulic cross section (trapezoidal channel): $y = \frac{b\sin\theta}{2(1-\cos\theta)}$

$y = b \sin\theta / (2 - 2\cos\theta)$

Best trapezoid angle: $\theta = 60°$

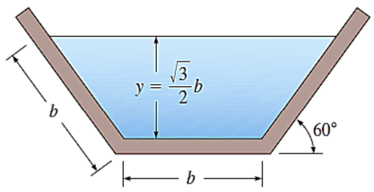
Best flow depth for $\theta = 60°$: $y = \frac{\sqrt{3}}{2} b$

$$s = \frac{y}{\sin 60°} = \frac{b\sqrt{3}/2}{\sqrt{3}/2} = b$$

$$p = 3b$$

$$A_c = \left(b + \frac{y}{\tan\theta}\right) y = \left(b + \frac{b\sqrt{3}/2}{\tan 60°}\right)(b\sqrt{3}/2) = \frac{3\sqrt{3}}{4} b^2$$

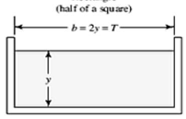
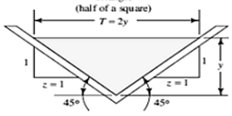
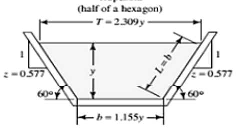
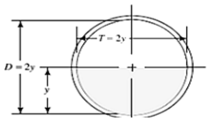
Hydraulic radius for the best cross section: $R_h = \frac{y}{2}$



$$R_h = \frac{y}{2} = \frac{\sqrt{3}}{4} b \qquad A_c = \frac{3\sqrt{3}}{4} b^2$$

The best cross section for trapezoidal channel is *half of a hexagon*.

Table 2.5 Most efficient sections for open channels

Section	Area A	Wetted Perimeter WP	Hydraulic Radius R
Rectangle (half of a square) 	$2.0y^2$	$4y$	$y/2$
Triangle (half of a square) 	y^2	$2.83y$	$0.354y$
Trapezoid (half of a hexagon) 	$1.73y^2$	$3.46y$	$y/2$
Semicircle* 	$\frac{1}{2}\pi y^2$	πy	$y/2$

Source: Applied Fluid Mechanics, Robert L Mott (page 458)

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Propositions of Some Most Efficient Sections

Channel Shape	A	P	b	R
Rectangular	$2y^2$	4y	2y	y/2
Trapezoidal (half regular hexagon, θ=60°)	$y^2\sqrt{3}$	$2y\sqrt{3}$	$2y/\sqrt{3}$	y/2
Circular (semicircle)	$\pi y^2/2$	πy	-	y/2
Triangle (vertex angle=90°, θ=45°)	y^2	$2y\sqrt{2}$	-	$y/2\sqrt{2}$

TABLE 7-2. BEST HYDRAULIC SECTIONS

Cross section	Area A	Wetted perimeter P	Hydraulic radius R	Top width T	Hydraulic depth D	Section factor Z
Trapezoid, half of a hexagon	$\sqrt{3}\,y^2$	$2\sqrt{3}\,y$	$\frac{1}{2}y$	$\frac{4}{3}\sqrt{3}\,y$	$\frac{3}{4}y$	$\frac{3}{2}y^{2.5}$
Rectangle, half of a square	$2y^2$	$4y$	$\frac{1}{2}y$	$2y$	y	$2y^{2.5}$
Triangle, half of a square	y^2	$2\sqrt{2}\,y$	$\frac{1}{4}\sqrt{2}\,y$	$2y$	$\frac{1}{2}y$	$\frac{\sqrt{2}}{2}y^{2.5}$
Semicircle	$\frac{\pi}{2}y^2$	πy	$\frac{1}{2}y$	$2y$	$\frac{\pi}{4}y$	$\frac{\pi}{4}y^{2.5}$
Parabola, $T = 2\sqrt{2}\,y$	$\frac{4}{3}\sqrt{2}\,y^2$	$\frac{8}{3}\sqrt{2}\,y$	$\frac{1}{2}y$	$2\sqrt{2}\,y$	$\frac{2}{3}y$	$\frac{8}{9}\sqrt{3}\,y^{2.5}$
Hydrostatic catenary	$1.39586y^2$	$2.9836y$	$0.46784y$	$1.917532y$	$0.72795y$	$1.19093y^{2.5}$

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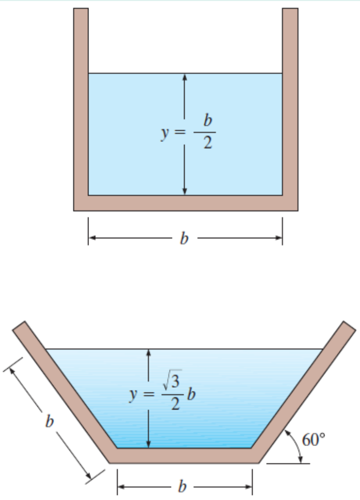
EXAMPLE 13–5 **Best Cross Section of an Open Channel**

Water is to be transported at a rate of 2 m^3/s in uniform flow in an open channel whose surfaces are asphalt lined. The bottom slope is 0.001. Determine the dimensions of the best cross section if the shape of the channel is (*a*) rectangular and (*b*) trapezoidal (Fig. 13–27).



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Properties The Manning coefficient for an open channel with asphalt lining is $n = 0.016$.

Analysis (*a*) The best cross section for a rectangular channel occurs when the flow height is half the channel width, $y = b/2$. Then the cross-sectional area, perimeter, and hydraulic radius of the channel are

$$A_c = by = \frac{b^2}{2} \qquad p = b + 2y = 2b \qquad R_h = \frac{A_c}{p} = \frac{b}{4}$$

Substituting into the Manning equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \rightarrow b = \left(\frac{2n\dot{V}4^{2/3}}{a\sqrt{S_0}}\right)^{3/8} = \left(\frac{2(0.016)(2\text{ m}^3/\text{s})4^{2/3}}{(1\text{ m}^{1/3}/\text{s})\sqrt{0.001}}\right)^{3/8}$$

which gives $b = 1.84$ m. Therefore, $A_c = 1.70\text{ m}^2$, $p = 3.68$ m, and the dimensions of the best rectangular channel are

$$b = \mathbf{1.84\ m} \quad \text{and} \quad y = \mathbf{0.92\ m}$$

(*b*) The best cross section for a trapezoidal channel occurs when the trapezoid angle is 60° and flow height is $y = b\sqrt{3}/2$. Then,

$$A_c = y(b + b\cos\theta) = 0.5\sqrt{3}b^2(1 + \cos 60°) = 0.75\sqrt{3}b^2$$

$$p = 3b \qquad R_h = \frac{y}{2} = \frac{\sqrt{3}}{4}b$$

Substituting into the Manning equation,

$$\dot{V} = \frac{a}{n} A_c R_h^{2/3} S_0^{1/2} \rightarrow b = \left(\frac{(0.016)(2\text{ m}^3/\text{s})}{0.75\sqrt{3}(\sqrt{3}/4)^{2/3}(1\text{ m}^{1/3}/\text{s})\sqrt{0.001}}\right)^{3/8}$$

which yields $b = 1.12$ m. Therefore, $A_c = 1.64\text{ m}^2$, $p = 3.37$ m, and the dimensions of the best trapezoidal channel are

$$b = \mathbf{1.12\ m} \quad y = \mathbf{0.973\ m} \quad \text{and} \quad \theta = \mathbf{60°}$$

Discussion

Note that the trapezoidal cross section is better since it has a smaller perimeter (3.37 m versus 3.68 m) and thus lower construction cost. This is why many man-made waterways are trapezoidal in shape. However, the average velocity through the trapezoidal channel is larger since A_c is smaller.

Example 2.7:

An open channel with n=0.011 is to be designed to carry 1.0 m^3/s at slope of 0.0065. Find the most efficient cross section for;

a. A rectangular

b. A semicircular

c. A triangular

d. A trapezoidal

Example 2.8:

An open channel is designed to carry 15m^3/s at a channel slope of 0.0009. If Chezy's coefficient is 66, determine the most efficient cross section for these channel:

i. a rectangular

ii. a trapezoidal

If the excavation cost is RM5 per m^3, determine the most efficient cross section (economically) between (i) and (ii) for the 50m channel's length

Lecturer Information (Authors)

Pn. Nadiatul Adilah bt Ahmad Abdul Ghani
Dr Nor Azlina bt. Alias
Pn. Wafty bt. Abd Rahman
Dr. Jacqueline Isabella